Appendix C

Procedure for Simulation

1.1 C Introduction
In the first section of this appendix, worksheet functions for normal and chi-square distributions with examples are discussed. Second section describes the procedure for simulation with the help of three flow diagrams.

Worksheet function for normal and chi-square distributions are explained as given under.

Normal distribution
Worksheet function is used in Microsoft Excel. It is used to get the probabilities in normal distribution and chi-square distribution, which can also be obtained directly from the standard Tables, available in statistical books.

As shown in section 3.3, worksheet functions for normal distribution can be expressed as under.

\[ a(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-x^2/2} \, dx \]

Probability that a variable with normal distribution \( N(0, 1) \) will not exceed ‘z’

Worksheet functions NORMSDIST (z) is equivalent of function, \( a(z) \)

Example 1
Function, NORMSDIST (z) returns the probability, that a variable with normal distribution \( N(0, 1) \) will be equal to or less than z.

For \( z = 2 \), NORMSDIST (2) = 0.97725 (This value can also be seen from standard normal distribution Table available in many statistical books)

Similarly, worksheet function, NORMSINV (p) is equivalent of function, \( a'(p) \), where,

\[ a'(p) \] returns z, that a variable with \( N(0,1) \) will have the value less than or equal to p.

Example 2
For \( p = 0.97725 \), NORMSINV (0.97725) = 2
Chi-square distribution

As shown in section 3.4, worksheet functions for chi-square distribution can be expressed as under.

Worksheet functions CHIDIST ($U^*$, $v$) is equivalent of function $\beta (U^*, v)$, where,

$\beta (U^*, v) = \text{probability of a variable with chi-square distribution with } v \text{ degrees of freedom, exceeding a value of } U^*.$

Example 1
For sample size (n) of 4, history (H) of 4 and $U^* = 26.3$
Degree of freedom ($v$) = $n*H = 16$, confidence level = 95%

CHIDIST (26.3, 16) = 0.05

Similarly, inverse worksheet function CHIINV ($p$, $v$) is equivalent of function, $\beta' (p, v)$, where,

$\beta' (p, v) = \text{returns } U^* , \text{ that a variable with chi-square distribution with } v \text{ degrees of freedom, exceeding a value of } U^*.$

Example 2
For sample size (n) of 4, history (H) of 4 and $p = 5$
Degree of freedom ($v$) = $n*H = 16$

CHIINV (0.05, 16) = 26.3

1.2C Procedure for simulation
The program is prepared to calculate the ARLs for new $\bar{X}$ chart, using strategy CSQ. The Simulation Run of 10000 is considered to calculate the ARLs of new $\bar{X}$ chart. Random numbers are generated, using Equation 4.2 (chapter 4) suggested by Box and Muller (1958). Simulation procedure to calculate the ARLs is shown with the help of three flow diagrams described as under.

1.2.1C Procedure for determining status of the process
Figure 1C describes the subroutine to return the state of system. It reads X1, X2, X3 and X4 and evaluates sample mean ($\bar{X}$). It takes values of UCL, LCL, UWL and LWL from
It compares $\bar{X}$ with UCL, LCL, UWL and LWL and decides the state of the process as per the following rules:

(i) If the value of $\bar{X}$ lies within warning limits, the process may be assumed to be under control.

(ii) If the value of $\bar{X}$ lies beyond control limits, the process may be assumed to be out-of-control.

(iii) If the value of $\bar{X}$ lies between warning limits and control limits, calculate statistic, $U$, and if $U < U^*$, the process may be assumed to be under control.

(iv) If the value of $\bar{X}$ lies between warning limits and control limits, calculate statistic, $U$ and if $U > U^*$, the process may be assumed to be out-of-control.

It returns the state of the system as ‘in-control’ and ‘Out-of-control’. Both are Boolean variables.

1.2.2C Procedure to compute run length (RL)

Figure 2C describes the subroutine to find the run length (RL). It reads $X_1$, $X_2$, $X_3$ and $X_4$ and evaluates sample mean ($\bar{X}$). Two states of the process are already obtained in subroutine shown in Figure 1. The subroutine shown in Figure 2 continues to run until out-of-control state is obtained and that is called run length (RL).

1.2.3C Procedure to compute average run length (ARL)

Figure 3C shows the main program to calculate the average run length (ARL) of the new $\bar{X}$ chart for predetermined simulation run (SR). One can assume any value of the simulation run but in the program, it is taken as 10000. Average Run Length (ARL) is computed by taking the average of Run Lengths (RLs) found in each simulation run. In the procedure (Figure 3C), it is calculated as:

$$ARL = \frac{\text{SUM}}{\text{Simulation Run}}.$$
Figure 1C Procedure for determining state (i) of the process

Start

Procedure Get state (i)

Read X1, X2, X3, X4

\[ \bar{x} = \frac{X1 + X2 + X3 + X4}{4} \]

Yes

LCLx < \( \bar{x} \) or
UCLx > \( \bar{x} \)

NO

\( \bar{x} \) lies between center line and warning limits

NO

\( \bar{x} \) lies between Warning limits and Control limits

Yes

Evaluate "U"

\[ U > U^* \]

NO

State = Out-of-Control

Yes

State = In-control

Return State
Figure 2C Procedure to compute run length (RL)
Figure 3C Procedure to compute average run length (ARL)

Start

Read $n$, $H$, $TM$, $TSD$, $K$, $L$, $U$ and Simulation Run (SR)

$UCL = TM + L \times TSD / \sqrt{n}$, $LCL = TM - L \times TSD / \sqrt{n}$,
$UWL = TM + K \times TSD / \sqrt{n}$ and $LCL = TM - K \times TSD / \sqrt{n}$,

$SUM = 0$

$i = 1$

$K = Get RL()$

$SUM = SUM + K$

Yes

$i < SR$

NO

$ARL = SUM / SR$

Stop