CHAPTER 1

Introduction and Summary

This thesis is devoted to the theoretical study of spin-dependence in hadron collisions involving polarized beams and polarized targets. The approach is empirical and model-independent and utilizes the power and elegance of irreducible tensorial formalism. The specific collision processes considered in this thesis are: pion production in $NN$ collisions, $np$ and $Nd$ radiative capture and double pion production in $pd$ collisions. These reaction processes involving the nucleons and/or deuterons are of primary interest to Nuclear Physics and hence are also currently of experimental interest. The formalism developed here is non-relativistic in the sense that the spin-operators employed transform as irreducible tensors with respect to the 3-dimensional rotation group.

Major portions of the work reported in this thesis have already been published [1-7] in leading international journals like Physical Review C, Journal of Physics G: Nuclear and Particle Physics and Nuclear Physics A.

Pion production in $NN$ collisions, to which half of this thesis is devoted, is the basic inelasticity channel in $NN$ collisions [8]. Close to threshold, it involves very large momentum transfers and serves to probe the $NN$ interaction at very short distances when the two nucleons start to overlap and their internal degrees of freedom come into effect. The setting up of the Cooler Ring technology at Indiana University Cyclotron Facility (IUCF), Indiana, USA [9] allowed for high-precision measurements [10, 11] of the total cross sections at threshold for $pp \rightarrow ppp\pi^0$. This measurement showed that the then existing theoretical model of Koltun and Rietan [12] underestimated the total cross section by as much as a factor of five. Subsequent measurements [13] of the total cross sections carried out at the CELSIUS
Accelerator Storage Ring [14] at Uppsala University, Uppsala, Sweden were found to be consistent with that of the IUCF group [11]. This triggered a flurry of activity on the theoretical side (discussed briefly in chapter 2). This in turn triggered experiments to employ polarized nucleon beams and polarized nucleon targets to measure spin-observables slightly above threshold, when higher partial-waves start to contribute and hence allow for a more incisive study of the theoretical models. The recent facility for Polarized INternal Target EXperiments (PINTEX) at IUCF [15] has made it possible, for the first time, to carry out precise measurements [16] of the two analyzing powers and the five initial spin-correlation coefficients for $\vec{p}\vec{p} \rightarrow pnn^0$ in a kinematically-complete experiment for bombarding energies between 325 and 400 MeV. Initial spin-correlation coefficients have also been measured at IUCF by Dearmick et al. [17] in a kinematically-complete experiment for $\vec{p}\vec{p} \rightarrow pnn^+$ for bombarding energies between 325 and 400 MeV. These experiments [16, 17] have revealed that there are serious discrepancies between the experiments and the most successful theoretical model hitherto, viz., the Jülich model [18]. As far as microscopic model calculations of the reaction $NN \rightarrow NN\pi$ are concerned one has to concede that theory is definitely lagging behind the development on the experimental sector ... Furthermore they take into account only the lowest partial wave(s). Therefore it is not possible to confront those models with the wealth of experimental information available nowadays, specifically with differential cross sections and with spin-dependent observables.

In this scenario, it is indeed of topical interest to develop a model-independent approach to $NN \rightarrow NN\pi$ to analyze the experimental data. The formalism discussed in this thesis is based on the irreducible tensor approach together with invariance requirements. This leads to a spin-structure of the $T$-matrix which is valid at all energies. Exact partial-wave expansions for the reaction amplitudes are also obtained. Moreover, we define channel-spin cross sections at the complete kinematical five-fold level itself and show how they can be determined empirically from appropriate measurements employing a polarized beam and a polarized target. This is in contrast to the earlier theoretical work of Bilenky and Ryndin [19], which leads to expressions for the singlet and triplet total cross sections, which have been made use of in discussing the recent experiments [20–23]. Further, we present here numerical estimates for the singlet and triplet differential cross sections using the data of [16, 17].

Another important process considered in this thesis is $Nd$ radiative capture on which polarized beam or polarized target experiments have also been carried out at very low en-
ergies (in the keV regime) at Triangle Universities Nuclear Laboratory (TUNL), North Carolina, USA [24–30]. In this thesis, we draw attention to an interesting, but perhaps overlooked, aspect of these experimental studies viz., that an empirical study of the energy-dependence of the tensor analyzing powers could determine the relative importance of the quartet amplitudes in comparison with the doublet amplitudes as a function of energy. Historically, we may recall that in the context of muon-catalyzed fusion (μCF) [31], where the atomic effects clouded the issue considerably, the conjectured no-quartet theorem [32] was followed by the Wolfenstein-Gerstein effect [33] which subsequently led to the Wolfenstein-Gerstein anomaly [34]. Although the calculations of Friar et al. [35] employing numerically-converged $^3$He bound states and $pd$ scattering wave functions gave a substantial value for the quartet capture rate, it is perhaps desirable to confirm this finding in an empirical model-independent way by making use of spin-observables, as they are more sensitive to small admixtures of amplitudes than the cross section. In fact, the more recent theoretical calculations of Schadow and Sandhas [36] for $pd$ fusion using several realistic potential models show that the angular distributions of the differential cross section are practically insensitive to the choice of the potentials, while the magnitudes differ slightly from potential to potential. The irreducible tensor formalism developed in this thesis for the fusion process shows clearly that a non-zero measurement (beyond experimental uncertainties) of the tensor analyzing power provides a clear signature for the participation of the quartet amplitudes, even if they are very small. Extending the existing experimental studies on the tensor analyzing powers [24–29] to different energies would enable one to identify the precise energy range, if any, for the validity of the no-quartet theorem. Such a determination without any attendant atomic ambiguities could be important at low energies of interest to μCF. We also show in this thesis that it is, in fact, possible to determine individually the doublet and quartet differential cross sections $(d\sigma(s_i, m_i))/(d\Omega)$ themselves by measuring specific spin-observables associated with $^3$He fusion.

A problem of considerable interest to nuclear physics as well as astrophysics [37] is the radiative capture of neutrons by protons. Recently [38] a measurement of the $\gamma$-anisotropy in polarized neutron capture by polarized proton targets has been reported. In this thesis, we draw attention to the fact that by extending the experimental facilities of [38] on $p(\vec{n}, \gamma)d$ to measure the spin-observables suggested by us, one can determine the channel-spin differential cross sections $(d\sigma_{s,m})/(d\Omega)$ individually and hence study the fusion reaction more incisively at any given energy.

We also present in this thesis another reaction viz., $pd \rightarrow ^3$He $\pi^+\pi^-$, studied recently [39] using the MOMO facility at COSY, Jülich to ascertain the dominance of p-wave $\pi^+\pi^-$ pairs.
1. Introduction and Summary

The irreducible formalism developed for this reaction in this thesis shows clearly that the form assumed for the reaction matrix $M$ in [39], indicates a clear dominance of the irreducible amplitude with rank $\lambda = 1$, which can involve contributions from both the initial doublet and quartet channel-spin states. As before, a non-vanishing tensor analyzing power provides a clear signature for the participation of the initial quartet states. We also show how under certain assumptions, conclusions can be drawn regarding the dominance of the quartet amplitudes with rank $\lambda = 1$.

It may be noted here that the formalism outlined in this thesis for $NN \rightarrow NN\pi$, $pd \rightarrow ^3He\pi^+\pi^-$ extends the irreducible tensor approach to discuss 3-body final states, while that for the fusion reactions involves a photon in the final state, whereas the earlier work of Johnson [40] and Ramachandran and Vidya [41] deals with 2-body final states involving massive particles.

Finally, we present here a generalization of the irreducible tensor formalism to $N$-particle final states in collisions where the initial particles have arbitrary spins $s_1$ and $s_2$. Our discussion has validity at the differential level and it leads, on integration with respect to the angles, to a generalization of the earlier results for total cross section of Bilenky and Ryndin [19] for $s_1 = s_2 = 1/2$. The model-independent formalism presented here may therefore be utilized to discuss polarized beam and polarized target experiments with arbitrary spins $s_1$, $s_2$ not only at the total cross section level, but also at the differential level. The advantage which one derives through an analysis at the differential level is that the c.m. energy as well as the momentum-transfer could both be specified. It may also be noted that the Fourier-transform with respect to the momentum-transfer permits one to study the interaction as a function of the relative distance between the two particles.

The chapter-wise summary of the thesis is as follows:

**Chapter 2: Model-independent irreducible tensor formalism for pion production in $NN$ collisions**

In this chapter, we develop a model-independent irreducible tensor formalism for $NN \rightarrow NN\pi$, after a brief introduction to the relativistic kinematics involved. A form for the reaction matrix containing 6 terms is obtained which is akin to that of the Wolfenstein-form for elastic-$NN$-scattering. The method of approach also enables us to obtain exact partial-wave expansion for these reaction amplitudes. The particular case of the reaction $pp \rightarrow pp\pi^+$, which is currently of experimental interest is discussed. The contents of this chapter have been published [5].
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1.1. INTRODUCTION

Granulite terranes represent the lithologic associations that have been equilibrated in the middle to lower crust at elevated temperature and pressure conditions (700 to 850°C and pressures of 7.0 to 8.5 kbar) (Newton, 1989; Bohlen, 1987; Vielzeuf, 1988; Harley, 1989 and references therein). Such P-T condition is observed at the deeper crustal level, approximately equivalent to 25 to 30 km depth. Granulites occupy most of the Precambrian terrane in different continents. Their study has attracted the attention of many geoscientists to understand the formation and evolution of the continental crust. Granulites which represent the high grade metamorphic rocks are generally agreed to originate in the lower to mid continental crust (Fountain, 1989, Newton and Perkins, 1982). The granulites represent a cross section of the deep continental crust, now exposed to the surface in many continents; offer an unique opportunity in understanding the composition and evolution of the lower continental crust.

Granulites have a widespread distribution in space and time with ages ranging from Archaean and Phanerozoic which have been reported from virtually every continents (Brown, 1983; Hensen and Warren, 1988; Van Reenen et al., 1988). The largest are the Archaean Ashuanipi complex of north-eastern Canada (Percival et al., 1992); Wheat belt of Western Australia (Wilson, 1978) and the southern Dharwar craton, South India (Radhakrishna and Naqvi, 1986; Janardhan et al., 1982; Raith et al., 1999). Few Phanerozoic terranes also contain granulite facies rocks, which are of smaller extent like the Moldanubian zone of central Europe (Petrakakis, 1997); Fiordland of southwest New Zealand (Oliver, 1980) and the Ivrea zone of northern Italy (Mehnert, 1975).

Granulites occupy more than 70% of the south Indian Precambrian terrane which are traversed by many shear zones with complex structural styles and grades of metamorphism. The Precambrian terrane in south India is composed of varied lithological association like meta-supracrustal rocks and different types of plutonic igneous rocks. Some of the igneous rocks have been formed due to intra-crustal melting (either by vapour absent or vapour present). Many lithologies have been
Chapter 3: Cross section and analyzing powers for $\bar{N}N \to NN\pi$

Continuing our discussion of the irreducible tensor formalism for $NN \to NN\pi$, we focus attention on polarized beam and polarized target experiments and derive explicit formulas for the density matrices characterizing the spin-states of the initial and final systems. Expressions are then obtained for differential cross sections, analyzing powers and total cross sections for collisions involving polarized nucleon beams and polarized nucleon targets which are relevant for the current ongoing experiments on $\bar{N}N \to NN\pi$. The contents of this chapter have been published [5].

Chapter 4: A sufficient set of experimental observables in $\bar{N}N \to NN\pi$ to determine the singlet and triplet differential cross sections in kinematically-complete experiments

A study of threshold pion production as a function of the momentum-transfer requires the measurement of the differential cross section rather than the total cross section. With this in view, we show in this chapter how the differential cross section for $\bar{N}N$ collisions for kinematically-complete experiments can be partitioned into a sum of singlet and triplet differential cross sections, $2\sigma_{i=0,1, m_i = -s_i, \ldots, s_i}$. A methodology to empirically determine them in kinematically-complete experiments by measuring appropriate spin-observables is then suggested. Relations between the irreducible bilinears and their Cartesian counterparts are derived. We also discuss the symmetry properties exhibited by these bilinears. Finally, we discuss some interesting aspects of the decomposition of the differential cross section when the beam and target polarizations are non-collinear. The contents of this chapter have been published [3].

Chapter 5: Numerical estimates for the singlet and triplet differential cross sections from existing data for the reactions $pp \to pp\pi^0$ and $pp \to pnn^+$

In this chapter, we obtain numerical estimates for the channel-spin cross sections from the existing data on $\vec{p}\vec{p} \to pp\pi^0$ and $\vec{p}\vec{p} \to pnn^+$ using the results of Meyer et al. [16] and Daehnick et al. [17] respectively. The numerical estimates for $pp \to pp\pi^0$ have been published [1].

Chapter 6: Channel-spin cross sections for radiative capture of polarized neutrons by polarized protons
Radiative capture of neutrons by protons is of interest not only to nuclear physics as a testing ground for theories of $NN$ interaction, but also to astrophysics, where the fusion reaction is part of the proton-proton chain responsible for the generation of solar energy and production of elements in the early universe. In this chapter, a model-independent irreducible tensor approach to $p(n,\gamma)d$ is presented and an explicit form for the spin-structure of the reaction matrix $M$ is obtained in terms of the Pauli spin-matrices $\sigma^p(n)$ and $\sigma^d(p)$. Expressing the multipole amplitudes in terms of the triplet $\to$ triplet and singlet $\to$ triplet transitions, we point out how the initial singlet and triplet contributions to the differential cross section can be determined empirically. This work has been communicated for publication [42].

**Chapter 7:** *pd radiative capture with polarized deuterons and the no-quartet theorem*

Historically, $pd$ radiative capture was first discussed in the context of Muon catalyzed fusion ($\mu$CF). This subsequently led to the controversy regarding the *no-quartet theorem*. After developing an irreducible tensorial formalism for the reaction, we show in this chapter, that a non-zero measurement (beyond experimental uncertainties) of the tensor analyzing power would clearly indicate the participation of the initial quartet states in the reaction. This observation has been published [7].

**Chapter 8:** *Channel-spin differential cross sections for $^3He$ fusion*

Continuing our analysis of $pd$ radiative capture, we show in this chapter how a model-independent decomposition of the differential cross section for $^3He$ fusion into doublet and quartet channel-spin cross sections can be effected. We also indicate how these quantities can be estimated empirically using appropriately chosen polarization-observables. The contents of this chapter have been published [2].

**Chapter 9:** *Irreducible phenomenology for the reaction $pd \to ^3He\pi^+\pi^-$*

Recently, a kinematically-complete experiment [39] on $pd \to ^3He\pi^+\pi^-$ at c.m. excess energies of $Q = 70$ MeV was carried out at the MOMO facility at COSY, Jülich, Germany to ascertain the dominance of p-wave $\pi^+\pi^-$ pairs. The irreducible formalism developed for this reaction in this chapter shows clearly that the form assumed for the reaction matrix $M$ in [39], indicates a clear dominance of the irreducible amplitude with $\lambda = 1$, which can involve
contributions from both the initial doublet and quartet channel-spin states. We point out, as in the case of \( pd \) fusion, that a non-vanishing tensor analyzing power provides a clear signature for the participation of the initial quartet states. We also show how under certain assumptions, conclusions can be drawn regarding the dominance of the quartet amplitudes with rank \( \lambda = 1 \). The contents of this chapter have been published [6].

Chapter 10: Cross sections for scattering and reactions with polarized beam and polarized target with arbitrary spins

In this chapter, we present a general irreducible tensorial formalism for handling scattering/reactions which involve polarized beams and polarized targets with arbitrary spins. We derive explicit expressions for the differential and total cross sections for a final state comprising of \( N \) particles. Integration over angles leads to a generalization of the results of Bilenky and Ryndin [19] at the total cross section level. The contents of this chapter have been published [4].