INTRODUCTION
INTRODUCTION

Let \( \{X_n, n \geq 1\} \) be a sequence of independent and identically distributed (i.i.d) random variables having common distribution function (d.f.) \( F \). Define

\[
M_n^{(r)} = \text{r}^{\text{th}} \text{ largest} \ \{X_1, X_2, \ldots, X_n\}
\]

\[
Z_n = (M_n^{(r)} - b(n))/a(n)
\]

where \( \{a(n)\} \) and \( \{b(n)\} \) are some sequences of real numbers with \( a(n) > 0 \). Study of extremes of random phenomena was initiated by the pioneering works of Frechet (1927), Fisher and Tippett (1928). Since then many researchers have enriched the field of asymptotic theory of order statistics. The details are available in the books by David (1981), Pancheva (1984), Galambos (1987), and Falk, Husler and Reiss (2004). Some interesting results are due to Mohan and Ravi (1993), Vasudeva (1999), Hebbar and Vadiraja (1999).

In most of the investigations found in the literature, \( F \) is assumed to be continuous. The lack of attention given to discrete case is partly due to the fact that the distribution of maximum of a sample of size ‘\( n \)’ from a distribution on the non-negative integers
cannot in general, be normalized so as to converge to a non degenerate distribution. In this case there are some partial results. Ferguson (1993) obtained a limit distribution of \( \{Z_n\} \) over an appropriate subsequence of integers. Recently Anderson et al (1997), Nadarajah and Mitov (2004) showed that non-degenerate limit for \( \{Z_n\} \) can be obtained if a parameter of \( F \) is allowed to vary with respect to 'n' in a suitable manner. More precisely one of their results is as follows.

Let \( (X_1, X_2, \ldots, X_n) \) be independent and identically distributed (i.i.d) binomial random variables with parameter \( N= N(n) \to \infty \) but parameter 'p' is fixed. If \( N(n) \) grows with 'n' according to \((\log n)^3/N(n) = o(1)\), then,

\[
\lim_{n \to \infty} P(Z_n \leq x) = \exp \left\{ -\exp(-x) \right\}, \quad -\infty < x < \infty,
\]

when,

\[
a(n) = \left( p(1-p) N(n) \right)^{\frac{1}{2}} (2 \log n)^{-\frac{1}{2}}
\]

\[
b(n) = pN(n) + \left( p(1-p) N(n) \right)^{\frac{1}{2}} \left( \frac{2 \log n}{2\sqrt{2} \log n} - \log \log n + \log 4\pi \right)
\]

They have derived non degenerate laws for discrete uniform, geometric, negative binomial and Poisson by
letting their parameters vary as \( n \to \infty \) by giving a separate proof in each case. If appears a unified proof is rather difficult. Similar difficulty has been faced by us while establishing almost sure limit points of the sequence \( \{M_n^{(0)}\} \).

A definition and some known results are given in Chapter - 1.

In Chapter - 2 of the Thesis, we deal with a particular distribution but more general sequence than \( \{M_n^{(r)}\} \). Ferguson (1993) obtained the almost sure limit of \( \frac{M_n^{(1)}}{\log n} \) assuming \( F \) to be discrete satisfying certain tail behaviour. Chapter - 3 contains a generalization of this result, wherein the random variables are not assumed to be identically distributed but are assumed to be independent observations from 's' (fixed positive integer) discrete distributions. Similar results pertaining to continuous distributions are due to Nayak (1986). This type of non-identical nature of random variables have been treated by Zinger (1965) and Sreehari (1970) while investigating limit distributions for normalized sums. Parallel results pertaining to normalized maximum can be found in Hebbar (1981) and Ravi (2000).
In Chapter - 2 we considered a sequence of random variables which are independent and identically distributed and we relaxed the identical nature for discussion in Chapter - 3. Next we retain the identical nature but replace independence by a particular dependence. To be precise, we deal with a sequence of stationary standard Gaussian random variables. Starting with Berman (1964), substantial research has been carried out with respect to this sequence in obtaining limiting distribution of $M_n^{(r)}$. Leadbetter, Lindgren and Rootzen (1983) provides detailed developments. In Hebbar (1980) almost sure limit set of suitably normalized $M_n^{(1)}$ were obtained. This result is generalized by considering what is called moving maximum sequence. That constitutes the Chapter - 4.

Next we return to the sequence of independent and identically distributed random variables. Klass (1984) dealt with the minimal growth of $M_n^{(1)}$ by considering certain subsequence $\{n_k, k \geq 1\}$ giving a characterization of this growth by lower and upper bound. More explicit results with respect to the growth of $M_n^{(1)}$ on subsequences $\{n_k\}$ are in Husler (1985). The latter results have been generalised by considering random subsequences. The results are due to Vasudeva and Savitha (1992). Recently
Mathew (2003) extended Husler's results by considering the limit points of independent copies of \( \{ M_{n_k}^{(1)} \} \). Our result in this direction is given in Chapter - 5.