Chapter 5

Tests of the Monte-Carlo code

The Monte-Carlo scheme to perform the radiative transfer computations in the polar cap region has been discussed in the last chapter. Before presenting the results of application of this code, in the next chapter to the problems of our interest, here we discuss some basic results that help understand the trends observed in the simulations, and also describe some of the tests and checks that we have performed. While two different geometrical settings - a slab and a cylinder - are in common use in similar computations, we have performed simulations only for the slab 1-0 geometry to test our code and to produce the results presented in this chapter. The final results including all the three processes of cyclotron absorption, cyclotron emission and magneto-compton scattering for slab 1-0 and cylindrical column will be presented in the next chapter.

5.1 Optical depth profiles

The cyclotron lines features produced in these two geometries are found to be significantly different, for which one reason is the different angular variation of optical depth $\tau(\omega_i, \theta_i)$ in these two cases.

We confine ourselves here to a description of the scattering optical depth. The dependence of the absorption cross section on $\omega$ and $\theta$ are very similar to those of scattering cross section and hence the absorption optical depth shares the same qualitative features. The scattering optical depth profile $\tau_{sc}(\omega_i, \theta_i)$ is related to the scattering line profile $P_{sc}(\omega_i, \theta_i)$ as:

$$
\tau_{sc}(\omega_i, \theta_i) = \frac{P_{sc}(\omega_i, \theta_i)}{\sigma_T} \tau_T(\theta_i)
$$

(5.1)
Figure 5.1: The scattering optical depth profile $\tau_{sc}(\omega_i, \mu_i) = (\sigma_{sc}(\omega_i, \mu_i)/\sigma_T)\tau_T(\mu_i)$ for a magnetic field $B' = .03$ and for five viewing angles $\mu_i = .1, .26, .5, .76, .985$. The top panel shows the optical depth for slab geometry with $\tau_T(\mu_i = 1) = 10^{-3}$ and the bottom panel shows the optical depth for the cylindrical geometry with $\tau_T(\mu_i = 0) = 10^{-3}$.
Figure 5.2: The figure is a zoomed view of the fundamental and the second harmonics in Fig.5.1 for slab and cylindrical geometry.
where $\tau_T(\theta_i)$ is the Thompson optical depth in a given direction $\theta_i$. It is clear from Eq.5.1 that there are two parts which contribute to the scattering optical depth profile $\tau_{sc}(\omega_i, \theta_i)$: one is $P_{sc}(\omega_i, \theta_i)/\sigma_T$ and the other is $\tau_T(\theta_i)$. The first part depends solely on the scattering cross-section $\sigma_{sc}(\omega_i, \theta_i)$ and has no connection to geometry and the second part $\tau_T(\theta_i)$, the angular dependence of which is determined only by which the geometry. For the slab geometry (with its normal parallel to the magnetic field) the Thompson optical depth $\tau_T(\theta_i)$ in any direction $\theta_i$ can be related to the Thompson optical depth along the slab normal $\tau_T(\theta_i = 0)$ by the following equation

$$\tau_T(\theta_i) = \frac{\tau_T(\theta_i = 0)}{\cos\theta_i} \text{ (Slab)} \quad (5.2)$$

The scattering line profile $P_{sc}(\omega_i, \theta_i)$ is resonant in energy $\omega_i$ and is strongly dependent on the viewing angle $\theta_i$. It is interesting to see the net effect of the variation of viewing angle $\theta_i$ on the optical depth profile $\tau_{sc}(\omega_i, \theta_i)$. Close to $\theta_i \to 0$ the line profile $P_{sc}(\omega_i, \theta_i)$ has shallower and broader features (Fig.4.2). The Thompson optical depth $\tau_T(\theta_i)$ has its minimum value (Eq.5.2) along $\theta_i \to 0$ so both factors tend to reduce the scattering optical depth $\tau_{sc}(\omega_i, \theta_i)$ along the magnetic field direction $\theta_i \to 0$. In the direction perpendicular to the field $\theta_i \to \pi/2$ the line profile $P_{sc}(\omega_i, \mu_i)$ has very sharp and strong resonance in energy $\omega_i$ and $\tau_T(\theta_i \to \pi/2) \to \infty$ so the optical depth perpendicular to the magnetic field is very high.

In the case of a cylinder geometry the Thompson optical depth $\tau_T(\theta_i)$ is written in term of optical depth $\tau_T(\theta_i = \pi/2)$ perpendicular to the cylindrical axis

$$\tau_T(\theta_i) = \frac{\tau_T(\theta_i = \pi/2)}{\sin\theta_i} \text{ (Cylinder)} \quad (5.3)$$

In this case the the two factors $P_{sc}(\omega_i, \mu_i)/\sigma_T$ and $\tau_T(\theta_i)$ have an opposite behaviour. Close to $\theta_i \to 0$, $P_{sc}(\omega_i, \mu_i)$ has broad and shallow features but in this direction the Thompson optical depth $\tau_T(\theta_i)$ has a large value. In the direction perpendicular to the magnetic field $P_{sc}(\omega_i, \mu_i)$ has very strong and sharp resonances and the Thompson optical depth is minimum in this direction. So in the cylindrical geometry these two factors compensate each other and the optical depth spans a range of intermediate values.

The behaviour of the optical depth profile $\tau_{sc}(\omega_i, \mu_i)$ for the slab and the cylindrical geometry are shown in Fig.5.1, for the Thompson optical depth $\tau_T(\theta_i = 0) = 10^{-3}$ (for slab), $\tau_T(\theta_i = \pi/2) = 10^{-3}$ (for cylinder). A zoomed view of the fundamental and the second harmonics is shown in Fig.5.2. The optical depth profiles $\tau_{sc}(\omega_i, \theta_i)$ are evaluated at magnetic field $B' = .03$ and for five viewing angles
5.2: Role of 0-0 and other scatterings in the formation of cyclotron lines

µi = .1, .26, .5, .76, .985. For the fundamental in the slab geometry (top left panel Fig.5.2) it can be seen that τ(ωi, θi → π/2) is greater than τ(ωi, θi → 0) in the resonance region 14 – 16 KeV. This trend changes somewhat in the case of cylindrical geometry (top right panel Fig.5.2). τ(ωi, θi → 0) rises and becomes comparable to τ(ωi, θi → π/2) in the energy range ωi ∈ (14 – 16) KeV. Considering these facts it is expected that the fundamental should be very broad at low angles (since τ(ωi, θi → 0) is broad) for a cylindrical geometry and should be very deep close to 90° in a slab geometry. For the second harmonic, however τ(ωi, θi → π/2) remains larger than τ(ωi, θi → 0) for both the cases (bottom left and bottom right panel Fig.5.2). In a cylindrical geometry relatively shallow dependence of optical depth on propagation angle near fundamental resonance causes the photon scattering to be more isotropic than in a slab geometry, where photons can escape more easily at low angles.

5.2 Role of 0-0 and other scatterings in the formation of cyclotron lines

To test and illustrate the role of scattering we performed a simulation for a slab 1-0 geometry and stored the intermediate results in order to study the photon history during radiative transfer. The simulation is performed for a slab height of h = 100 cm and radius r = 1 Km, a constant magnetic field B' = .03, a Thompson optical depth of 10⁻³ normal to the slab, and an electron temperature Te = 5 KeV. In this simulation, only the scattering process is incorporated and absorption is not taken into account. To see the effect of pure scattering, transition photons are also not injected. The parameters corresponding to each scattering are stored. To show how many times a particular number of scatterings nscat occur, we plot the frequency of occurrence f(nscat) of the number of scatterings nscat in fig. 5.3. A total of 30000 scatterings have been used in this figure. It is seen that in general the frequency f(nscat) decreases sharply with the number of scatterings. f(nscat) ≈ 100 for 1 scattering ~10 for nscat = 10, and dropping to ~ 1 for nscat = 300.

It is clear from the Fig.5.3 that small number of scatterings nscat < 10 occur more frequently. These correspond to scattering in the wings of the cyclotron lines. Larger number of scatterings in the range 10 – 300 occur with moderate frequency, primarily near the centre of the cyclotron lines where the average optical depth τsc is of order 10 leading to hundreds of scatterings as the number of scatterings scale as τsc² + τsc.
5.2: Role of 0-0 and other scatterings in the formation of cyclotron lines

Figure 5.3: In this figure the X axis represents the number of scatterings $n_{\text{scatt}}$ of an injected photon before escape. Y axis is the frequency of a given $n_{\text{scatt}}$. The simulation is performed in a slab 1-0 geometry for uniform magnetic field $B' = 0.03$, $T_e = 5$ KeV and $\tau_T(\mu_i = 1) = 10^{-3}$. The absorption process is not considered and transition photons are not re-injected in the Monte-Carlo scheme.
5.2: Role of 0-0 and other scatterings in the formation of cyclotron lines

The contribution to the formation of cyclotron lines features is different for scatterings involving different final states. The scattering in which electron’s initial \( n_i \) and final state \( n_f \) are the ground state, is called the 0-0 scattering. This scattering is mainly responsible for the very complex shape of the fundamental. In 0-0 scattering photons exchange energy with only the parallel motion of the electron while in other scatterings \( (n_f \neq 0) \) a photon loses a part of its energy in the excitation of the Landau state \( \omega_{nf}^{\text{res}} \) and the rest is exchanged with the parallel motion of the electron. A plot of the number of scatterings as a function of \( n_f \) is shown in Fig.5.4. The figure shows that the number of scatterings for \( n_f = 0 \) far exceeds that for any other level for the typical values of physical parameters \( (B' = 0.03, T_e = 5 \text{ KeV}) \). For the total of 30000 scatterings in our simulation the ratio of number of scatterings corresponding to final Landau levels \( n_0 : n_1 : n_2 : n_3 : n_4 : n_5 \) is \( 29697 : 236 : 47 : 13 : 6 : 1 \), clearly indicating the dominance of 0-0 scattering over others.

It is also interesting to see which scatterings \( (n_i = 0 \rightarrow n_f = 0,1,3..) \) dominantly occur on which harmonic. This may be inferred seen from the individual cross-sections of these scatterings (Fig.3.4). The 0-0 scattering, 0-1 scattering and 0-2

![Figure 5.4: This figure shows a plot of number of scatterings in which a particular Landau level \( n_f \) is excited. The number of scatterings is plotted on Y axis and \( n_f \) is plotted on the X-axis. The input parameters for the simulation are the same as in Fig.5.3.](image)
scattering dominate at the first, second and third harmonics respectively over a wide range of angles except at $\theta_i \to 0$. In exciting higher Landau states, the loss of energy causes the scattered photon to move to lower resonances. Photons near higher harmonics disappear from higher harmonics in very few scatterings and appear at the fundamental. The energy exchange of the photon in a 0-0 scattering is the minimum and at the fundamental the 0-0 scattering has a very high probability. Therefore once the photon enters the central part of the fundamental resonance it remains trapped there resulting in a large number of scatterings until a scattering by an electron of high parallel momentum throws it out of resonance. The important conclusion can be drawn from the above discussion is that the 0-0 scattering occurs the maximum number of times and it occurs mostly at the fundamental, so it is the fundamental where most of the Comptonization and angular re-distribution of photons take place.

5.3 Energy and angular re-distribution due to 0-0 and other scatterings

After the scattering photons are redistributed in energy and angle. The distribution of the post-scattering energy of photons for 30000 scatterings is shown in Fig.5.5. It is interesting to note that we injected a flat continuum $n_p(\omega_i) = 1$ but the scattered photons are re-distributed in energy such a way that the resultant energy distribution after scatterings peaks at the resonance energies $\omega_{1\text{res}}, \omega_{2\text{res}}, \ldots$. This happens because among the scattered photons only those with energy close to resonance are further scattered many times. Scattered photons with energy away from resonance peaks escape after a few scatterings because of low optical depth far from resonances. From Fig.5.5 it can be seen that energy distribution has a much larger peak at the first resonance $\omega_{1\text{res}}$ than at higher ones ($\omega_{2\text{res}}, \omega_{3\text{res}}, \ldots$). This is due to the high optical depth at first resonance caused by 0-0 scattering. Additionally, some photons after 0-1 scattering at second harmonics can also jump to the fundamental. If transition photons are injected than they too would further add to the first peak since most of the transition photons generated have energy close to the fundamental energy $\omega_{1\text{res}}$.

The distribution of initial electron parallel momentum $p_i$ selected for the 30000 scatterings is shown in the Fig.5.7. We display two different curves: 1) the distribution of parallel momentum $p_i$ of the electrons for the first scattering of injected photons and 2) that for all the scatterings. It should be noted that only those pho-
Figure 5.5: This figure shows the distribution of energy $\omega_f$ of the photons after scattering. The input parameters for the simulation are the same as in Fig. 5.3.

Figure 5.6: The distribution of the cosine of the photon propagation angle $\mu_f$ after scatterings. The input parameters for the simulation are the same as in Fig. 5.3.
5.3: Energy and angular re-distribution due to 0-0 and other scatterings

Figure 5.7: The distribution of the momentum $p_i$ of the electrons chosen to interact with the photons at the first scattering after injection (blue solid line) and in all scatterings (green dashed line). The input parameters for the simulation are the same as in Fig. 5.3.

Photons are scattered heavily which have rest frame energy $\omega'_i$ close to the resonance energy $\omega_{\text{res}}^i$. As mentioned earlier the distribution of the energy after scattering $\omega_f$ peaks at resonance energies in the lab frame. This suggest that in a large number of scattering events the parallel momentum $p_i$ is close to zero, for which the resonance energies in the rest frame and the lab frame are nearly equal. This is corroborated by the green dashed curve for all scatterings in Fig. 5.7, which shows a large excess near $p_i \to 0$ over the blue solid curve representing only the first scattering. In conclusion it can be stated that very large number of scatterings the electrons has very small parallel momentum $p_i \to 0$.

Photons are re-distributed in angle after scattering. The distribution of the cosine of angle $\mu_f$ after scatterings is plotted in Fig. 5.6. We have injected an isotropic continuum which is flat in $\mu_i$, but after scatterings photons are depleted in the angle $\mu_i \to 0$, $\theta_i \to 90^\circ$ and peak develop in the directions $\mu_i = \pm 1$. 
5.4 Evolution of line depth with number of scatterings

We will now describe the evolution of spectra with increasing number of allowed scatterings in our simulations. Even if the transition photons are not injected in the Monte-Carlo scheme the large number of scatterings at the fundamental start filling it in. To see how the filling of the cyclotron line occurs we have run a set of simulations without the injection of transition photons and ignoring absorption. As in the previous section, the simulations are performed for slab $1 - 0$ ($h_c = 100$ cm, $r_c = 1$ Km) for a constant magnetic field $B' = 0.03$, $T_e = 5$ KeV and constant electron number density $n_e = 10^{19}$ cm$^{-3}$. Three simulations are performed: in one of these a maximum of 10 scatterings were allowed and in the second a maximum of 11 scatterings were allowed. If the number of scatterings exceed this then the photon is forced to escape the simulation volume. The third simulation is performed with all scattering allowed. Results of these three simulations are compared in Fig. 5.8 for the fundamental and in Fig. 5.9 for the second harmonic. Four different viewing angle bins are shown. It can be seen that in Fig. 5.8 the curves for 10 and 11 scatterings have noticeable difference and for all scattering the fundamental is filled significantly. Comparing the different angles bins it can be observed that the filling of the line is more for smaller viewing angles. The reason for this is the low optical depth of the slab for smaller angles. If a large number of scatterings are allowed then in small angle bins (Fig. 5.8) the photon escape not only in the wings but also at the central part of the line due to low optical depth. However at larger angles the optical depth is quite high, preventing the escape of photons near the line centre. It can also be seen that the filling of the lines at higher angles are not symmetric. For example in the angle bin $0.25 < \mu < 0.50$ asymmetry in the filling of the line is very clear when all scatterings are allowed. The asymmetric filling occurs due to the relativistic cutoff in the line profiles $P_{sc}(\omega_i, \mu_i)$. The line profile $P_{sc}(\omega_i, \mu_i)$ rises gradually below the relativistic cutoff energy $\omega_{cut}^i$ and drops abruptly just above the relativistic cutoff energy. This feature is clearly reflected in the spectra for high angle bins shown in fig 5.8, for all allowed scatterings.

Spectra corresponding to the second harmonic are plotted in Fig. 5.9. The behaviour of the second harmonic is clearly significantly different from that of the fundamental (Fig. 5.8). In the case of the second harmonic it can be seen that results for maximum 10, 11 and all scatterings are almost the same indicating that very small number of scatterings occur at the second harmonic in comparison to the fundamental.
Figure 5.8: Comparison of spectra after different number of scatterings. The spectra are plotted for the fundamental in 4 angle bins, in the four different panels. The blue solid curves represent spectra in which maximum 10 scatterings are allowed, the green dashed curves are the spectra for maximum of 11 scatterings. The red dotted curves result when all scatterings are allowed. The simulation is performed in a slab 1-0 with constant magnetic field $B' = 0.03$, $T_e = 5$ KeV, $\tau_T(\mu_i = 1) = 10^{-3}$. Absorption is not considered and transition photons are not re-injected in the Monte-Carlo scheme.
Figure 5.9: The same as Fig. 5.8 but for the second harmonic.
5.5 Effect of transition photons

To see the effect of transition photons on cyclotron line features the simulation for slab 1-0 is performed with transition photons injected into the Monte-Carlo Scheme. The simulation is performed for the same input parameters as in the previous sections. The spectra from two different simulations without and with transition photons are plotted in Fig.5.10 (fundamental) and Fig.5.11 (second harmonic). It can be seen from the Fig.5.10 that the filling of the fundamental due to transition photons is more in lower angle bins than at higher angles and the reason is the same as already discussed that the optical depth in low angle bins is low so the transition photons can easily escape. At the second harmonic (Fig.5.11) the spectra with and without transition photons have very similar shapes implying that relatively small number of transition photons are generated at the second harmonic for the parameter space chosen in our simulations.

5.6 Tests of the Monte carlo code

5.6.1 Escape probability

First, we performed a test to confirm that the escape probability is modelled properly in our code. The simulation was performed for a thin slab 1-0 ($r_c = 1$ Km, $h_c = 100$ cm) for a uniform magnetic field $B' = .03$, $T_c = 5$ Kev, $\tau_T(\mu_i = 1) = 10^{-3}$. The escape probability of a photon traversing an optical depth $\delta \tau(\omega_i, \mu_i)$ is given by $\exp(-\delta \tau(\omega_i, \mu_i))$. We perform our simulations in real space so this corresponds to $\delta \tau(\omega_i, \mu_i) = \frac{\delta \lambda}{\lambda} = n_e \langle \sigma_{sc}(\omega_i, \mu_i) \rangle f(\omega) \delta \lambda$. We generate the free paths $\delta \lambda$ in our simulation and store the corresponding $\delta \tau(\omega_i, \mu_i)$ values, the distribution of which is plotted in Fig.5.12. The blue curve is the actual distribution of $\delta \tau$ produced by the Monte-Carlo simulations and green dots represent the evaluation of the function $\exp(-\delta \tau(\omega_i, \mu_i))$. The two results match well, indicating that the escape probability is modelled accurately in our code.

5.6.2 Pencil beam injection test

Next, we performed a pencil beam injection test to confirm that the intensity removed from it is in accordance with the analytical expectation from radiative transfer. We injected a beam of photons in a very small angle bin $\mu_i \pm \delta \mu_i$ with the energy
Figure 5.10: The spectra, with transition photons injected, plotted for fundamental in 4 angle bins, one in each panel. The solid lines show the spectra without transition photons and the dashed lines are for those including transition photons.
Figure 5.11: The same as Fig. 5.10 but for the second harmonic.
Figure 5.12: The distribution of the optical depth $\tau$ corresponding to the free paths generated for different scatterings. The solid blue line shows the distribution generated in the Monte-Carlo simulation. The points in green represent the probability function $\exp(-\tau)$ scaled to the total number of scatterings. The input parameters for the simulation are the same as in Fig. 5.3.
5.6: Tests of the Monte carlo code

in full continuum range (1-100 KeV). The escaping photons were collected in the same angle range $\mu_i \pm \delta \mu_i$. During the propagation many photons are scattered out of the beam. The output spectrum of the photons remaining in the pencil beam is expected to be given by just the optical depth profile as

$$I(\omega_i, \mu_i) = I_0 \exp(-\tau(\omega_i, \mu_i))$$

where $I_0$ is the intensity of the injected continuum. The optical depth $\tau$ depends upon the energy $\omega_i$, and the viewing angle $\mu_i$ as discussed in sec.5.1. We have here

$$\tau(\omega_i, \mu_i) = n_e \langle \sigma_{sc}(\omega_i, \mu_i) \rangle \frac{h}{\mu_i}$$

where $h$ is vertical height of the slab and other symbols have their usual meaning.

Fig. 5.13 compares the result of the analytical expectation from Eq. 5.4 with that of a Monte Carlo run for constant magnetic field $B' = 0.03$, $\tau_T = 10^{-3}$ and electron temperature $T_e = 5$ KeV. The left panel of the figure is for $\mu_i = 0.3$ and the right panel is for $\mu_i = 0.5$. The dashed lines represent the photon counts corresponding to the specific Intensity $I(\omega_i, \mu_i)$ derived from (Eq. 5.1). The spectra obtained from the Monte-Carlo radiative transfer (solid lines) match well with the analytical expectation. This test confirms that photon removal from the beam due to scattering is accurately modelled in our code.

5.6.3 Comparison with the results of other authors

One way to validate our Monte carlo code would be to cross compare our results with those obtained by other authors. There are two published works with similar computations - Araya and Harding (1999) and Schonherr (2007), both of which use the same code as the computation kernal. Our results resemble those presented in the above papers very closely, but are not an exact match. However certain errors have recently been found in the computation kernal used in the above papers (Jorn, Wilms, private communication) and hence the differences seen from our results may not be unexpected. During the past few months we became aware of another similar code in development by Fritz schwarm, a Ph.D student at Sternwarte Astronomical Institute Bamberg, that attempts to correct the errors in the kernal used earlier. While this is still a work in progress, we were able to make some preliminary cross comparisons of our results with those produced from the code by Schwarm. In Fig. 5.14 we compare the spectra produced by both codes for a slab
Figure 5.13: (a) Comparison of the monte-Carlo and analytical estimates of the pencil beam spectra. The solid line shows the Monte Carlo results and the dashed line the analytical expectation. Panel (a) is for $\mu_i = 0.1$ and panel (b) for $\mu_i = 0.5$.
1-0 geometry, $B' = 0.03$, $T_e = 5$ KeV, $\tau_T(\mu_i = 1) = 10^{-3}$. Except for a slightly more prominent emission wings produced by our code near $\mu = 1$, the match is excellent. Considering that the code by Fritz Schwarm is still under development and not all issues have been fully addressed, we consider this match satisfactory enough to proceed with further application of our code, as presented in the next chapter.
Figure 5.14: Comparison of the results of our code (red curves) with those of Fritz Schwarm (black curves). The simulation is performed for slab 1-0 with uniform magnetic field $B' = 0.03$, $T_e = 5$ KeV, $\tau_T(\mu = 1) = 10^{-3}$. Absorption is not included. The transition photons are re-injected and propagated in the simulations.