CHAPTER V

EFFECT OF SLOPING FLOOR AND DEPRESSED FLOOR

INTRODUCTION

In the previous chapter, the 'Second Approximation' for calculating the uplift pressures due to seepage at various points in the foundations of a horizontal floor with two or more vertical piles, has been outlined. However, in practice, most of the floors of weirs on permeable foundations have one or more sloping components. Melchenko's method of "eliminated cut-offs" cannot be applied to the case of a floor with sloping components. Malhotra gave an exact mathematical analysis for a straight sloping floor which has not been extended for a composite floor comprising of sloping and horizontal components. Khosla has attempted to incorporate the effect of the sloping floor in his method of "independent variables" by applying a correction to the pressures at the end points of the sloping floor. However he did not consider the effect of the presence of sloping floor on the points upstream and downstream of the end points of the sloping length. Thus it may be observed that no analytical solution exists for the determination of pressures under a composite floor.
Therefore, an attempt has been made to determine the effect of a sloping component of floor on the pressures at various points under the composite floor.

**TRANSFORMATION OF A COMPOSITE FLOOR**

The floor ABCD in the $z$-plane in Fig. 5.1 consists of two horizontal components AB and CD and a sloping component BC inclined at an angle $\theta = \pi$ to the horizontal. The floor ABCD can be transformed into a straight line abed in $\zeta$-plane by the Schwarz-Christoffel transformation. The equation of transformation is given by

$$Z = A_0 \int (\zeta - \zeta_b)^{-n} (\zeta - \zeta_c)^{n} d\zeta \quad \ldots (5.1)$$

where $A_0$ is an arbitrary constant which may be taken as unity. Taking the origin at the point 'b' and assuming be equal to one unit, so that

$$\zeta_b = 0 \quad \text{and} \quad \zeta_c = 1,$$

the equation (5.1) is simplified to the form:

$$Z = \int (1 - \frac{1}{\zeta})^{n} d\zeta \quad \ldots (5.2)$$

Similarly taking the origin at the point 'c' i.e.

$$\zeta_c = 0 \quad \text{and} \quad \zeta_b = -1$$
the equation (5.1) reduces to the form:

\[ z = \int \left( 1 + \frac{1}{\xi} \right)^{-n} d\xi \quad \ldots (5.3) \]

From equation (5.2),

\[ \zeta_c = \int_{\xi_b}^{\xi_c} \left( 1 - \frac{1}{\xi} \right)^n d\xi \]

\[ = \int_{\xi_b}^{1} \left( 1 - \frac{1}{\xi} \right)^n d\xi \]

\[ = (-1)^n \int_{0}^{1} \xi^{-n} (1 - \xi) d\xi \]

\[ = (-1)^n \frac{n \pi}{\sin n \pi} \]

so that \[ |\zeta_c| = \frac{n \pi}{\sin n \pi} \]

or be

\[ |BC| = \left| \frac{S_{\xi_b} \xi_c \pi}{\sin n \pi} \right| \quad \ldots (5.4) \]

Also \[ AB = \int_{\xi_a}^{\xi_b} \left( 1 - \frac{1}{\xi} \right)^n d\xi \]

\[ = \int_{0}^{1} (-\xi)^{-n} (1 - \xi) d\xi \]

\[ = \int_{0}^{\xi_a} \xi^{-n} (1 + \xi) d\xi \]

For \( 0 < |\xi_a| \leq 1 \),

\[ AB = \int_{0}^{\xi_a} \xi^{-n} \left( 1 + \xi \cdot \frac{n(n-1)}{2} \xi^{2} + \frac{n(n-1)(n-2)}{6} \xi^{3} + \ldots \right) d\xi \]

\[ = \int_{0}^{\xi_a} \left( \xi^{-n} + \frac{n(n-1)}{2} \xi^{2} + \frac{n(n-1)(n-2)}{6} \xi^{3} + \ldots \right) d\xi \]
\[
(AB) = \left[ \frac{\xi^{n-1}}{1-n} + \frac{\eta n \xi^{2-n}}{2-n} + \frac{\eta (n-1) \xi^{3-n}}{2 (3-n)} + \cdots \right]_{\xi=1}^{\xi_a} \quad \cdots (5.5)
\]

For \( |\xi_a| \geq 1 \),

\[
(AB)_{|\xi_a| = 1} = \int_{|\xi_a| = 1} \left[ \frac{\xi_{\xi}}{(1+\frac{\xi}{\xi})} \right] d\xi
\]

\[
= \int [\xi + n \log \xi - \frac{n(n-1)}{2} \xi - \frac{n(n-1)(n-2)}{6} \xi^2 - \cdots] d\xi
\]

From equation 5.3,

\[
\mathcal{C}^d = \int_{0}^{\xi_a} \left( \xi^n \xi^{i+n} + \frac{n(n+1)}{2} \xi^{2+n} - \frac{n(n+1)(n+2)}{6} \xi^{3+n} + \cdots \right) d\xi
\]

\[
= \left[ \frac{\xi_{\xi}^{n+1}}{1+n} - \frac{\eta \xi_{\xi}^{2+n}}{2+n} + \frac{\eta (n+1) \xi_{\xi}^{3+n}}{2 (3+n)} + \cdots \right] \quad \cdots (6.7)
\]

For \( \xi_a \geq 1 \),

\[
\mathcal{C}^d = (C D)_{|\xi_a| = 1} = \int_{|\xi_a| = 1} \left[ \xi - n \log \xi - \frac{n(n+1)}{2} \xi - \frac{n(n+1)(n+2)}{6} \xi^2 - \cdots \right] d\xi
\]

Let 'X' be any point in BC,

\[
\text{then } \{3X\} = \int_{0}^{\xi_a} \xi^n (1-\xi)^n d\xi
\]

\[
= \int_{0}^{\xi_a} \left( \xi^n - n \xi^{1-n} + \frac{n(n-1)}{2} \xi^{2-n} - \cdots \right) d\xi
\]
\[ \langle x \rangle = \left[ \frac{\xi_x^{1-n}}{1-n} - \frac{\xi_x^{2-n}}{2-n} + \frac{\eta(n-1)\xi_x^{3-n}}{2(3-n)} \cdots \right] \quad \cdots (5.9) \]

Substituting different values of \( n \) and \( \zeta \), the lengths of the three components of the floor corresponding to the lengths in the \( \zeta \)-plane have been worked out and are tabulated in Tables 5.1 to 5.3. The curves between the lengths in \( \zeta \)-plane versus the lengths of floor components in \( z \)-plane are shown in Figures 5.2 to 5.4, for different values of \( \theta \). All these tables and curves correspond to the length being taken as unity.

**EFFECT OF SLOPING COMPONENT ON FLOOR**

**ON HORIZONTAL COMPONENTS**

To determine the effect of the sloping component of floor on the horizontal components on upstream and downstream sides, first transform the sloping component on to the \( \zeta \)-plane, such that

\[ b = |BC| \frac{\sin \theta}{\theta} \]

Then to find the effect of the sloping component on the length \( AB \) where \( A \) is any point in the higher horizontal floor (which may be upstream or downstream) calculate the ratio \( \frac{R_A}{R_B} = \frac{|AB|}{R_B} \) and read the value of \( R_A \) against this from Figure 5.2 or Table 5.1. This gives the length of the higher floor at in the transformed plane taking \( b \) equal to unity. Similarly the length of the lower floor
ed in the transformed plane can be found from Figure 5.3 or Table 5.2.

EFFECT OF TWO OR MORE SLOPING COMPONENTS OF FLOOR

If there are two or more sloping components in the floor their effect can be found by an approximation similar to the 'Second Approximation' in the case of two or more vertical piles under a horizontal floor, discussed in detail in Chapter IV.

First transform the sloping components into horizontal lengths, treating each sloping component as independent of the other sloping components. Then find the effect of each sloping component, one by one as in the case of the vertical piles.

EFFECT OF SLOPING COMPONENTS ON FLOOR WITH VERTICAL PILES

The effect of sloping components of floor on the floor with one or more vertical piles can be found by first transforming the vertical piles into horizontal lengths, then determining the effect of the sloping components and retransforming the piles one by one and finding their effect.
DEPRESSED FLOOR

If the floor is depressed as shown in Figure 5.5, its transformation can be achieved by treating the vertical components AB and FG as inclined components of floor with \( \theta = 90^\circ \).

For a depressed floor without any pile, exact theoretical solution is available. In Table 5.4, the values of percentage pressures \( P_2 \) at the downstream corner of a depressed floor worked out by this method for different ratios of \( \frac{b}{d} \) (length/depth) have been compared with the exact theoretical values \( P_e \).

It may be observed that in all cases, the values of percentage pressures obtained from the Approximation method discussed above tally very closely with those obtained from the exact theory, the difference in any case being not more than 0.1 per cent.

A STEPPED HORIZONTAL FLOOR WITH A VERTICAL PILE AT THE STEP

Consider a hypothetical stepped horizontal floor with a vertical pile at the step AB with dimensions as shown in Fig. 5.6. For such a floor, exact analytical solution is available. The percentage pressures at the points B and E are computed below by the approximate
method outlined above and compared with the exact theoretical pressures at these points. The part B C is treated as a vertical sloping component of floor and the part CDE as a vertical pile.

Step 1: Effect of sloping component BC

\[
\begin{align*}
BC &= 1.0 \\
be &= 1.0 \times \frac{\sin \pi/2}{\pi/2} = 0.637 \\
AB &= 3.0 \\
CE &= 2.0 \\
CF &= 2 + 2 = 4.0 \\
R_a &= \frac{2.0}{0.637} = 3.14 \\
R_e &= \frac{2.0}{0.637} = 3.14 \\
R_f &= \frac{4.0}{0.637} = 6.28
\end{align*}
\]

From the curve (\(\theta = 90^\circ\)) in Fig. 5.2, \(R'_a = 2.94\)

From the curve (\(\theta = 90^\circ\)) in Fig. 5.3, \(R'_e = 4.12\)

and \(R'_f = 7.83\)

Step 2: Effect of pile CDE

\[
\begin{align*}
ac' &= \frac{4.12}{2} \times 0.637 = 1.31 \\
be &= 0.637 \\
as &= 0.637 + 0.637 \times 2.94 = 2.51
\end{align*}
\]
\[
\begin{align*}
\text{ef} & \quad = \quad (7.53 - 4.12) \times 0.637 \quad = \quad 2.17 \\
\text{so} \quad R_a & \quad = \quad \frac{2.61}{1.31} \quad = \quad 1.915 \\
R_b & \quad = \quad \frac{0.637}{1.31} \quad = \quad 0.485 \\
R_c & \quad = \quad \frac{2.17}{1.31} \quad = \quad 1.665 \\
R_d' & \quad = \quad 1.16 \\
R_b' & \quad = \quad 0.11 \\
R_c & \quad = \quad 0.84 \\
\text{Length b'c'} & \quad = \quad 0.11 \times 1.31 \quad = \quad 0.14 \\
\text{a'b'} & \quad = \quad (1.16 - 0.11) \times 1.31 \quad = \quad 1.23 \\
e'f' & \quad = \quad 0.94 \times 1.31 \quad = \quad 1.23 \\
\text{Length upto b'} & \quad = \quad 1.38 \\
\text{Length upto c'} & \quad = \quad 1.38 + 0.14 + 2.62 \quad = \quad 4.14 \\
\text{Total Length} & \quad = \quad 4.14 + 1.23 \quad = \quad 5.37 \\
\text{Ratio} \quad \frac{R_b}{5.37} & \quad = \quad 0.267 \\
R_c & \quad = \quad \frac{4.14}{5.37} \quad = \quad 0.771 \\
P_b & \quad = \quad 66.2\% \\
P_c & \quad = \quad 31.8\%
\end{align*}
\]

RESULTS AND DISCUSSION

The exact theoretical solution gives the values of \( P_b = 66.0\% \) and \( P_c = 32.1\% \). Thus the difference between the percentage pressures obtained by the Approximation Method given herein and the theoretical
values is only 0.2 to 0.3. This indicates that the maximum error introduced in the case of a floor with a number of sloping components and a number of vertical piles usually encountered in practice is not likely to exceed 1 or 2 per cent in any case.
Relation between $R_a$ and $R'_a$ for a composite floor

**TABLE - 5.1**

<table>
<thead>
<tr>
<th>$R'_a = ab/be$</th>
<th>$R_a = AB/be$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta = 90^\circ$</td>
</tr>
<tr>
<td></td>
<td>$n = \frac{1}{2}$</td>
</tr>
<tr>
<td>0.1</td>
<td>0.64</td>
</tr>
<tr>
<td>0.2</td>
<td>0.92</td>
</tr>
<tr>
<td>0.4</td>
<td>1.35</td>
</tr>
<tr>
<td>0.6</td>
<td>1.69</td>
</tr>
<tr>
<td>0.8</td>
<td>2.00</td>
</tr>
<tr>
<td>1.0</td>
<td>2.30</td>
</tr>
<tr>
<td>2.0</td>
<td>3.60</td>
</tr>
<tr>
<td>3.0</td>
<td>4.73</td>
</tr>
<tr>
<td>4.0</td>
<td>5.92</td>
</tr>
<tr>
<td>5.0</td>
<td>7.02</td>
</tr>
<tr>
<td>6.0</td>
<td>8.11</td>
</tr>
<tr>
<td>8.0</td>
<td>10.23</td>
</tr>
<tr>
<td>10.0</td>
<td>12.36</td>
</tr>
</tbody>
</table>
Relation between $R_d$ and $R'^d$ for a composite floor

**TABLE - 5.2**

<table>
<thead>
<tr>
<th>$R'^d = \frac{ed}{bc}$</th>
<th>$R_d = \frac{CD}{bc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 90^\circ$</td>
<td>$\theta = 60^\circ$</td>
</tr>
<tr>
<td>$n = \frac{1}{2}$</td>
<td>$n = \frac{1}{3}$</td>
</tr>
<tr>
<td>0.2</td>
<td>0.06</td>
</tr>
<tr>
<td>0.4</td>
<td>0.15</td>
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<td>0.6</td>
<td>0.27</td>
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</tr>
<tr>
<td>1.0</td>
<td>0.63</td>
</tr>
<tr>
<td>2.0</td>
<td>1.30</td>
</tr>
<tr>
<td>3.0</td>
<td>2.15</td>
</tr>
<tr>
<td>4.0</td>
<td>3.03</td>
</tr>
<tr>
<td>5.0</td>
<td>3.93</td>
</tr>
<tr>
<td>6.0</td>
<td>4.85</td>
</tr>
<tr>
<td>8.0</td>
<td>6.72</td>
</tr>
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</table>
Relation between $R_x$ and $R'_x$ for a sloping component of floor

**Table - 5.3**

<table>
<thead>
<tr>
<th>$R'_x = bx/be$</th>
<th>$R_x = bx/be$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 90^\circ$</td>
<td>$\theta = 60^\circ$</td>
</tr>
<tr>
<td>$n = 1$</td>
<td>$n = 1/3$</td>
</tr>
<tr>
<td>0.1</td>
<td>0.62</td>
</tr>
<tr>
<td>0.3</td>
<td>1.04</td>
</tr>
<tr>
<td>0.5</td>
<td>1.28</td>
</tr>
<tr>
<td>0.7</td>
<td>1.45</td>
</tr>
<tr>
<td>0.9</td>
<td>1.55</td>
</tr>
<tr>
<td>1.0</td>
<td>1.67</td>
</tr>
</tbody>
</table>
Pressures at downstream corner of a depressed floor

**Table 5.4**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>b/d</th>
<th>( \bar{r} )</th>
<th>( P_2 )</th>
<th>( \zeta_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.0</td>
<td>9.8</td>
<td>9.8</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>12.0</td>
<td>13.1</td>
<td>13.2</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>6.0</td>
<td>17.4</td>
<td>17.3</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>4.0</td>
<td>20.1</td>
<td>20.1</td>
<td>0.0</td>
</tr>
<tr>
<td>5</td>
<td>3.0</td>
<td>22.3</td>
<td>22.2</td>
<td>0.0</td>
</tr>
<tr>
<td>6</td>
<td>2.4</td>
<td>23.7</td>
<td>23.7</td>
<td>0.0</td>
</tr>
<tr>
<td>7</td>
<td>2.0</td>
<td>25.0</td>
<td>25.0</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>1.0</td>
<td>30.0</td>
<td>30.1</td>
<td>0.1</td>
</tr>
<tr>
<td>9</td>
<td>0.5</td>
<td>34.6</td>
<td>34.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>
(Z-PLANE)

(Ω-PLANE)

FIG. 5.1

TRANSFORMATION OF A COMPOSITE FLOOR
FIG. 5.2

RELATION BETWEEN $R_a$ AND $R'_a$
A COMPOSITE FLOOR
FIG. 5.3
RELATION BETWEEN \( R_d \) AND \( R_d' \) FOR A COMPOSITE FLOOR.
FIG. 5.4

RELATION BETWEEN $R_x$ AND $R'_x$ FOR
THE SLOPING FLOOR.
FIG. 5.5

A DEPRESSED FLOOR
FIG. 5.6
A STEPPED FLOOR WITH A VERTICAL PILE AT THE STEP

FIG. 5.7
TRANSFORMATION OF THE FLOOR IN FIGURE 5.6