Chapter 5

Adaptive Wavelet Transforms for Sensorineural Impairment

5.1 Introduction

Convergence performance of the standard LMS algorithm can be improved by using frequency domain filtering. The orthogonal property of frequency domain filtering is implemented using different types of transforms. The performance of TLMS depends on the orthogonal capabilities of the data independent transform used to preprocess the input. In the previous chapter, TLMS is implemented with real transforms like DWHT, DHT and DCT, which are having good decorrelation efficiency and also having less computational complexity. But, still the SNR improvement is less. Hence, in this chapter, real transforms, which are having good decorrelation efficiency and less computational complexity are used to improve SNR. The DWT is good orthonormal, orthogonal, separable, and has multi resolution property. The use of the wavelet transform (WT) in adaptive filtering has been proposed in [3] and is very much analogous to the DFT-LMS of Narayan and Peterson [110]. This chapter describes adaptive Discrete Wavelet Transform (DWT-LMS) to improve the SNR and to reduce the convergence time of the LMS for SNHL patients.

5.2 Discrete Wavelet Transform

A wavelet is a “small wave”, which has its energy concentrated in time. It gives a tool for the analysis of transient, non-stationary process. It has an oscillating wave like characteristic but also has the ability to allow simulations time and frequency analysis with a
flexible mathematical foundation. Traditional signal processing techniques such as Fourier transform and short Fourier transform are poorly suited for analyzing signals which have abrupt transitions superimposed on lower frequency backgrounds such as speech, music, and bio-electrical signals. The wavelet transform has a multi-resolution capability. The multi-resolution signal processing is used in computer vision and sub-band coding developed for speech and image processing. The wavelet theory provides a unified framework for a number of techniques, which had been developed independently for various signal-processing applications [3].

The continuous wavelet transform (CWT) is similar to the short time Fourier transform (STFT) because it also involves the multiplication of the signal with a function, and the transform is computed separately for different segments of the signal. The difference between the CWT and the STFT is that the width of the window is changed as the transform is computed for each spectral component. The continuous wavelet transform of a time domain signal $x(t)$ is defined as

$$\gamma(s, \tau) = \int x(t) \psi^*_{s, \tau}(t) dt$$  \hspace{1cm} 5.1

Where $\ast$ denotes complex conjugation and

$$\psi_{s, \tau}(t) = \frac{1}{\sqrt{s}} \psi \left( \frac{t - \tau}{s} \right)$$  \hspace{1cm} 5.2

The transformed signal is a function of $s$ and $\tau$, which are the scale and translation parameters respectively. $\psi_{s, \tau}$ is the transforming function, otherwise known as the mother wavelet and the factor $\frac{1}{\sqrt{s}}$ is for energy normalization across the different scales. The continuous wavelet transform is a reversible transform and the inverse wavelet transform is defined as
\[ x(t) = \int \int \gamma(s, \tau) \psi_{s, \tau}(t) d\tau ds \quad 5.3 \]

Discrete wavelets are not continuously scalable and translatable, but can only be scaled and translated in discrete steps. This is achieved by modifying the wavelet representation in equation 5.3 as

\[ \psi_{j, k}(t) = \frac{1}{\sqrt{s_0^j}} \psi \left( \frac{t - k \tau_0 s_0^j}{s_0^j} \right) \quad 5.4 \]

Although it is called a discrete wavelet, it normally is a (piecewise) continuous function. In equation 5.4, \( j \) and \( k \) are integers and \( s_0 > 1 \) is a fixed dilation step. The translation factor \( \tau_0 \) depends on the dilation step. The effect of discretizing the wavelet is that the time-scale space is now sampled at discrete intervals. If \( s_0 = 2 \), then the sampling of the frequency axis corresponds to dyadic sampling. This is a very natural choice for computers, the human ear, non-stationary signals etc. For the translation factor, \( \tau_0 = 1 \), then we have dyadic sampling of the time axis. Then the equation 5.4 becomes,

\[ \psi_{j, k}(t) = \frac{1}{\sqrt{2^j}} \psi \left( \frac{t - k 2^j}{2^j} \right) \quad 5.5 \]

The discrete wavelets can be made orthogonal to their own dilations and translations by special choices of the mother wavelet, which means,

\[ \int \psi_{j, k}(t) \psi_{m, n}^*(t) dt = \begin{cases} 1 & \text{if } j = m \text{ and } k = n \\ 0 & \text{otherwise} \end{cases} \quad 5.6 \]

An arbitrary signal can be reconstructed by summing the orthogonal wavelet basis functions, weighted by the wavelet transform coefficients. The inverse transform for discrete wavelet is given by

\[ x(t) = \sum_{j, k} \gamma(j, k) \psi_{j, k}(t) \quad 5.7 \]
When the signal has infinite energy, it is impossible to cover its frequency spectrum and its time duration with wavelets. Usually this constraint is stated as,

\[ \int_{-\infty}^{\infty} |x(t)|^2 dt < \infty \]  

Because of the low-pass nature of the scaling function spectrum, it is sometimes referred to as the *averaging filter*. If we consider the scaling function as a signal with a low-pass spectrum, then we can decompose it in wavelet components and can be expressed it, like equation 5.7

\[ \varphi(t) = \sum_{j,k} \gamma(j,k) \psi_{j,k}(t) \]  

The low-pass spectrum of the scaling function admissibility condition is given by

\[ \int \varphi(t) dt = 1 \]  

Which shows that the 0\(^{\text{th}}\) moment of the scaling function cannot vanish. If one wavelet can be seen as a band-pass filter and a scaling function is a low-pass filter, then a series of dilated wavelets together with a scaling function can be seen as a filter bank [71].

Equation 5.9 shows that the scaling function could be expressed in wavelets from minus infinity up to a certain scale \(j\). Express the first scaling function in terms of the second, because all the information needed to do this is contained in the second scaling function. We can express this formally in the so-called multi resolution formulation or two-scale relation

\[ \varphi(2^j t) = \sum_k h_{j+1}(k) \varphi(2^{j+1} t - k) \]  

5.11
The two-scale relation states that the scaling function at a certain scale can be expressed in terms of translated scaling functions at the next smaller scale. Smaller scale means more detail.

The first scaling function replaced a set of wavelets and therefore we can also express the wavelets in this set in terms of translated scaling functions at the next scale. More specifically we can write for the wavelet at level $j$

$$\psi(2^j t) = \sum_k g_{j,1}(k) \varphi(2^{j+1} t - k)$$  \hspace{1cm} 5.12$$

which is the two-scale relation between the scaling function and the wavelet.

Since the signal $x(t)$ could be expressed in terms of dilated and translated wavelets up to a scale $j-1$, this leads to the result that $x(t)$ can also be expressed in terms of dilated and translated scaling functions at a scale $j$

$$x(t) = \sum_k \lambda_j(k) \varphi(2^j t - k)$$  \hspace{1cm} 5.13$$

If in this equation we step up a scale to $j-1$, we have to add wavelets in order to keep the same level of detail. We can then express the signal $x(t)$ as

$$x(t) = \sum_k \lambda_{j-1}(k) \varphi(2^{j-1} t - k) + \sum_k \gamma_{j-1}(k) \psi(2^{j-1} t - k)$$  \hspace{1cm} 5.14$$

If the scaling function $\varphi_{j,k}(t)$ and the wavelets $\psi_{j,k}(t)$ are orthonormal, then the coefficients $\lambda_{j-1}(k)$ and $\gamma_{j-1}(k)$ are found by taking the inner products

$$\lambda_{j-1}(k) = \langle x(t), \varphi_{j,k}(t) \rangle$$
$$\gamma_{j-1}(k) = \langle x(t), \psi_{j,k}(t) \rangle$$  \hspace{1cm} 5.15$$
If we now replace $\varphi_{j,k}(t)$ and $\psi_{j,k}(t)$ in the inner products by suitably scaled and translated versions of equations 5.11 and 5.12 (replace $2^t$ in equation 5.11 and equation 5.12 by $2^t-k$ ) and are manipulated as follows

$$\lambda_{j-1}(k) = \sum_m h(m-2k)\lambda_j(m) \quad 5.16$$

$$\gamma_{j-1}(k) = \sum_m g(m-2k)\gamma_j(m) \quad 5.17$$

These two equations state that the wavelet and scaling function coefficients on a certain scale can be found by calculating a weighted sum of the scaling function coefficients from the previous scale. Scaling function coefficients are derived from low-pass filter and in subband coding the filter banks are obtained by repeatedly splitting the low-pass spectrum into low-pass and high-pass components.

![Figure 5.1 First stage sub band coding tree](image)

This means that equations 5.16 and 5.17 together form one stage of an iterated digital filter bank and the coefficients $h(k)$ is referred to as the scaling filter and the coefficients $g(k)$ as the wavelet filter as shown in the Fig 5.1. Normally the iteration will stop at the point where the number of samples has become smaller than the length of the scaling filter or the wavelet filter, whichever is the longest, so the length of the longest filter determines the width of the spectrum of the scaling function [126].
5.3 Adaptive Discrete Wavelet Transform

In frequency domain, decomposition of the input signal is obtained by dividing into sub bands. The outputs of filters are approximation and detail components of the wavelet transforms represent the low and high band portion of the original input signal. The quality of filters and the computational complexity necessary for implementation depends on the type of wavelet used. There are many types of wavelets. Typically, the input signals properties and system computational power determines the type of wavelet.

A common approach for performing and analyzing wavelet transform is to express them in their matrix form. The matrix contains low pass and high pass filters. The size of the matrix is $N \times N$, where $N$ is length of the adaptive filter. The length of the filter is used to obtain wavelet transform depends on the choice of wavelet. For the Daubechies 4 wavelets, an 8x8 matrix, with a size of 4-wavelet coefficient low pass filter $h$ and 4-coefficient high pass filter $g$ is given by (Similar structuring can be applied to any other smaller or larger size filters or matrix size.)

$$T_i = \begin{pmatrix}
  h(1) & h(2) & h(3) & h(4) & 0 & 0 & 0 & 0 \\
  0 & 0 & h(1) & h(2) & h(3) & h(4) & 0 & 0 \\
  0 & 0 & 0 & 0 & h(1) & h(2) & h(3) & h(4) \\
  h(3) & h(4) & 0 & 0 & 0 & 0 & h(1) & h(2) \\
  g(1) & g(2) & g(3) & g(4) & 0 & 0 & 0 & 0 \\
  0 & 0 & g(1) & g(2) & g(3) & g(4) & 0 & 0 \\
  0 & 0 & 0 & 0 & g(1) & g(2) & g(3) & g(4) \\
  g(3) & g(4) & 0 & 0 & 0 & 0 & g(1) & g(2)
\end{pmatrix}$$

Shift by two in each row is due to the sub sampling by two done at the each level of the subband-filtering tree. Additionally the wrapping of the data done at last rows of each filter set produces an orthogonal matrix. Further down sampling the subband-filtering tree, another transformation is done, after the first stage and this will continue up to the highest subband level. Subsequent transformations up to highest level is given by
\[
T_2 = \begin{pmatrix}
  h(1) & h(2) & h(3) & h(4) & 0 & 0 & 0 & 0 \\
  h(3) & h(4) & h(1) & h(2) & 0 & 0 & 0 & 0 \\
  g(1) & g(2) & g(3) & g(4) & 0 & 0 & 0 & 0 \\
  g(3) & g(4) & g(1) & g(2) & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

and can be represented as

\[
= \begin{pmatrix}
  T_1(N/2 \times N/2) & 0 \\
  0 & I(N/2 \times N/2) \\
\end{pmatrix}
\]

\(N\) is the size of the matrix. The transformed input vector is produced by product of each of the matrices for each level. The above structuring corresponds to the successive dyadic decomposition of the low-pass subband. We can also have a dyadic form on the high pass side of the tree only. If we consider both the sides, it becomes a uniform tree. For example if we had a uniform structure then the second level decomposition is given by

\[
T_{2\text{(uniform)}} = \begin{pmatrix}
  h(1) & h(2) & h(3) & h(4) & 0 & 0 & 0 & 0 \\
  h(3) & h(4) & h(1) & h(2) & 0 & 0 & 0 & 0 \\
  g(1) & g(2) & g(3) & g(4) & 0 & 0 & 0 & 0 \\
  g(3) & g(4) & g(1) & g(2) & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & h(1) & h(2) & h(3) & h(4) \\
  0 & 0 & 0 & 0 & h(3) & h(4) & h(1) & h(2) \\
  0 & 0 & 0 & 0 & g(1) & g(2) & g(3) & g(4) \\
  0 & 0 & 0 & 0 & g(3) & g(4) & g(1) & g(2) \\
\end{pmatrix}
\]

and can be represented as.
The transformed input vector can be produced by product of each of the matrices for each level so that the transformation matrix of the TLMS would be \( T = T_1 \ldots T_i \). At each step a division of the frequency domain is achieved and depending on whether it is a dyadic or uniform structure and the low or high pass components has to be retained. The DWT can be implemented using many types of wavelets and in this work Symmlet, Coif, Biorthogonal, and Daubechies are used. The DWT-LMS algorithm uses sub-band coding which filters out noise by convolving with high pass and low pass filter. This produces the coefficients, that is, the row vector normalized and is fed to adaptive linear combiner where weight update is done by LMS algorithm as, explained in the section 3.2.

5.4 Wavelet families

The Harr, Daubechies, Symmlets, coiflets and Biorthogonal wavelets are used in this work.

Haar:

Any discussion of wavelets begins with Haar wavelet, the first and simplest. Haar wavelet is discontinuous, and resembles a step function. It is compact, orthogonal, symmetric and represents the same wavelet as Daubechies db1.

\[
T_i(N/2 \times N/2) \\
0 \\
T_i(N/2 \times N/2)
\]

\[5.19\]

\textbf{Figure 5.2 Haar Wavelet function}
Daubechies:

Ingrid Daubechies, one of the brightest stars in the world of wavelet research, invented what are called compactly supported orthonormal wavelets, thus making discrete wavelet analysis practicable. Mathematical statement of the conditions for orthogonality by Ingrid Daubechies led to the construction of the continuous wavelets called Daubs. They are compact, orthogonal, nearly symmetric and continuous.

The names of the Daubechies family wavelets are written $\text{db}N$, where $N$ is the order, and $\text{db}$ the "surname" of the wavelet. The $\text{db}1$ wavelet, as mentioned above, is the same as Haar wavelet. Here, wavelet functions of the next nine members of the family are shown in Fig. 5.3.

![db2 to db10 wavelets](image)

*Figure 5.3 Daubechies wavelet family*

Symlets:

Associated scaling filters are near linear-phase filters. Short name 'sym', order $N$, where $N = 2, 3, \ldots$ e.g. sym2, sym8 etc. as shown in Fig 5.4. These are not unique, and more nearly symmetric, compact, orthogonal, and continuous.
Figure 5.4 Symlets wavelet family

Coiflets:

Short name of this family is coif with order N. Where N = 1, 2, ..., 5, e.g. coif2, coif4 etc as shown in Fig. 5.5. It is compact orthogonal and even biorthogonal. This family can be used in DWT and also in CWT.

Figure 5.5 Coiflets wavelet family

Biorthogonal:

This family of wavelets exhibits the property of linear phase, which is needed for signal and image reconstruction. Fig. 5.6 shows the members of biorthogonal family.
In this chapter DWT-LMS has been implemented for the enhancement of the speech signal in digital hearing aid for SNHL persons. The performance of the algorithms has been evaluated using output SNR, eigenvalue ratio, time plots and intelligibility tests.
5.5.1 Performance evaluation by using output SNR, eigenvalue ratio and time plots

The algorithm is evaluated for corrupted speech signals with different types of noises like cafeteria, low frequency and babble noise with different SNR. The various parameters like $\beta$, filter order and also different types of wavelets were changed and the performance of the algorithm was evaluated. We have found that both parameters SNR and eigenvalue ratio are strongly depending on $\beta$, filter order and types of filters in the discrete wavelet. Harr, Daubechies, Symmlets, coiflets and Biorthogonal wavelets are used in this work. Which is one of the advantages of the DWT, it is not limited to a single wavelet type.

The input signal is a speech sentence in English and is recorded with sampling frequency 22050 Hz in different noisy conditions to evaluate the performance of the algorithm. The performance of the algorithm was studied, for different values of $\beta$ and also for different filter order. From the results obtained, we have noticed that for $\beta = 0.45$ and filter order = 10, DWT-LMS gives better SNR and intelligibility improvement.

DWT-LMS methods are evaluated for speech corrupted in different noisy environments. Table 5.3 and Table 5.4 shows that the noise is reduced from the corrupted speech signal and the speech quality is also improved. The eigenvalue distribution of the input auto correlation matrix has been derived after DWT and power normalization.

For different input SNR, the output SNR and eigenvalue ratios are calculated as shown in Table 5.1 for DWT-LMS. The Table 5.1 shows that, the output SNR improvement up to 13.12 dB and the eigenvalue ratio up to 40.5 for 0 dB input SNR in Symmlet 8 wavelet. Hence, the performance of Sym8 is better compared to other wavelet families. Even Coif1 shows good improvement in SNR and eigenvalue ratio.
Results shows that the improvement of output SNR in DWT-LMS is excellent, but the eigenvalue ratio is high compared to FMDCT-LMS. FMDCT-LMS shows the significant improvement in both output SNR and eigenvalue ratio. Fig. 5.7 shows time plots for the pure signal, corrupted signal with -5dB SNR, and the DWT-LMS filtered signal. Fig. 5.8 shows the autocorrelation of the input signal after DWT.

<table>
<thead>
<tr>
<th>Filter type</th>
<th>For -5dB input SNR</th>
<th>For 0 dB input SNR</th>
<th>For 5 dB input SNR</th>
<th>Eigenvalue ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harr</td>
<td>10.03</td>
<td>12.11</td>
<td>13.96</td>
<td>101.2</td>
</tr>
<tr>
<td>Db40</td>
<td>10.78</td>
<td>12.91</td>
<td>14.80</td>
<td>50.40</td>
</tr>
<tr>
<td>Bior 2.2</td>
<td>10.63</td>
<td>12.54</td>
<td>14.20</td>
<td>63.52</td>
</tr>
<tr>
<td>Coif 1</td>
<td>10.94</td>
<td>13.01</td>
<td>15.31</td>
<td>60.65</td>
</tr>
<tr>
<td>Sym 8</td>
<td>11.08</td>
<td>13.12</td>
<td>15.31</td>
<td>40.50</td>
</tr>
</tbody>
</table>

*Table 5.1 SNR of the output signals to different Wavelet filter types.*

*Figure 5.7 Autocorrelation of the input corrupted signal after DWT (Harr)*
Figure 5.8 Pure signal, contaminated signal and DWT-LMS filtered signal
5.5.2 Intelligibility Test

In order to measure the performance of clinical intelligibility of the algorithms listening tests were carried out. The tests were conducted as explained in the section 2.6.3. The results are displayed in Tables 5.2, 5.3 and 5.4 for -5dB input SNR. The result indicates that a considerable improvement is obtained, particularly for moderate to severe SNHL subjects. The result of recognition test for DWT-LMS filtered signals are displayed in Table 5.4. It can be seen that, after processing intelligibility improvement is achieved in DWT-LMS. Filtered signal showed an average intelligibility improvement of 0.5 % with normal subjects, 6.5 % with mild to moderate SNHL subjects and 4.5 % with moderate to severe SNHL subjects as compared to FMDCT-LMS with cocktail party noise.

<table>
<thead>
<tr>
<th>Group1</th>
<th>Group 2</th>
<th>Group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>96 %</td>
<td>78 %</td>
<td>63 %</td>
</tr>
</tbody>
</table>

*Table 5.2 Average intelligibility score for the noiseless signal*

<table>
<thead>
<tr>
<th>Types of noise</th>
<th>Cocktail party noise</th>
<th>Babble noise</th>
<th>Low frequency noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group1</td>
<td>73 %</td>
<td>78 %</td>
<td>83 %</td>
</tr>
<tr>
<td>Group 2</td>
<td>31 %</td>
<td>34 %</td>
<td>38 %</td>
</tr>
<tr>
<td>Group 3</td>
<td>15 %</td>
<td>13 %</td>
<td>16 %</td>
</tr>
</tbody>
</table>

*Table 5.3 Average intelligibility score for the signal plus noise*

<table>
<thead>
<tr>
<th>Types of noise</th>
<th>Cocktail party noise</th>
<th>Babble noise</th>
<th>Low frequency noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group1</td>
<td>DWT-LMS (sym8)</td>
<td>96 %</td>
<td>95 %</td>
</tr>
<tr>
<td>Group2</td>
<td>DWT-LMS (sym8)</td>
<td>83 %</td>
<td>81 %</td>
</tr>
<tr>
<td>Group3</td>
<td>DWT-LMS (sym8)</td>
<td>73 %</td>
<td>76 %</td>
</tr>
</tbody>
</table>

*Table 5.4 Intelligibility improvements by DWT-LMS (sym8) for three groups of subjects.*
5.6 Conclusion

Results show that, DWT-LMS is best compared to all the other methods, which are discussed in the previous chapters and even with FMDCT-LMS in terms of SNR improvement. The performance of Symmlet 8 wavelet is best compared to other wavelet families. Even Coif 1 shows good improvement in SNR and eigenvalue ratio. Output SNR improvement in DWT-LMS is excellent compared to all the other methods discussed in the previous chapters.