CHAPTER 3

THEORETICAL ANALYSIS

3.1 INTRODUCTION

Though a number of methods as described in Chapter 1 are in existence for the production of hexagonal nut blanks, cold forming is now becoming a widely accepted process, the reason being that the wastage of the material in the process is very small. The method suggested by the author (1978) in which a cylindrical piece from a rod is placed into a closed forming die, and the material from the center is forced to fill in the corners, results in a very low wastage of the material. The wastage is amounting to between 5 to 10% depending upon the type of the punch used.

Even though the above method has been accepted as a basic production process and special machines for the purpose are now available, (VDMA, 1986), the analysis of the process for predicting the forging variables has never been performed. Because of the complexity of the available methods, research in closed die forging has mostly been limited to plane strain and axisymmetric components. Forging of hexagonal blank without central hole in a closed die has been studied by Sagar (1980) using the upper bound technique.

In the present work a study is made for predicting
the forging variables while producing a nut blank with a circular hole at the center in a closed die (Fig. 3.1). An upper bound technique is used to analyse the problem.

3.2 THE UPPER BOUND APPROACH

The general description of the upper bound approach as an application of limit analysis to metal forming processes is given by Avitzur (1977). The theory formulated by Prager & Hodge (1951) states that among all kinematically admissible strain rate fields, the one that occurs minimizes the expression.

\[ J = \dot{W}_d + \dot{W}_f + \dot{W}_b \]

where

- \( J \) is the externally supplied power.
- \( \dot{W}_d \) is the internal power of deformation.
- \( \dot{W}_f \) is the power which accounts for the shear losses (for both velocity discontinuities within the material and friction at the tool - workpiece interface).
- \( \dot{W}_b \) is the power associated with external tractions.

A kinematically admissible strain rate field is one which is derived from a kinematically admissible velocity field, that is, one which satisfies the geometrical
boundary conditions and the volume consistency requirement.

If Von Mise's yield criterion is assumed to be obeyed by the material with a flow stress of \( \sigma_0 \), then

\[
\dot{W}_d = \frac{2\sigma_0}{\sqrt{3}} \int_v \sqrt{1/2} \hat{\varepsilon}_{ij} \hat{\varepsilon}_{ij} \, dv
\]  

(3.1)

where \( \hat{\varepsilon}_{ij} \) are the strain rate tensor components and the integration is over \( v \), the volume of the material. The repeated index in the above equation is a standard tensor notation to indicate summation.

The strain rates are given as under:

\[
\varepsilon_{ij} = -\left( \frac{\partial \dot{U}_i}{\partial x_j} + \frac{\partial \dot{U}_j}{\partial x_i} \right)
\]  

(3.2)

where the \( \dot{U}_i \)'s are components of the velocity vector and the \( x_j \)'s are components of the coordinate system. In cylindrical coordinates (i.e. \( R, \theta , y \)) the strain rates as a function of the velocity components are

\[
\begin{align*}
\dot{\varepsilon}_{RR} &= \frac{\partial \dot{U}_R}{\partial R} \\
\dot{\varepsilon}_{YY} &= \frac{\partial \dot{U}_y}{\partial y} \\
\dot{\varepsilon}_{\theta \theta} &= \frac{\dot{U}_R}{R} + \frac{1}{R} \frac{\partial \dot{U}_\theta}{\partial \theta} \\
\dot{\varepsilon}_{Ry} &= -\left( \frac{\partial \dot{U}_R}{\partial y} + \frac{\partial \dot{U}_y}{\partial R} \right)
\end{align*}
\]
If the interfacial friction is given by the friction factor $m$ and if there are no internal surfaces of discontinuity, then as given by Avitzur (1977),

$$W_m = m \sigma_\theta \int_s |\Delta v| \, ds \quad (3.4)$$

where $\Delta v$ is the difference in velocities at the interface, and the integration is performed over the surface, where friction is acting. The power associated with the external tractions is represented by

$$\dot{W}_b = \int_{st} v_i \, ds \quad (3.5)$$

where $v_i$ is the velocity of the surface on which the external stress $\tau_i$, is applied, and the integration is over the surface st. However, if $v_i$ and $\tau_i$ are in the same direction, then this external force aids the process and thus decreases the actual power needed to deform the material. The $\dot{W}_b$ will be negative. Otherwise if $v_i$ and $\tau_i$ are in opposite directions then $\dot{W}_b$ would yield a positive value.
In order to describe the velocity field of the deformation process realistically, generally extra parameters in the analysis are introduced. The flow pattern then varies with the assigned values of these parameters. Although these extra parameters are treated as if they are independent, they are not, since their values are not controlled by the operator. These variables are termed pseudo-independent parameters by Avitzur (1977). When chosen so as to represent reality, the values that they come to possess are determined within the analysis. The determination is made by using the principle of minimum energy or least resistance. The material will assume the flow pattern which requires the least amount of energy. This minimum energy flow pattern will be characterized by specific values of the pseudo-independent parameters. In effect, the analysis will choose these values to minimize the computed energy needed for the deformation to occur.

Computations for these are generally carried out on a digital computer.

3.3 ANALYSIS OF PROPOSED METHOD

In order to analyze the process by upper bound technique it is essential to assume a kinematically admissible velocity field. The flow pattern was first studied by drawing radial as well as concentric circular
grid lines on an aluminium circular disc specimen before forging it in a closed hexagonal die with the help of identical flat hexagonal shaped platens having cylindrical punches protruding at the center with flat noses. Since the flow pattern goes on changing with the travel of the punch, in this work the total load is calculated when the hexagonal die cavity is nearly full and the movement of the material is directed towards the six corners of the hexagonal shaped cavity. The load will be maximum at the point of filling the cavity. It may be noted that the complete filling of the cavity will tend to make the load infinite. By drawing normals to the sides of the hexagon from the center 0, the complete disc is divided into six regions such as ABCEF (Fig. 3.3) which are symmetrical as far as flow is concerned. This region is further subdivided into two symmetrical regions ABCD in which it is assumed that during compression the planar velocity of all particles, near the end of the process, is directed towards the corner A. However the material being compressed by the punch portion OCD will flow in the region CDA along with that being compressed in the region CDA itself, towards the corner A. Thus the analysis will consist of finding three separate kinematically admissible velocity fields conforming to the respective boundary conditions for the three regions ABC, ACD and GJIM (Fig. 3.2) and satisfying the plastic incompressibility condition.
3.3.1 Velocity and Strain Rate Fields in The Region ABC

Point A (Fig. 3.2) is taken as the origin, and the coordinates are cylindrical. The components of the velocity for the material flow are denoted by \( U_\theta \) (\( \theta \) measured from AB anticlockwise) and \( U_y \) (taken positive towards the central plane HI at the depth 'a'). For the flow directed towards the corner, we then have

\[
U_\theta = 0 \quad (3.6)
\]

and

\[
U_y = u \left( 1 - \frac{y}{a} \right) \quad (3.7)
\]

Where \( u \) is the velocity of the punch, 'a' the thickness of the disc and \( y \) is the depth of any point in the material in this region. These equations satisfy the boundary conditions since at the depth 'a' along the plane HI, \( U_y = 0 \) and at the top surface where \( y = 0 \), \( U_y = u \).

For the continuity of material flow it is easily seen that the rate of flow across section \( Q_1 R_1 \) (Fig 3.3) towards the corner A is equal to the rate at which the material is compressed over the area \( Q_1 R_1 T_1 S_1 \). For small angle \( d\theta \), areas \( T_1 T'_1 P_1 \) and \( S_1 S'_1 P_1 \) are equal where \( S'_1 P_1 T'_1 \) is an arc with A as center and A P_1 as radius.

Therefore area \( Q_1 R_1 T_1 S_1 \) = area \( Q_1 R_1 T'_1 S'_1 \)

Now area of sector \( Q_1 R_1 T'_1 S'_1 \) = \( \frac{1}{2} AP'_1^2 \frac{d\theta}{2} - \frac{1}{2} r'^2 d\theta \)
Thus for the continuity of metal flow, if $U_r$ is the radial velocity (positive radially outwards), we have

$$\frac{1}{2} \left( \frac{R_o^2}{\cos^2 \theta} - r^2 \right) d\theta = u + (rd\theta) a U_r = 0$$

i.e. $U_r = -\left( \frac{R_o^2}{\cos^2 \theta} \right) \frac{u}{2ra}$ (3.8)

The associated strain rate fields from equations 3.6, 3.7 and 3.8 are then as follows

$$\varepsilon_{rr} = \frac{3 U_r}{3r} = \frac{R_o^2}{2a} \left[ \frac{1}{r^2 \cos^2 \theta} + 1 \right]$$

(3.9)

$$\varepsilon_{\theta \theta} = \frac{U_r}{r} + \frac{1}{r} \frac{3 U_\theta}{3\theta} = \frac{U_r}{r} \quad \text{[U_\theta being 0]}$$

(3.10)

$$\varepsilon_{yy} = \frac{3 U_y}{3y} = \frac{u}{a}$$

(3.11)

$$\varepsilon_{r \theta} = \varepsilon_{\theta r} = \frac{1}{2r} \frac{3 U_r}{3\theta} + \frac{1}{3r} \frac{3 U_\theta}{3r} - \frac{U_\theta}{r}$$

= \frac{1}{2r} \frac{3 U_r}{3\theta} = \frac{u}{2ar^2} R_o^2 \sec^2 \theta \tan \theta$$

(3.12)
From equations (3.9), (3.10) and (3.11) it is seen that following condition of plastic incompressibility is satisfied for the region ABC

\[ \dot{\varepsilon}_{rr} + \dot{\varepsilon}_{\theta\theta} + \dot{\varepsilon}_{yy} = 0 \]

Further for the region ABC, \( 0 \leq \theta \leq \theta_1 \)

where

\[ \theta_1 = \tan^{-1} \left[ \frac{2R_o \cos \left( \frac{\pi}{6} \right) - \frac{d}{2}}{R_o} \right] \]

where \( d \) is the diameter of the cylindrical portion of the punch (i.e., blank cavity)

3.3.2 Velocity and Strain Rate Fields In Region ACD

Taking the geometry of Fig. 3.3 we have, if

\[ A S_2 = R \quad \text{and} \quad AB = R_o \]

\[ AC = R_o / \cos \theta_1 \]
\[
R = \frac{AC \cos \left( \frac{\pi}{3} - \theta_1 \right)}{\cos \left( \frac{\pi}{3} - \theta_1 - \phi \right)}
\]

\[
= \frac{R_0 \cos \left( \frac{\pi}{3} - \theta_1 \right) \sec \left( \frac{\pi}{3} - \theta_1 - \phi \right)}{\cos \theta_1}
\]

(3.16)

and

\[
\tan \left( \frac{\pi}{3} - \theta_1 - \phi \right) = \frac{d - \sin \left( \frac{\pi}{6} - \psi \right)}{2 - \frac{\sqrt{3}}{6}}
\]

\[
= \frac{AC \cos \left( \frac{\pi}{3} - \theta_1 \right) + \frac{\sqrt{3}}{4} d \frac{d}{2} \cos \left( \frac{\pi}{6} - \psi \right)}{6}
\]

(3.17)

Again taking A as the origin of cylindrical co-ordinates, the velocity components \( U_\phi \) (\( \phi \) measured from AC anticlockwise) and \( U_y \) can be assumed, for the flow directed towards the corner A.
Therefore,
\[ U_\phi = 0 \]  \hspace{1cm} (3.18)
and
\[ U_y = u \left(1 - \frac{y}{a} \right) \]  \hspace{1cm} (3.19)

For the continuity of material flow in this region as explained above
\[ u \left[ \text{area } Q_2 R_2 T_2 S_2 \right] + u \left[ \text{area } OS_2 T_2 \right] + U_r \left( rd \phi \right) a = 0 \]  \hspace{1cm} (3.20)

If \( P_2 \) is the mid point of arc \( S_2 T_2 \) and \( S'_2 P_2 T'_2 \) is an arc of radius \( AP_2 \) with center \( A \), then approximately areas \( S_2 P_2 S'_2 \) and \( T_2 P_2 T'_2 \) will be equal. This will render

area \( Q_2 R_2 T_2 S_2 = area \ Q_2 R_2 T'_2 S'_2 \)
\[ = - \frac{1}{2} AP_2^2 d\phi - \frac{1}{2} r^2 d\phi \]
\[ = - \frac{1}{2} \left( R^2 - r^2 \right) d\phi \]  \hspace{1cm} (3.21)

And geometrically
\[ \text{arc } S_2 T_2 = \frac{d}{2} \text{d} \psi = R \text{d} \phi \]
\[ \text{i.e. } \text{d} \psi = \frac{2R}{d} \text{d} \phi \]  \hspace{1cm} (3.22)

Also area \( OS_2 T_2 = \frac{1}{2} \left( \frac{d}{2} \right)^2 \text{d} \psi = \frac{1}{8} d^2 \text{d} \psi \)  \hspace{1cm} (3.23)
Therefore from equations (3.20) through (3.22)

\[ U_r = - \frac{u}{2a} \left[ \frac{R (2R + d)}{2r} \right] \]  \hspace{1cm} (3.24)

The associated strain rate field in the region ACD can then be derived from equations (3.16), (3.18), (3.19) and (3.24) as follows

\[ \varepsilon_{\phi\phi} = \frac{U_r}{r} + \frac{1}{r} \frac{\partial U_\phi}{\partial \phi} = \frac{U_r}{r} \]  \hspace{1cm} (3.25)

\[ \varepsilon_{rr} = \frac{\partial U_r}{\partial r} = - \frac{u}{2a} \left[ \frac{R (2R + d)}{2r^2} + 1 \right] \] \hspace{1cm} (3.26)

\[ \varepsilon_{yy} = \frac{\partial U_y}{\partial y} = - \frac{u}{a} \] \hspace{1cm} (3.27)

\[ \varepsilon_{r\phi} = \varepsilon_{\phi r} = \frac{1}{2r \partial \phi} \left[ \frac{\partial U_r}{\partial r} + \frac{\partial U_\phi}{\partial \phi} - \frac{U_\phi}{r} \right] \]

\[ = \frac{1}{2r} \frac{\partial U_r}{\partial \phi} \]

\[ = \frac{U}{8ar^2} \frac{R_0}{\cos \theta_{1}} \cos \left( \frac{\pi}{3} - \theta_{1} \right) \]

\[ \times \sec \left( \frac{\pi}{3} - \theta_{1} - \phi \right) \tan \left( \frac{\pi}{3} - \theta_{1} - \phi \right) \] \hspace{1cm} (3.28)
\[
\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{yy} = 0
\]  

(3.29)

From equations (3.25) through (3.27) it is seen that the following condition of plastic incompressibility is satisfied

\[
\varepsilon_{\theta\theta} + \varepsilon_{rr} + \varepsilon_{yy} = 0
\]

3.3.3 Velocity and Strain Rate Fields in Region Under the Punch Nose for a Flat Shape

In Fig. 3.2 the equation of the flat portion GK is

\[ Y = c \]  

(3.30)

where \( c \) is a constant

at the extreme bottom position

\[ C = (a - 1) \]

where \( l \) is the length of the central punch

hence

\[ Y = (a - 1) \]  

(3.31)

Taking the origin at \( I \) for the cylindrical coordinates, assume the velocity field for region below the punch nose

\[ U_y = 0 \]  

(3.32)

\[ U_y = \frac{uy}{a-l} \]  

(3.33)

The velocity field defined by the equations (3.32) and (3.33) satisfies the velocity boundary conditions of \( U_y = u \).
along the nose surface GK where \( y = (a-1) \) and \( U_y = 0 \) at \( y = 0 \) along the central plane HI.

The kinematically admissible velocity field should also satisfy the plastic incompressibility condition and the radial velocity \( U_r \) (positive radially inwards) should be such that

\[
\epsilon_{yy} + \epsilon_{rr} + \epsilon_{yy} = 0 \tag{3.34}
\]

But

\[
\epsilon_{yy} = \frac{U_r}{r} + \frac{1}{r^2} \frac{\partial U_y}{\partial y} = \frac{U_r}{r} \tag{3.35}
\]

\[
\epsilon_{rr} = \frac{\partial U_r}{\partial r} \tag{3.36}
\]

\[
\epsilon_{yy} = \frac{\partial U_y}{\partial y} = \frac{u}{(a-1)} \tag{3.37}
\]

Thus substituting equations (3.35) through (3.37) in equation (3.34) we get

\[
\frac{U_r}{r} + \frac{\partial U_r}{\partial r} + \frac{u}{(a-1)} = 0
\]

or

\[
\frac{\partial U_r}{\partial r} + U_r \left( - \right) = - \frac{u}{(a-1)}
\]

or

\[
U_r = - \frac{u \cdot r}{(a-1) \cdot 2}
\]

The associated strain rate field for this region can then be
derived from equations (3.32), (3.33) and (3.38) as follows

\[
\varepsilon_{yy} = -\frac{u}{2(a-1)} \quad (3.39)
\]

\[
\varepsilon_{rr} = \frac{\partial U_r}{\partial r} = -\frac{u}{2(a-1)} \quad (3.40)
\]

\[
\varepsilon_{yy} = \frac{\partial U_y}{\partial y} = -\frac{u}{(a-1)} \quad (3.41)
\]

\[
\varepsilon_{r\psi} = \varepsilon_{\psi r} = \frac{1}{2} \left( \frac{\partial U_r}{\partial r} - \frac{\partial U_\psi}{\partial \psi} \right) = 0 \quad (3.42)
\]

\[
\varepsilon_{r\psi} = \varepsilon_{\psi r} = \frac{1}{2} \left( -\frac{\partial U_\psi}{\partial r} - \frac{\partial U_y}{\partial \psi} \right) = 0 \quad (3.43)
\]

\[
\varepsilon_{\psi \psi} = \varepsilon_{\psi \psi} = \frac{1}{2} \left( \frac{\partial U_\psi}{\partial \psi} - \frac{\partial U_y}{\partial \psi} \right) = 0 \quad (3.44)
\]

3.3.4 Velocity & Strain Rate Fields in Region Under the Punch Nose for a Conical Shape

Taking origin at L for cylindrical co-ordinates (Fig. 3.4), assume the following velocity field for the region below the punch nose having \( \beta \) as the semi-cone angle \( \alpha \)

\[
U_\psi = 0 \quad (3.45)
\]

\[
U_y = \frac{u_y}{(y_2 + r \tan \alpha)} = \frac{u_y}{(y_2 + r \tan \beta)} \quad (3.46)
\]

where

\[
\alpha = \frac{\pi}{2} - \beta
\]
\[ y_2 = (a - h_1 - \frac{d}{2} \tan \alpha) \]

\[ h_1 = \text{length of cylindrical portion of punch} \]

The velocity field defined by equations (3.45) and (3.46) satisfies the boundary conditions of \( U_y = u \) along the conical surface where \( y = y_2 + r \tan \alpha \) at any radius \( r \) and \( U_y = 0 \) at \( y = 0 \) along the central plane LM. Since the kinematically admissible velocity field should also satisfy the plastic incompressibility condition, \( U_r \) should be such that

\[ \varepsilon_{yy} + \varepsilon_{rr} + \varepsilon_{yy} = 0 \]  

(3.47)

But

\[ \varepsilon_{yy} = \frac{1}{r} \frac{\partial U_y}{\partial y} = \frac{U_r}{r} \]  

(3.48)

\[ \varepsilon_{yy} = \frac{\partial U_y}{\partial y} = \frac{u}{y_2 + r \tan \alpha} \]  

(3.49)

\[ \varepsilon_{rr} = \frac{\partial U_r}{\partial r} \]  

(3.50)

Thus substituting equations (3.48) through (3.50) in equation (3.47) and solving the resultant differential equation we get

\[ U_r = -\frac{u}{\tan \alpha} + \frac{u}{r \tan^2 \alpha} \ln \left( \frac{y_2 + r}{r \tan \alpha} \right) \]  

(3.51)
Then the strain rate elements for this region are

\[
\varepsilon_{ry} = \frac{1}{2} \left( \frac{1}{r} \frac{\partial U_r}{\partial r} + \frac{\partial U_y}{\partial y} - \frac{U_r}{r} \right) = 0 \quad (3.52)
\]

\[
\varepsilon_{xy} = \frac{1}{2} \left( \frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} \right) = \frac{1}{2} \frac{\partial U_y}{\partial r} \quad (3.53)
\]

\[
\varepsilon_{yy} = \frac{1}{2} \left( \frac{\partial U_y}{\partial y} + \frac{\partial U_y}{\partial y} \right) = 0 \quad (3.54)
\]

3.3.5 Velocity and Strain Rate Fields in Region under the Punch Nose for a Spherical Shape

Geometrically from fig (3.6)

\[
\sin \gamma = \frac{r}{\rho} \quad (3.55)
\]

\[
Y_{max} = \beta_2 \quad (3.56)
\]

Nose Radius \( \rho = \frac{d}{2 \sin \beta_2} \quad (3.57) \)

Height of Spherical nose \( h_3 \)

\[
= \rho \left( 1 - \cos \beta_2 \right) \quad (3.58)
\]

\[
K_2J_2 = \rho \left( 1 - \cos \gamma \right) \quad (3.59)
\]

\[
Y_3 = (a - h_2) - h_3 \]

\[
= (a - h_2) - \rho \left( 1 - \cos \beta_2 \right) \quad (3.60)
\]
Taking the origin at $L_2$ for cylindrical co-ordinate Fig. (3.5) assume the following velocity field for the region below the punch nose, with $u$ vertically downwards towards $L_2$ and $U_r$ radially towards $L_2$ as positive

$$U_y = 0$$ (3.61)

$$U_y = \frac{u}{y_3 + K_2 J_2} = \frac{u}{P - \rho \cos \gamma}$$ (3.62)

where

$$P = y_3 + \rho$$ (3.63)

Equation (3.62) satisfies the velocity boundary conditions of $U_y = 0$ at $y = 0$ and $U_y = u$ at any radius $r$. Since the velocity field should also satisfy the plastic incompressibility condition, $U_r$ should be such that

$$\varepsilon_{yy} + \varepsilon_{rr} + \varepsilon_{yy} = 0$$ (3.64)

$$\varepsilon_{yy} = \frac{U_r}{r} + \frac{1}{r} \frac{\partial U_y}{\partial y} \frac{U_r}{r}$$ (3.65)

$$\varepsilon_{rr} = \frac{\partial U_r}{\partial r}$$ (3.66)

$$\varepsilon_{yy} = \frac{u}{P - \rho \cos \gamma}$$ (3.67)

From equations (3.64) through (3.67) we have
\[ \frac{\partial U_r}{\partial r} + \frac{1}{r} U_r = -\frac{u}{P - p \cos \gamma} \] (3.68)

The solution of the differential equation (3.68) gives

\[ U_r = -\frac{u}{r} \left( \frac{P}{P - (\rho^2 - r^2)^{\frac{1}{2}}} - \frac{P}{P - (\rho^2 - r^2)^{\frac{1}{2}}} \ln \left[ \frac{P - (\rho^2 - r^2)^{\frac{1}{2}}}{P - (\rho^2 - r^2)^{\frac{1}{2}}} \right] \right) \] (3.69)

Then the other strain rate elements are

\[ \varepsilon_{rr} = \frac{1}{\partial r} \frac{\partial U_r}{\partial r} = \frac{1}{r} \left( \frac{\partial U_r}{\partial r} \right) \] (3.70)

\[ \varepsilon_{\psi \psi} = \frac{1}{\partial \psi} \frac{\partial U_\psi}{\partial \psi} - \frac{1}{\partial \psi} \frac{\partial U_\psi}{\partial \psi} = 0 \]

\[ \varepsilon_{x \psi} = \frac{1}{\partial x} \frac{\partial U_x}{\partial x} + \frac{1}{\partial \psi} \frac{\partial U_\psi}{\partial \psi} = \frac{1}{\partial \psi} \frac{\partial U_\psi}{\partial \psi} + \frac{1}{\partial \psi} \frac{\partial U_\psi}{\partial \psi} = 0 \]

3.3.6 Velocity and Strain Rate Fields in Region under the Punch Nose for a Parabolic Shape

In Fig. 3.6 the equation of the parabolic nose PQR, with Ox and Oy as axis of reference is
\[ x^2 = \frac{d^2 y}{4 h_4} \]  
\[ (3.73) \]

where \( h_4 = QS \) = height of the parabolic nose

The slope of the parabolic curve at \( T \), which is at a distance \( r \) from the \( y \)-axis is

\[ \tan \delta_2 = \frac{\frac{dy}{dx}}{\frac{d^2}{d^2}} = \frac{8h_4r}{d^2} \]  
\[ (3.74) \]

and

\[ TV = \frac{4 r^2 h}{d^2} \]  
\[ (3.75) \]

Taking the origin at \( W \) for cylindrical coordinates, assume the following velocity field for the region below the punch nose

\[ U_y = 0 \]  
\[ (3.76) \]

\[ U_y = \frac{Y u}{Y_4 + TV} = \frac{Y u}{Y_4 + h_6 r^2} \]  
\[ (3.77) \]

where

\[ Y_4 = a - h_4 - h_5 \]  
\[ (3.78) \]

\[ h_5 = \text{length of cylindrical portion of punch} \]

\[ h_6 = \frac{4 h_4}{d^2} \]  
\[ (3.79) \]

The velocity field defined by equations (3.76) and (3.77) satisfies the velocity boundary conditions of \( U_y = u \) along the nose surface where \( y = Y_4 + h_6 r^2 \) and \( U_y = 0 \) at \( y = 0 \)
along the central plane \( XZ \). The kinematically admissible velocity field should also satisfy the plastic incompressibility condition and the radial velocity \( U \) (positive radially inwards) should be such that

\[
\varepsilon_{\psi\psi} + \varepsilon_{rr} + \varepsilon_{yy} = 0 \tag{3.80}
\]

but

\[
\dot{\varepsilon}_{\psi\psi} = \frac{U_r}{r} - \frac{\partial U_r}{\partial y} \frac{U_r}{r} \tag{3.81}
\]

\[
\varepsilon_{rr} = \frac{\partial U_r}{\partial r} \tag{3.82}
\]

\[
\dot{\varepsilon}_{yy} = \frac{\partial U_y}{\partial y} = \frac{u}{y + h^6 r^2} \tag{3.83}
\]

Thus substituting equations (3.81) through (3.83) in equation (3.80) and solving the differential equation we get

\[
U_r = - \frac{u}{2 rh^6} \ln \left( y + h^6 r^2 \right) \tag{3.84}
\]

Then the other strain rate elements are

\[
\varepsilon_{\psi\psi} = - \frac{u}{2r^2 h^6} \ln \left( y + h^6 r^2 \right) \tag{3.85}
\]

\[
\varepsilon_{rr} = \frac{u}{2h^6 r^2} \ln \left( y + h^6 r^2 \right) - \frac{u}{y + h^6 r^2} \tag{3.86}
\]

\[
\varepsilon_{\psi y} = \frac{1}{2} \left[ - \frac{\partial U_r}{\partial r} + \frac{\partial U_y}{\partial y} \frac{U_r}{r} \right] \tag{3.87}
\]
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\[
\begin{align*}
1 \frac{\partial U_r}{\partial r} &= -\frac{\partial \psi}{\partial \psi} = 0 \\
2r \frac{\partial v}{\partial \psi} \\
\varepsilon_{xy} &= -\left[ \frac{\partial U_r}{\partial y} + \frac{\partial U_y}{\partial r} \right] = -\frac{\partial U_y}{\partial r} \\
2 \frac{\partial y}{\partial r} \frac{\partial r}{\partial \psi} \\
\frac{h \xi u}{(y_4 + h \xi r^2)^2} \\
3 \Psi \\
\varepsilon_{y\psi} &= -\left[ \frac{\partial U_y}{\partial \psi} + \frac{1}{2} \frac{\partial U_y}{\partial \psi} \right] = 0 \\
2 \frac{\partial \psi}{\partial \psi} \frac{\partial \psi}{\partial \psi}
\end{align*}
\]

3.3.7 **Upper Bound Approach**

As the velocity fields are kinematically admissible, the upper bound theorem (Avitzur, 1977) can be used assuming the material to follow Von Mises yield criterion

\[
J = \frac{2}{\sqrt{3}} \sigma_0 \int \left( \frac{1}{2} \varepsilon_{ij} \varepsilon_{ij} \right)^{\frac{1}{2}} dv + \int_{T} |\Delta \psi| ds
\]

\[
= \int_{ST} T_{ij} U_i ds
\]

The first term on the right hand side of equation (3.90) represents the plastic work dissipation rate due to strain rate fields \(\dot{\varepsilon}_{ij}\). The second term denotes the power dissipation at the surfaces of discontinuity \(\dot{\psi}\) and the last term which signifies the power dissipation due to surface tractions is zero as the die is stationary and
assumed rigid.

Therefore

\[ J = \dot{W}_p + \dot{W}_f \tag{3.91} \]

and

\[ \dot{W}_p = 24 ( \dot{W}_{p1} + \dot{W}_{p2}) + 2 \dot{W}_{p3} \tag{3.92} \]

\[ \dot{W}_f = 24 ( \dot{W}_{f1} + \dot{W}_{f2} + \dot{W}_{f3}) + ( \dot{W}_{f4} + \dot{W}_{f5}) + 24 \dot{W}_{f6} + \dot{W}_{f7} \tag{3.93} \]

where various elements of \((\dot{W}_p)\) and \((\dot{W}_f)\) are defined in the nomenclature, and are evaluated as below.

### 3.3.7.1 Evaluation of \(\dot{W}_{p1}\) for region ABC

The plastic work dissipation rate due to strain rate field is given by

\[ \dot{W}_{p1} = \frac{2\sigma_0}{\sqrt{3}} \int \frac{1}{v} \left[ - \varepsilon_{ij} \varepsilon_{ij} \right]^\frac{1}{2} \, dv \]

\[ = \frac{2\sigma_0}{\sqrt{3}} \int \frac{1}{v} \left[ - \left( \varepsilon_{rr}^2 + \varepsilon_{\theta\theta}^2 + \varepsilon_{yy}^2 + 2\varepsilon_{r\theta}^2 \right) \right]^\frac{1}{2} \, dv \]

Substituting in above the values from equations (3.9) through (3.12) we get

\[ \dot{W}_{p1} = \frac{2\sigma_0 \sqrt{2} u}{\sqrt{3}} \int_0^1 \int_0^{\theta_1} \int_0^{\frac{R_0}{\cos \theta}} \int_0^{\frac{R_0}{\sec \theta}} \frac{R_0}{r} \ \frac{1}{4} \left( 3 + \frac{R_0^4}{r^4} \sec^6 \theta \right)^\frac{1}{2} \, r \, dr \, d\theta \, d\phi \]
While numerically integrating equation (3.94), the lower limit of 'r' as zero will render the integral infinite. Therefore the lower limit of 'r' is kept (after Sagar, 1980) as 0.1 mm. Thus equation (3.94) can be written as

\[
\dot{W}_{p1} = - \frac{1}{2} \sigma_0 \theta_1 \frac{R_0}{\cos \theta} \int_{0}^{\pi/2} \int_{0}^{0.1} r (3 + \frac{R^4}{r^4} \sec^6 \theta)^{\frac{1}{4}} \, d\theta \, dr
\]

(3.95)

3.3.7.2 Evaluation of \( \dot{W}_{p2} \) for region ACD

Similarly for the equation ACD we have from equations (3.25) through (3.29)

\[
\dot{W}_{p2} = 2 \sigma_0 \frac{\theta_2}{\sqrt{3}} \frac{R}{\phi = 0} \int_{0}^{\pi} \int_{0}^{0.1} \frac{1}{2} (\varepsilon_{ij}^2 + \varepsilon_{ii}^2 + \varepsilon_{yy}^2 + 2 \varepsilon_{r\phi}^2 )^{\frac{1}{2}} (r d\phi \, dz)
\]

\[
= 2 \sigma_0 \frac{\theta_2}{\sqrt{3}} \frac{R}{\phi = 0} \int_{0}^{\pi} \int_{0}^{0.1} \frac{1}{2} (\varepsilon_{\phi\phi} + \varepsilon_{\phi r}^2 + \varepsilon_{y y}^2 + 2 \varepsilon_{r \phi}^2 )^{\frac{1}{2}} (r d\phi \, dz)
\]

\[
= \frac{\sigma_0 u \theta_2}{2/3} \int_{\phi = 0}^{\pi} \int_{r = 0.1}^{\pi} \left( \frac{F_1}{r} \right)^{\frac{1}{2}} r d\phi \, dz
\]

(3.96)

where \( \theta_2 = \frac{\pi}{3} - \theta_1 \)
\[ \Phi_1 = \frac{R^2 (4R+d)^2}{4 r^4} \cos^2 \theta \frac{\sec^2 (\theta_2 - \phi) \tan^2 (\theta_2 - \phi)}{\cos^2 \theta_1} \]

and

\[ F_1 = \frac{R^2 (2R+d)^2}{r^4} \]

3.3.7.3 Evaluation of \( \dot{W}_{p3} \) for the region below central punch

(a) FOR FLAT PUNCH

From equations (3.32) through (3.44)

\[ (\dot{W}_{p3})_1 = \frac{2 \sigma_0}{\sqrt{3}} \int \int \int \left[ \varepsilon_{ij} \varepsilon_{ij} \right] (\rho \psi dr) \] (a-1)

\[ = \frac{\pi}{4} \frac{d^2 \sigma_0 u}{d} \] (3.99)

(b) FOR CONICAL PUNCH

\[ (\dot{W}_{p3})_2 = \frac{2 \sigma_0}{\sqrt{3}} \int \int \int \left[ - \varepsilon_{ij} \varepsilon_{ij} \right] (\rho \psi dr) dy \]

By substituting values of various strain rates and simplifying, this equation reduces to

\[ (\dot{W}_{p3})_2 = \frac{4 \pi \sigma_0 u}{\sqrt{6}} \int \int \int r A_5 \left( A_2 \right)^{3/2} dr \] (3.100)

where

\[ A_5 = Y_2 + r \tan \theta \]

\[ E_1 = \frac{Y_2}{r \tan \theta} \ln \left( \frac{A_5}{\tan \theta} \right) \] (3.101)
\[ E_2 = \frac{Y_2}{r A_5 \tan \alpha} \]  

\[ A_1 = (E_2 - E_1)^2 + \left( E_1 - \frac{1}{r \tan \alpha} \right) + \frac{1}{A_5^2} \]  

\[ A_2 = \frac{\tan^2 \alpha}{2 A_5^4} \]  

\[ A_3 = \left( \frac{A_1}{A_2} \right)^{\frac{1}{2}} \]  

\[ A_4 = (A_5^2 + A_3^2) \]  

\[ A_6 = \ln \left[ \frac{A_5 + A_4}{A_3} \right] \]  

(c) FOR SPHERICAL PUNCH

\[ (\dot{W}_{p3})_3 = \frac{2 \sigma_0}{\sqrt{3}} \int_{r=0}^{d/2} \frac{1}{(\sigma^2 - \tau^2)^{\frac{1}{2}}} \left( \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} \left[ -\dot{\varepsilon}_{i j} \dot{\varepsilon}_{i j} \right]^{\frac{1}{2}} (r d\varphi dr) d\theta \right) \]

Substituting the values from equations (3.65) through (3.72) we get

\[ (\dot{W}_{p3})_3 = \frac{4 \pi \sigma_0}{\sqrt{3}} \int_{r=0}^{d/2} \left[ \frac{1}{2} \dot{\varepsilon}_{rr}^2 + \frac{1}{2} \dot{\varepsilon}_{\varphi\varphi}^2 + \frac{1}{2} \dot{\varepsilon}_{yy}^2 + \frac{1}{2} \dot{\varepsilon}_{rr}^2 + \frac{1}{2} \dot{\varepsilon}_{\varphi\varphi}^2 + \frac{1}{2} \dot{\varepsilon}_{yy}^2 \right] \frac{1}{(\sigma^2 - \tau^2)^{\frac{1}{2}}} \frac{1}{r^2} \left[ P - (\sigma^2 - \tau^2)^{\frac{1}{2}} \right] dr \]

where

\[ P = Y_3 + \sigma \]
(d) FOR PARABOLIC PUNCH

For the region under the punch nose \((\dot{W}_{p3})_4\) is obtained by substituting equations (3.83) through (3.89) in the following equation and simplifying

\[
(\dot{W}_{p3})_4 = \frac{2\sigma_0}{\sqrt{3}} \frac{d/2}{r} (y_4 + h_6 r^2)^{2\pi} \frac{1}{y_4} \int_0^{\eta} \int_0^{\varphi} \left[ -\frac{\epsilon_{rr}}{2} - \frac{\epsilon_{\varphi\varphi}}{2} \right] \eta \, d\eta \, d\varphi
\]

\[
= \frac{4\pi\sigma_0}{\sqrt{3}} \frac{d/2}{r} \frac{1}{S_1} \int_0^{1} \left[ -\frac{S_3 S_4}{2} + \frac{S_5 S_6}{2} - \frac{S_3 S_5}{4} \right] \, d\eta
\]

\[
(3.110)
\]

where

\[
S_o = Y_4 + h_6 r^2
\]

\[
S_1 = \frac{h_6 r u}{S_o}
\]

\[
S_2 = \frac{1}{2} \left( \frac{\epsilon_{rr}}{E} + \frac{\epsilon_{\varphi\varphi}}{E} + \frac{\epsilon_{yy}}{E} \right)
\]

\[
S_3 = \frac{S_2}{S_1^2}
\]

\[
S_4 = \left[ S_3 + (S_o)^2 \right]^{1/2}
\]

\[
S_5 = \ln (S_3)
\]

\[
S_6 = \ln (S_o + S_4)
\]

\[
(3.111) \quad (3.112) \quad (3.113) \quad (3.114) \quad (3.115) \quad (3.116) \quad (3.117)
\]
3.3.7.4 Evaluation of frictional work dissipation rate at ABC ($\dot{W}_{f1}$)

$$\dot{W}_{f1} = \int T |\Delta v| \, ds$$

Here

$$|\Delta v| = |[u_r^2 + u_\theta^2]|$$

as $u_\theta = 0$

Therefore

$$|\Delta v| = |u_r|$$

Hence

$$\dot{W}_{f1} = \frac{m \sigma_0}{\sqrt{3}} \int \int \left(\frac{R_0}{\cos \theta} - r^2\right) \, rdrd\theta$$

On integration it reduces to

$$\dot{W}_{f1} = \frac{m \sigma_0}{2\sqrt{3}} \frac{u}{a} F_2 \quad (3.118)$$

where

$$F_2 = \frac{R_o^3}{3} \left[ \tan \theta_1 \sec \theta_1 + \ln (\sec \theta_1 + \tan \theta_1) \right]$$

$$- 0.1 R_o^2 \tan \theta_1 + \frac{0.001}{3} \quad \theta_1$$

along the

3.3.7.5 Frictional power dissipation vertical surface AB ($\dot{W}_{f2}$)

In equations (3.7) and (3.8) $\theta = 0$ along AB.
Therefore we get

\[ |\Delta v| = | \left[ (U_y)^2_{\theta=0} + (U_\phi)^2_{\theta=0} \right]^{1/4} | \]

and as

\[ \dot{W}_{f2} = \frac{m \sigma_0}{\sqrt{3}} \int |\Delta v| \, ds \]

\[ \Rightarrow \dot{W}_{f2} = -\frac{m \sigma_0 \cdot u}{3} \int_{r=0.1}^{R_0} \int_{y=0}^{a} \left[ (1 - \frac{y}{a})^2 + \frac{1}{4 \cdot \rho \cdot a \cdot (R_0^2 - r^2)^2} \right] dy \, dr \]

on simplification reduces to

\[ \dot{W}_{f2} = \frac{m \sigma_0 \cdot u \cdot a}{2 \sqrt{3}} (F_3) \]  \hspace{1cm} (3.120)

where

\[ F_3 = R_0 \int_{r=0.1}^{R_0} \left[ e^{\ln (1 + \sqrt{1 + B}) - \frac{B}{2}} \ln (1 + \sqrt{1 + B}) - \frac{B}{2} \right] \, dr \]  \hspace{1cm} (3.121)

where

\[ B = \left( \frac{R_0^2 - r^2}{2 \pi} \right)^2 \]  \hspace{1cm} (3.122)

3.3.7.6 Frictional power dissipation at surface ACD (\( \dot{W}_{f3} \))

At this surface since \( U_{\phi} = 0 \), we have

\[ |\Delta v| = | \left[ U_r^2 + U_{\phi}^2 \right]^{1/4} = | U_r | \]

and

\[ \dot{W}_{f3} = \frac{m \sigma_0}{\sqrt{3}} \int_{\phi=0}^{\pi/2 - \theta_1} \int_{r=0.1}^{R} |U_r| \, r \, dr \, d\phi \]

Substituting the values of \( R \) and \( U_r \) from equations (3.16) and (3.24) and then integrating we get
\[ \dot{W}_{f3} = \frac{m_0 u}{12/3 a} \left( \frac{\pi}{3-\beta_1} \right) \int_{\phi=0}^{2\pi} (4R^3 + 3R^3 d) \, d\phi \]  
(3.123)

where

\[ R = \frac{R_0}{\cos \left( \frac{\pi}{3} - \beta_1 \right) \sec \left( \frac{\pi}{3} - \beta_1 - \phi \right)} \]  
(3.124)

3.3.7.7 Frictional power dissipation on the cylindrical surface of Punch \((W_f)\)

(a) FOR FLAT PUNCH (Fig. 3.2)

Since the punch and the adjoining metal along this surface of the cylinder are moving with different velocities, the frictional power dissipation is given by

\[ \dot{W}_{f4(1)} = \frac{m_0 u}{\sqrt{3} a} \int_{a}^{\infty} |\Delta v| \, ds \]

and

\[ |\Delta v| \leq |U - U_y| \]

\[ = |u - u (1 - \frac{y}{a})| \text{ from equation (3.19)} \]

\[ y = \frac{u - u}{a} \]

Then

\[ \dot{W}_{f4(1)} = \frac{m_0 u}{\sqrt{3} a} \int_{y=-1}^{u} |(u - y)| \, |y| \, d\]

or

\[ \dot{W}_{f4(1)} = \frac{m_0 u l^2 d}{2 \sqrt{3} a} \]  
(3.125)

(b) FOR CONICAL PUNCH (Fig. 3.4)

\[ \dot{W}_{f4(2)} = \frac{\pi m_0 u h^2 d}{2 \sqrt{3} a} \]  
(3.126)
(c) FOR SPHERICAL PUNCH (Fig. 3.5)

\[ \dot{W}_{f4(3)} = \frac{\pi m \sigma_0 u h^2 d}{2 \sqrt{3} a} \]  (3.127)

(d) FOR PARABOLIC PUNCH (Fig. 3.6)

\[ \dot{W}_{f4(4)} = \frac{\pi m \sigma_0 u h^2 d}{2/3 a} \]  (3.128)

3.3.7.8 Frictional power dissipation at the punch nose surface \( \dot{W}_{f5} \)

(a) FOR FLAT PUNCH (Fig. 3.2) \( \dot{W}_{f5(1)} \)

As \[ \dot{W}_{f5(1)} = \frac{m \sigma_0}{\sqrt{3}} \int_0^1 \Delta v | \Delta v | ds \]

and \[ | \Delta v | = | \left( U_x^2 + U_y^2 \right) | \] and as \( U_y = 0 \)

\[ | \Delta v | = | U_x | \]

Substituting \( U_x \) from equation (3.38)

\[ \dot{W}_{f5(1)} = \frac{m \sigma_0}{\sqrt{3}} \int_0^1 \int_{\psi=0}^{2\pi} \frac{d^2}{2} \frac{u r}{r=0} \frac{\left| r dr d\psi \right|}{2 \left( a-1 \right)} \]

\[ \dot{W}_{f5(1)} = \frac{\pi m \sigma_0 u d^3}{24 \sqrt{3} \left( a-1 \right)} \]  (3.129)

(b) FOR CONICAL PUNCH (Fig. 3.4) \( \dot{W}_{f5(2)} \)

At any point on the conical surface at radius \( r \), \( y = y_2 + rt\alpha \) and \( U_y = u \). The inward radial velocity is given
by equation (3.51). Along the conical surface $\Delta v$ is given by

$$| \Delta v | = | [- U_r \cos \alpha - \sin \alpha ]|$$

Hence

$$\dot{W}_{f5(2)} = \frac{m \sigma_0}{\sqrt{3}} \int_{r=0}^{\infty} \int_{\psi=0}^{\pi/2} | \Delta v | \rho d\rho d\psi$$

$$= \frac{m \sigma_0}{\sqrt{3}} \int_{r=0}^{\infty} \int_{\psi=0}^{\pi/2} \left| \frac{u \psi_2}{\tan \alpha} + \ln \left( \frac{\psi_2}{\tan \alpha} + r \right) \right| \cos \psi d\psi$$

$$= \frac{2\pi m \sigma_0 u}{\sqrt{3}} \int_{r=0}^{\infty} \left| \frac{1}{\tan \alpha} + \frac{\psi_2}{\tan^2 \alpha} \ln \left( \frac{\psi_2}{\tan \alpha} + r \right) \right| \cos \psi d\psi$$

where

$$\psi_2 = \frac{d}{r^2}$$

(c) FOR SPHERICAL PUNCH (Fig. 3.5) ($\dot{W}_{f5(3)}$)

For this nose shape

$$| \Delta v | = | -U_r \cos \gamma - u \sin \gamma |$$

and

$$\dot{W}_{f5(3)} = \frac{m \sigma_0}{\sqrt{3}} \int_{r=0.1}^{\infty} \int_{\psi=0}^{\pi/2} \left| [-U_r \cos \gamma - u \sin \gamma] r \right| \cos \gamma d\psi$$

$$= \frac{2\pi m \sigma_0}{\sqrt{3}} \int_{r=0.1}^{\infty} \left| -U_r - u \tan \gamma \right| r dr$$

Since $\tan \gamma = \frac{d}{\rho_0^2 - r^2}$ and also from equation (3.69), we get
\[ \dot{W}_{ES(3)} = \frac{2 \pi \sigma_0 \mu}{\sqrt{3}} \int_{r=0.1}^{r=0} \left[ \left( \rho^2 - r^2 \right)^{\frac{3}{2}} + \ln \left( \frac{P}{P - (\rho^2 - r^2)^{\frac{3}{2}}} \right) \right] \frac{r^2}{\sqrt{\rho^2 - r^2}} \, dr \]  

\[ (3.131) \]

(d) FOR PARABOLIC PUNCH (Fig. 3.6) \( (\dot{W}_{ES(4)}) \)

\[ |\Delta v| = \left| -U_x \cos \delta_2 - (U_y \sin \delta_2) \right| \quad \text{and} \]

\[ \dot{W}_{ES(4)} = \frac{m \sigma_0 d/2}{\sqrt{3}} \int_{r=0}^{r=0} |\Delta v| \frac{r dr}{\cos \delta_2} \]

From equations (3.74) and (3.84), we get

\[ \dot{W}_{ES(4)} = \frac{2 \pi m \sigma_0 \mu d/2}{\sqrt{3}} \int_{r=0}^{1} \left| \frac{1}{2h_6} \ln \left( y_h + h_6 r^2 \right) - \frac{8 h_4 r^2}{d^2} \right| \, dr \]  

\[ (3.132) \]

3.37.9 Shear work dissipation rate along CA due to velocity discontinuity \( (\dot{W}_{e6}) \)

For finding the shear work dissipation rate \( (\dot{W}_{e6}) \) along AC, we have from equation (3.8) the value of radial velocity along CA for the region ABC, when \( \theta = \theta_1 \). Similarly, the radial velocity along CA for the region ACD will be obtained from equation (3.24) for \( (R)_{\theta=0} \)

Thus along AC

\[ (U_r)_{ABC} = - \frac{R_0^2}{\cos^2 \theta_1} - \frac{r^2}{2ra} \]  

\[ (3.133) \]

and from equations (3.16) and (3.24) at \( \theta = 0 \)
(U_r)_{ACD} = \frac{u}{2a} \left[ \frac{R_0}{\cos \theta_1} \left( \frac{2R_0}{2a} + d \right) - r \right] - \frac{1}{2ra} \\
= \frac{u}{2a} \left[ \frac{R_0}{2r \cos \theta_1} \cos \theta_1 \left( \frac{2R_0}{2a} + d \right) - r \right] \quad (3.134)

Therefore from equations (3.133) and (3.134), the radial component of velocity \( \Delta v \) is

\[ |\Delta v|_r = |(U_r)_{ABC} - (U_r)_{ACD}| = \frac{ud}{2a} \quad (3.135) \]

Similarly for surface CA from equations (3.7) and (3.19) we obtain the relative velocity component parallel to \( y \)-axis as

\[ |\Delta v|_y = |(U_y)_{ABC} - (U_y)_{ACD}| = 0 \quad (3.136) \]

Thus from equations (3.135) and (3.136), we get

\[ |\Delta v| = \frac{ud}{2a} \]

and

\[ \dot{W}_{f6} = \frac{m \sigma_0 d/2}{\sqrt{3}} \int_0^{r_0} \int_0^{y_0} \frac{ud}{2a} \, dy \, dr \]

Therefore

\[ \dot{W}_{f6} = \frac{m \sigma_0 u d^2}{4 \sqrt{3}} \quad (3.137) \]

3.3.7.10 Shearing work dissipation rate along cylindrical surface GH for a flat nose (Fig.3.2) \((\dot{W}_{f7(1)})\)

The velocity \( (U_y)_{ACD} \) in the region ACD is given by equation (3.19) and velocity \( (U_y)_{JIHG} \) in the region below the
punch nose by equation (3.33). Due to the difference in these velocities along the surface GH, the shearing of the metal will occur along this cylindrical surface. If we measure \( y \) positive upwards from the central plane IH then the value of \((U_y)_{ACD}\) from equation (3.19) reduces to

\[
(U_y)_{ACD} = \frac{u_y}{a} \quad (3.138)
\]

and

\[
(U_y)_{JIHG} = \frac{u_y}{(a - 1)}
\]

Therefore

\[
| \Delta v | = u_y \left( \frac{1}{a-1} - \frac{1}{a} \right)
\]

and therefore

\[
\dot{W}_{\xi(1)} = \frac{m_o u}{\sqrt{3}} \int_{y=0}^{1} u_y \cdot \frac{\pi d}{a(a - 1)} \, d y
\]

\[
= \frac{\pi d m_o u}{2\sqrt{3}} \frac{1}{a(a - 1)} \quad (3.139)
\]

3.3.7.11 Shearing work dissipation rate along cylindrical surface MN for conical nose (Fig. 3.4) \((\dot{W}_{\xi(2)})\)

Here

\[
| \Delta v | = | (U_y)_{ACD} - (U_y)_{KLNM} |
\]

(3.140)

For the region KLNM \( U_y \) along MN is obtained from equation (3.46) by replacing \( r \) with \( d/2 \).
Therefore

\[
(U_y)_{KLMN} = \frac{u_y}{(y_2 + d/2 \tan \alpha)} \quad (3.141)
\]

From equations (3.138), (3.140) and (3.141), we have

\[
| \Delta v | = \left| \frac{u_y}{a} - \frac{u_y}{(y_2 + d/2 \tan \alpha)} \right| \quad (3.142)
\]

Then

\[
\dot{W}_{\epsilon?2} = \frac{m}{\sqrt{3}} \int_{y=0}^{(a-h_1)} \left| \frac{u_y}{a} - \frac{u_y}{(y_2 + d/2 \tan \alpha)} \right| \pi d \, dy
\]

\[
= \frac{m \sigma_0}{2/3} \int_{y_2 + d/2 \tan \alpha}^{1} \left( \frac{1}{a} \right) \, dy
\]

\[
= \frac{m \sigma_0}{2/3} \frac{\pi d}{a} \left( \frac{1}{y_2 + d/2 \tan \alpha} - 1 \right) \quad (3.143)
\]

3.3.7.12 Shearing work dissipation rate along cylindrical surface \( M_2N_2 \) for spherical nose (Fig. 3.5) \( \dot{W}_{\epsilon?3} \)

Here

\[
| \Delta v | = | (U_y)_{ACD} - (U_y)_{J_2L_2M_2N_2} | \quad (3.144)
\]

From equation (3.62), the velocity \( U_y \) at \( r = d/2 \) is given by

\[
(U_y)_{J_2L_2M_2N_2} = \frac{u_y}{\sqrt{\frac{3}{4} \left( \frac{d^2}{p} \right)^2 \left( \rho^2 - \frac{d^2}{4} \right)}} \quad (3.145)
\]

From (3.138), (3.144) and (3.145), we get.

\[
| \Delta v | = \left| \frac{u_y}{a} - \frac{u_y}{p - (\rho^2 - d^2/4)^2} \right| \quad (3.146)
\]
\[
\dot{W}_{f7(3)} = \frac{m}{\sqrt{3}} \int_{y=0}^{(a-h_2)} u \cdot y \quad \frac{u \cdot y}{y_4 + h_4} \quad \pi d\ dy
\]

\[
= u \cdot m \sigma_0 \cdot \pi d \left( \frac{1}{a} - \frac{1}{y_4 + h_4} \right) \left( a - h_5 \right)^2
\]

3.3.7.13 Shearing work dissipation rate along cylindrical surface PX for parabolic nose (Fig. 3.6) \((\dot{W}_{f7(4)})\)

Here \(|\Delta v| = |(U_y)_{ACD} - (U_y)_{QWXP}|\) (3.148)

For the region QWXP, \(U_y\) along PX is obtained from equation (3.77) for \(r = d/2\).

\[
(U_y)_{QWXP} = \frac{u \cdot y}{y_4 + h_4}
\]

\[
= \frac{u \cdot y}{y_4 + h_4}
\]

From equation (3.138), (3.148) and (3.149), we get

\[
|\Delta v| = \frac{u \cdot y}{a} - \frac{u \cdot y}{y_4 + h_4}
\]

\[
\dot{W}_{f7(4)} = \frac{m \sigma_0}{\sqrt{3}} \int_{y=0}^{a-h_5} u \cdot y \quad \frac{u \cdot y}{y_4 + h_4} \quad \pi d\ dy
\]

\[
= u \cdot m \sigma_0 \cdot \pi d \left( \frac{1}{a} - \frac{1}{y_4 + h_4} \right) \left( a - h_5 \right)^2
\]

3.3.8 Evaluation of Mean Effective Pressure \(p_m\)

Substituting the values of various terms in
equations (3.91), (3.92) and (3.93), the total work dissipation rate can be calculated. Further, if \( p_m \) is the mean pressure supplied by the press, the work dissipation rate can also be written as

\[
J = u p_m \text{(Area of hexagon of side } 2R_o) \\
= u p_m \times 6 \sqrt[4]{3} R_o^2 \\
\text{or } \quad \frac{J}{u} = \frac{p_m}{6 \sqrt[4]{3} R_o^2}
\]

If \( \frac{p}{p_m} = \frac{p}{\sigma_o} \),

Then \( \bar{p}_m = \frac{J}{6 \sqrt[4]{3} \ u \sigma_o R_o^2} \) \hspace{1cm} (3.152)

3.3.8.1 Evaluation of \( l \) for flat punch

The indentation is to be such that it just expands the circular disc of diameter \( d_2 \) (max. \( 2/3 R_o \)) (i.e. flat to flat distance of hexagon inside) to hexagonal shape of side \( 2R_o \). Thus equating the volume of the cylindrical punch to the volume of the space between disc and the hexagonal cavity, we get.

(for max. dia disc)

\[
l = \frac{a}{\pi d^2} (24/3 R_o^2 - \pi x 12 R_o^2)
\]

\[
= \frac{12a}{\pi d^2} (2\sqrt[4]{3} R_o^2 - \pi R_o^2)
\]
\[ \frac{12aR_0^2}{\pi d^2} = \frac{2\sqrt{3} - \pi}{2/3} \]

\[ = \frac{12 (2\sqrt{3} - \pi) a R_0^2}{\pi d^2} \]  \hspace{1cm} (3.153)

For Disk dia \( d_1 = \frac{2}{3} R' - \pi d^2 \)

Also, volume of web = \( \frac{\pi}{4} d^2 (a - 1) \) \hspace{1cm} (3.155)

3.3.8.2 Evaluation of \( h_1 \) and flash volume for conical punch

The penetration of the punch is to be such that \( h_1 \) just make the circular disc to expand to hexagonal shape.

The volume of space to be filled = \( \left[ \frac{6\sqrt{3} R_0^2}{4} - \frac{\pi}{4} \times 12 R_0^2 \right] a \) (for maximum dia disk)

For blank dia \( d_2 = \left[ \frac{6\sqrt{3} R_0^2}{4} - \frac{\pi}{4} d_2^2 \right] a \)

The volume of penetrated punch

\[ = \frac{\pi}{4} d^2 h_1 + \frac{1}{3} (\frac{\pi}{4} d^2) \times c \]

\[ = \frac{\pi}{4} d^2 \left[ h_1 + \frac{d}{6} \tan \alpha \right] \]

\[ = \frac{\pi}{4} d^2 \left[ h_1 + \frac{d}{6} \tan \alpha \right] = \left[ \frac{6\sqrt{3} R_0^2}{4} - \frac{\pi}{4} 12 R_0^2 \right] a \]
or \( h_1 = \frac{(24\sqrt{3} R_0^2 - \pi x 12 R_0^2)}{\pi d^2} \) a 
\( - \frac{d}{6} \tan \alpha \) (3.156)

\( \frac{(24\sqrt{3} R_0^2 - \pi d^2)}{\pi d^2} \) a 
\( - \frac{d}{6} \tan \alpha \) (3.157)

Further
\[
C_{\text{max}} = \frac{d}{2} \tan \alpha_{\text{max}} = a - h_1
\]

or \( \alpha_{\text{max}} = \tan^{-1} \left( \frac{2 (a - h_1)}{d} \right) \) (3.158)

also \( \beta_{\text{min}} = \frac{\pi}{2} - \alpha_{\text{max}} \) (3.159)

Flash volume \( V_f \)
\[
= \frac{\pi}{4} d^2 a - \left[ \frac{\pi}{4} d^2 h_1 + \frac{1}{3} \left( \frac{\pi}{4} d^2 \right) \times c \right]
\]
\[
= \frac{\pi}{4} d^2 \left[ a - h_1 - \frac{d}{6} \tan \alpha \right] (3.160)
\]

Inserting the value of \( h_1 \) from (3.157) the flash volume reduces to
\[
V_{\text{flash}} = \frac{a}{4} \left[ \pi d^2 - (24\sqrt{3} R_0^2 - \pi d^2) \right] \] (3.161)

Since flash volume cannot be zero, we conclude from equation (3.161) that
\[
d_2 \geq \frac{24\sqrt{3}}{2} \left( \frac{R_0^2}{\pi} - d_2^2 \right) \] (3.162)
It has already been seen before that \( \text{max. dia of disc} \) can be
\[
d_{2(\text{max})} = 2\sqrt{3} R_o
\] (3.163)

The volume of the blank with out central cavity is
\[
V_{\text{nut}} = \frac{6}{\sqrt{3}} R_o^3 \quad \text{a}
\] (3.164)

Therefore from equations (3.161) and (3.164) we get the
flash volume as
\[
\% V_{\text{flash}} = \left[ \frac{\pi (d^2 + d^2)}{24\sqrt{3} R_o^3} - 1 \right] \times 100
\] (3.165)

### 3.3.8.3 Evaluation of \( h_2 \) and flash volume for spherical punch

The penetration of the punch is to be such that it just makes the circular disc to expand to hexagonal shape

The volume of space to be filled = \[
\left[ \frac{6}{\sqrt{3}} R_o^2 - \frac{\pi d^2}{4} \right] \quad \text{a}
\]

The volume of the penetration punch
\[
= \frac{\pi}{4} d^2 h_2 + (\text{vol. corresponding to height } h_3 \text{ for the sphere of radius } \rho)
\]
\[
\frac{\pi}{4} d^2 h_2 + \pi h_3^2 \left( \rho - \frac{h_3}{3} \right) = \frac{6\sqrt{3} R_o^2}{4} - \frac{\pi}{4} d^2 \quad a
\]

or

\[ h_2 = \frac{4a}{\pi d^2} \left( \frac{6\sqrt{3} R_o^2}{4} - \frac{\pi}{4} d^2 \right) - \frac{4}{d} h_3^2 \left( \rho - \frac{h_3}{3} \right) \quad (3.166) \]

The flash volume

\[
\frac{\pi}{4} d^2 a - \left[ \frac{\pi}{4} d^2 h + \pi h_3^2 \left( \rho - \frac{h_3}{3} \right) \right]
\]

\[
= \frac{\pi}{4} d^2 a - \left[ a \left( \frac{6\sqrt{3} R_o^2}{4} - \frac{\pi}{4} d^2 \right) \right]
\]

\[
= a \left[ \frac{\pi}{4} \left( d^2 + d^2 \right) - \frac{6\sqrt{3} R_o^2}{4} \right]
\]

\[
= a \left[ \frac{\pi}{4} \left( d^2 + d^2 \right) - 6\sqrt{3} R_o^2 \right] \quad (3.167)
\]

From equation (3.83), we conclude that

\[
d_2 > \frac{24\sqrt{3} R_o^2}{\pi} \quad (3.168)
\]

Also

\[ d_{2\text{max}} = 2\sqrt{3} R_o \quad (3.169) \]

The percentage of the flash volume

\[
\frac{\pi (d^2 + d^2)}{24\sqrt{3} R_o^2} \left[ \frac{d^2 + d^2}{24\sqrt{3} R_o^2} - 1 \right] \times 100 \quad (3.170)
\]
3.3.8.4 Determination of Flash Volume and \( h_5 \) for Parabolic Punch

The Volume of the penetrated Punch

\[
V = \frac{\pi}{4} d^2 h_5 + \frac{\pi}{4} \left( \frac{d}{2} \right)^2 h_4
\]

\[
\therefore \quad \frac{\pi}{4} d^2 h_5 + \frac{\pi}{4} d^2 h_4 = \left( \frac{6}{3} R_o^2 - \frac{\pi}{4} d_2^2 \right) a
\]

or

\[
\frac{h_5}{\pi d_2^2} = \frac{a}{\left( \frac{24}{3} R_o^2 - \pi d_2^2 \right)} - \frac{h_4}{2}
\]

(3.171)

The Volume of flash

\[
\frac{\pi}{4} d^2 a - \left( \frac{\pi}{4} d^2 h_5 + \frac{\pi}{4} \left( \frac{d}{2} \right)^2 h_4 \right)
\]

\[
= \frac{\pi}{4} d^2 \left[ a - \left( h_5 + \frac{h_4}{2} \right) \right]
\]

\[
= a \left[ \frac{\pi}{4} \left( d^2 + d_2^2 \right) - 6\sqrt{3} R_o^2 \right]
\]

(3.172)

We conclude from this that

\[
d_2 \geq \left[ \frac{24\sqrt{3}}{\pi} R_o^2 - d^2 \right]^{\frac{1}{2}}
\]

(3.173)

Also the flash volume

\[
\text{Percentage of } \frac{\pi (d^2 + d_2^2)}{24\sqrt{3} R_o^2}
\]

\[
= \left[ \frac{\pi (d^2 + d_2^2)}{24\sqrt{3} R_o^2} - 1 \right] \times 100
\]

(3.174)
3.4 THEORETICAL RESULTS

3.4.1 Flat Shaped Central Punch

Numerical integration has been done to evaluate \( \dot{W}_{p1}, \dot{W}_{p2} \) and \( \dot{W}_{p3} \). While integrating over the range 'r' the lower limit of r was kept at 0.1 mm as suggested by Sagar (1980). Fig. 3.7 shows the variation of \( \bar{p}_m \) with \( d_2 \). Fig. 3.8 illustrates the variation of \( \beta \) with the change in \( d_2 \). Fig. 3.11 shows the web volume variation with \( d_2 \) and Fig. 3.12 gives the variation of \( h_4 \) with \( \beta \).

3.4.2 Conical Shaped Central Punch

The terms involving integrals have been solved by numerical integration. Fig. 3.9 shows the variation of \( \bar{p}_m \) with \( \beta \) for a given set of values of the parameters \( a, d \) and \( R_0 \). Fig. 3.10 contains curves of \( (\bar{p}_m)_{\text{min}} \) versus \( d_2 \). Fig. 3.11 shows the web volume variation with \( d_2 \) and Fig. 3.12 gives the variation of \( h_4 \) with \( \beta \).

3.4.3 Parabolic Shaped Central Punch

Fig. 3.13 shows the variation of \( \bar{p}_m \) with \( h_4 \). In Fig. 3.14 is plotted the variation of \( h_4 \) with \( h_4 \).
3.4.4 Spherical Shaped Central Punch

Fig. 3.15 shows the variation of $P_m$ with $\beta_2$. In Fig. 3.16 is drawn a curve depicting variation of $P_m$ with $d_2$ for a set of values of the parameters $a$, $d$ and $R_0$ with $\beta_2 = 90^\circ$ which is optimum value.

3.4.5 Comparison of Forging Pressures for various punch shapes

Fig. 3.17 shows the variation of $P_m$ with flash volume for various punch shapes. It is seen that for a given set of values of the parameters $d$, $a$, $m$ and $R_0$ the mean working pressure $P_m$ increases with decrease in flash volume. After a particular value of the flash volume the pressure increases tremendously. There are two possible reasons. One is either the void/gap of the hexagon cavity is completely filled and then there is no space left for the material to move anywhere. The other reason is that the gap is not completely filled and the punch touches the bottom die. Fig. 3.17 has been constructed by taking $\beta = 75^\circ$ for conical punch, $h_4 = 0.3d$ for parabolic punch and $\beta_2 = 90^\circ$ for a spherical punch.

3.5 COMPARISON OF THEORETICAL RESULTS WITH EXPERIMENTAL DATA

As the punch descends the load increases. The
maximum load at the end of the operation is the criterion for
the press selection and other engineering applications. In
order to determine the maximum working load the experimental
set up was provided with a cut off which stopped the press
when a desired height was achieved. The load on the press at
this point was noted. The load is dependent upon the
friction factor \( m \). Its value was experimentally determined
as suggested by Male and Cockcroft (1964), Pierre and Burney
(1974) and Avitzur and Kosher (1977). The theoretical and
average experimental non-dimensional forging pressures at
the point of cut off are given in tables 3.1 to 3.4. The
theoretical and experimental results showed good
agreement.

3.6 DISCUSSION

The acceptance of the proposed method as a
competitive production strategy is determined by the forging
pressure which dictates the capacity of the forging press
and its associated initial cost as well as the maintenance
cost. The forging pressure depends upon the shape of the
central punch as well as the thickness of the central disc
which is removed by trimming. An analysis is given below for
the proper selection of the shape of the punch.

Various punch nose shapes have been utilised in
the proposed method as given earlier. The nose shapes as
discussed earlier are conical, parabolic, spherical and flat. Table 2.11 shows the non-dimensional forging pressure for forging of nut blanks with each of the four different type of central punches. Analysis of the data reveals that the forging pressures for the conical, parabolic, spherical and flat punches are approximately in the ratio of 100 : 110 : 120 : 125. Table 2.12 shows that for these shapes the relative wastage of material in web is in the approximate ratio of 100 : 75 : 65 : 60. Thus the conical punch which requires the minimum pressure contributes to the maximum wastage of material.

The flat punch which is most efficient when material utilization is considered, requires higher press capacity because of increased forging pressure. The twin objectives of reduced forging pressure and material utilization are conflicting in nature and further investigation is required before finally selecting the optimal punch parameters.

Increase in working pressure increases the machinery cost as well as maintenance cost. Further increase in forging pressure increases die wear and decreases production rate as it necessitates frequent die changes. Wear of the die is caused by the rubbing action of the metal with the die surface. Wear is directly proportional to the normal pressure between the mating surfaces and the relative rubbing velocity (1959). Since the velocity is determined by
the production rate and is independent of the punch parameters, the die life may be assumed to be inversely proportional to the working pressure. However practical investigations of the data collected from industry reveals that in practice the die wears at a rate faster than what is predicted by this assumption.

The relation between die life and punch pressure is depicted in Table 3.5 and Figure 3.18. The information was collected from Chamber of Commerce and Industries, Northern Region (India).

**Suppose** Let

\[ p \quad \text{= working pressure on the die} \]
\[ C_m \quad \text{= Maintenance cost of the machine (per year)} \]
\[ H \quad \text{= Number of working hours per year} \]
\[ t_p \quad \text{= Meantime between two consecutive die replacements} \]
\[ r_1 \quad \text{= Production rate of the machine} \]
\[ C_d \quad \text{= Cost of the die (only the part being replaced)} \]
\[ t_d \quad \text{= Time to replace a die (set up time)} \]
\[ M_{cu} \quad \text{= Material cost per unit} \]

Then

\[ \text{Cycle time} = t_p + t_d \]
\[ \text{Production in one cycle} = r_1 \cdot t_p \]
Cost per cycle = \((t_p + t_d) \frac{C_m}{H} + cd\)

Cost per unit = \[\frac{(t_p + t_d) \frac{C_m}{H} + cd}{r_1 \cdot tp}\]

Total cost = \[-\left(1 + \frac{t_d}{tp} \frac{C_m}{H} + \frac{cd}{tp}\right) + M_{cu}\] (3.175)

The unit cost for nut blanks with various punches is given in Table 3.6. The punch shape does not contribute significantly to the unit cost of the nut blank. However, the flat punch is preferred because it leaves a clear line along which the material is subsequently sheared. The other shapes do not leave any clear line and makes the trimming operation more difficult.
Table 3.1: Theoretical and average experimental forging pressures of various heights for conical nose punch

<table>
<thead>
<tr>
<th>Sr.No.</th>
<th>Nut height (a) (mm)</th>
<th>Flat to flat dist. (3 x b) (mm)</th>
<th>Dia of the punch (mm)</th>
<th>Experimental non-dimensional forging pressure (average)</th>
<th>Theoretical non-dimensional forging pressure ($p_m/a_0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>17.30</td>
<td>6.90</td>
<td>2.00</td>
<td>2.25</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>17.30</td>
<td>6.90</td>
<td>2.25</td>
<td>2.50</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>17.30</td>
<td>6.90</td>
<td>2.50</td>
<td>2.80</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>19.25</td>
<td>9.40</td>
<td>2.60</td>
<td>3.10</td>
</tr>
<tr>
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<td>9</td>
<td>19.25</td>
<td>9.40</td>
<td>3.00</td>
<td>3.40</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>19.25</td>
<td>9.40</td>
<td>3.30</td>
<td>3.80</td>
</tr>
</tbody>
</table>

* Disc dia also.

Oil used: S.A.E. 40
Material: Aluminium

Punch Nose Shape: Conical
Cone Angle $\beta$: 75°
$m$: 0.2
Table 3.2: Theoretical and average experimental forging pressures of various heights for parabolic nose punch

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Nut height (a) (mm)</th>
<th>Flat to flat dist. (3 x b) (mm)</th>
<th>Dia of the punch (mm)</th>
<th>Experimental non-dimensional forging pressure (average)</th>
<th>Theoretical non-dimensional forging pressure ($p_m/\sigma_o$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>17.30</td>
<td>6.90</td>
<td>2.20</td>
<td>2.30</td>
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<td>17.30</td>
<td>6.90</td>
<td>2.40</td>
<td>2.60</td>
</tr>
<tr>
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<td>11</td>
<td>17.30</td>
<td>6.90</td>
<td>2.70</td>
<td>3.00</td>
</tr>
<tr>
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<td>8</td>
<td>19.25</td>
<td>9.40</td>
<td>2.80</td>
<td>3.10</td>
</tr>
<tr>
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<td>9</td>
<td>19.25</td>
<td>9.40</td>
<td>3.30</td>
<td>3.70</td>
</tr>
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<td>19.25</td>
<td>9.40</td>
<td>3.60</td>
<td>4.20</td>
</tr>
</tbody>
</table>

* Disc dia also.

Oil used: S.A.E. 40
Material: Aluminium
### Table 3.3: Theoretical and average experimental forging pressures at various heights for spherical nose punch

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Nut height (a) (mm)</th>
<th>Flat to flat dist. (3 x b) (mm)</th>
<th>Dia of the punch (mm)</th>
<th>Experimental non-dimensional forging pressure (average)</th>
<th>Theoretical non-dimensional forging pressure ($p_m/\sigma_o$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>6.90</td>
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<td>2.70</td>
</tr>
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<td>2.90</td>
</tr>
<tr>
<td>3</td>
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<td>6.90</td>
<td>2.90</td>
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</tr>
<tr>
<td>4</td>
<td>8</td>
<td>19.25</td>
<td>9.40</td>
<td>3.10</td>
<td>3.60</td>
</tr>
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<td>9</td>
<td>19.25</td>
<td>9.40</td>
<td>3.50</td>
<td>4.00</td>
</tr>
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<td>10</td>
<td>19.25</td>
<td>9.40</td>
<td>3.90</td>
<td>4.50</td>
</tr>
</tbody>
</table>

* Disc dia also.

Oil used: S.A.E. 40
Material: Aluminium

Punch Nose shape: Spherical
Height $h_j$: 0.5d
$\mu$: 0.2

Note: Nut height $a$, flat to flat distance $3 \times b$, Dia of the punch, Experimental non-dimensional forging pressure (average), Theoretical non-dimensional forging pressure $p_m/\sigma_o$. 

Theoretical non-dimensional forging pressure ($p_m/\sigma_o$) is calculated using the formula:

$$ p_m/\sigma_o = \frac{F}{A\sigma_o} $$

where $F$ is the force applied, $A$ is the area of the punch, and $\sigma_o$ is the experimental value of the forging pressure.
Table 3.4: Theoretical and average experimental forging pressures at various heights for flat nose punch

<table>
<thead>
<tr>
<th>Sr.No.</th>
<th>Nut height (a) (mm)</th>
<th>Flat to flat dist. (3 x b) (mm)</th>
<th>Dia of the punch (mm)</th>
<th>Experimental non-dimensional forging pressure (average)</th>
<th>Theoretical non-dimensional forging pressure ($p_m / \sigma_o$)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>6.90</td>
<td>2.50</td>
<td>2.70</td>
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<td>3.70</td>
<td>4.20</td>
</tr>
<tr>
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<td>19.25</td>
<td>9.40</td>
<td>4.10</td>
<td>4.70</td>
</tr>
</tbody>
</table>

* Disc dia also.
Table 3.5 Die life with respect to increase in load

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Load increase in percentage</th>
<th>Die life in percentage</th>
</tr>
</thead>
<tbody>
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<td>1</td>
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<td>100</td>
</tr>
<tr>
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<td>5</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
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<td>60</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 3.6 Unit cost of nut blank for various nose shaped punches (calculations in Appendix 1)

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Unit cost in paise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conical punch</td>
</tr>
<tr>
<td>1</td>
<td>6.34065</td>
</tr>
</tbody>
</table>
1. Central upper intender punch
2. Outer punch (hexagonal)
3. Work piece
4. Central lower punch
5. Lower die (hexagonal)

SECTION A-A
(NUT FORMING OPERATION)

FIG. 3.1
FIG. 3.2

Hexagonal Die with Flat Shape Punch.
FIG. 3.3

Die Geometry
FIG. 3.4

Hexagonal die with conical nose punch.
FIG. 3.6

Hexagonal die with parabolic nose punch
Hexagonal Die with Spherical Nose Punch
FIG. 3.7

Variation of $T_m$ with $d_2$ for flatnose punch.

$a = 10.0$
$d = 6.9$
$n = 0.2$
$R_0 = 5.0$
FIG. 3.8

Web/Flash volume $V$ vs. $d_2$ for flat nose punch.

$V \cdot d_2$

$a = 10.0$
$d = 6.9$
Variation of $\bar{F}_{M}$ with $\beta$ for conical nose profile.
FIG. 3.10

curve of $\bar{F}_m$ vs. $d_2$ for conical nose punch.
FIG. 3.11
percentage web volume v/s d_2 for conical nose punch
a = 10.0
\(d = 6.9\)
\(d_2 = 17.30\)

Variation of \(h_1\) with \(\beta\) for conical nose punch.
Variation of $\bar{F}_m$ with $h_4$ for parabolic nose profile

$\bar{F}_m$, $h_4$

$\bar{F}_m$ vs $h_4$

$\bar{F}_m = \frac{1}{m} \theta$

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FIG. 3.14

Variation of $h_5$ with $h_4$ for parabolic nose punch

\begin{align*}
a &= 10.0 \\
d &= 6.9 \\
d_2 &= 17.3
\end{align*}
Variation of \( F_n \) with \( \beta_2 \) for spherical nose punch
From \( \frac{V}{\pi} \) vs. \( d_2 \) for spherical nose prisms.
From Fig. 3.17 flash volume for conical, parabolic, spherical and flat punches.

Parameters:
- $a = 10.0$
- $d = 8.0$
- $R_o = 5.0$
- $\theta = 75^\circ$
- $h_3 = 0.5d$
- $h_4 = 0.3d$
- $m = 0.2$

Legend:
1. Conical
2. Parabolic
3. Spherical
4. Flat
RELATION BETWEEN LOAD AND DIE LIFE.

FIG. 3.18