CHAPTER—II

REVIEW OF RELATED LITERATURE
CHAPTER 2

REVIEW OF RELATED LITERATURE

This chapter presents review of literature relevant to the study. Education of Children with Visual Handicap in general and mathematics in particular is primarily discussed under:

2.1 Studies on Education of Children with Visual Handicap

- Studies on Language Comprehension
- Studies on Braille Reading
- Studies on Cognitive Mapping
- Studies on Mathematics
- Studies on Assessment of Learning

The main focus of the study was to describe/analyse and compare the data of Arithmetic Skills among Children with Visual Handicap in terms of certain selected basic arithmetic concepts and operations with that of Sighted children at lower primary level. Hence, relevant studies are presented under:

2.2 Development of Arithmetic abilities

- Studies on Cognitive Arithmetic
- Studies on Gender Differences in relation to achievement of arithmetic skills
- Studies on Teaching-learning in mathematics
- Studies on Assessment in Mathematics

A secondary focus of the study was to study certain selected cognitive capabilities among Children with Visual Handicap. Piaget’s theory for cognitive development was considered for benchmarking the emergence of concrete operational stage (7/8 years of age) among the selected sample of the study. Hence, studies at concrete operational stage with respect to both Sighted and Children with Visual Handicap were reviewed and is presented under:

2.3 Studies on Piaget’s Stages for Cognitive Development

- Studies on Cognitive Capabilities
- Studies on Piagetian tasks in relation to development of number
- Studies on Cognitive development stages

Further, a separate review on ‘inclusive education’ is also added at the end of the chapter to throw light on the emerging trends on the education of Children with Visual Handicap under
2.4 Studies on Inclusive Education

- Studies on general practice
- Teacher preparation and attitude towards inclusive education
- Approaches to inclusive educational practices
- Inclusive Education for Children with Visual Handicap

Finally, review of literature is summarized and the significance of study in relation to the literature reviewed as well as the justification of the study is presented at the end of the chapter.

2.1 STUDIES ON EDUCATION OF CHILDREN WITH VISUAL HANDICAP

The goal of providing education and rehabilitation to the disabled in general and persons with Visual Handicap in particular, has been in focus in most of the developed countries for the last two centuries. But the development of research in the area of education of the persons with Visual Handicap is neither systematic nor comprehensive. Considering the recent developments in the education of the persons with visually handicap, it could be observed that there is an increase in school enrolment, training of teachers for persons with visually handicap and the implementation of inclusive education via integrated education for the disabled in a number of general schools, but not in-depth research on learning aspects of person with visually handicap.

Research on Children with Visual Handicap is comparatively recent in India and review of the research indicates that many areas that are related to
visually handicap have been considered. Singh and Kulshreshta (1986) have thoroughly reviewed the available research studies and have classified them into various categories these categories provide a bird’s eye view of the status of research in India on visually handicapped. Categories of research reviewed by them include: Psycho-educational assessment and psychological researches; Guidance, counselling and crisis management; Rehabilitation Vocational training and employment; Community based extension services; Development of aids and Status of women with visual handicap. Even though, the above mentioned broad categories includes education of visually handicapped, in depth studies on development processes in acquiring knowledge are not available in India till 1988. Internationally, review of research related to persons with visual impairment provides a broad spectrum of research areas ranging from studies on orientation and mobility to that of providing computer related services to persons with visual handicap. An online review of studies available on the Educational Resource Information Centre online digest (ERIC Digest) with 'visually impaired' as the search criteria returns approximately 1820 (retrieved as on date 06 Feb 07). A review of the abstracts available from the results of the search reveals that most of the research related to education of visually impaired (8.57% - with 'education' as the keyword for search within results) is focused on the orientation and mobility (1.15% - with 'orientation and mobility' as keyword for search within 1820 results), computer education (1.5% - with 'computer' as keyword for search within 1820 results), tactile information (0.71% - with 'tactile' as keyword for search within 1820 results). Keyword search on 'Mathematics' within the results...
returned 10 abstracts (0.71%). The abstracts available on the online ERIC Digest are a mix of studies, book reviews, journal articles and other publication (thesis; dissertation). From 1820 results, the investigator has reviewed some of the noteworthy studies.

In the absence of detailed studies on education of persons with visual impairment in India, a brief note on research studies available in India and abroad is presented below. Main focus of the review is to present studies on education of persons with visual impairment. As the field of education is too broad hence, review of studies has been narrowed down to areas that are related to the current study. Development of language is essential for learning any content area, hence few studies on language comprehension among blind has been presented in this review. Also, as Braille being the gateway to literacy among blind, review of studies on Braille reading and writing is also covered. As the main focus of the study was to enquire development of arithmetic skills among Children with Visual Handicap, hence studies in relation to Mathematics among visually impaired has been reviewed and presented. Further, instruments of assessment for assessing the learning outcomes, intelligence level and even assessment instruments for determining the visual impairment to determine the eligibility of a person for educational services has attracted few researchers. Thus, a brief review of some of the studies in this area is also presented.
2.1.1 Studies on language comprehension

Molloy, (1983) in her study on an exploration of the role of oral language in the education of blind children, with special reference to junior school, remarks that oral language forms the foundation stones on which other forms of learning are built. She also states that the recognition of centrality of oral language in the total life of the individual has special relevance in the education of blind children, particularly because of the prevalence of oral methods throughout all areas of the school curriculum in Schools for the Blind within British Isles. As development and comprehension of language is important for learning any content matter including arithmetic skills. Hence, some noteworthy studies in relation to development and comprehension of language in general are presented below.

Cutsforth (1951), who investigated word associations in 26 congenitally blind children. He found that nearly one-half of their responses contained the names of visual qualities. Only about 7 percent referred to the qualities of taste or smell and approximately 3 percent to qualities of hearing. The remainder referred to abstract qualities not referring to particular sensory modalities. He concluded that the high percentage of visual responses was evidence that the children were developing language to meet social approval Nolan (1960) obtained free and controlled associative responses to the stimulus words used by Cutsforth. He obtained a somewhat smaller number of visual responses, but concluded that the use of visual terms of blind children was not a significant problem for them. Hayes (1938) studied 443 blind children (ages 10 to 23+), using Terman’s English Group Vocabulary Test. The results indicated that, meng the blind,
inferiority in the understanding of words was about equal to their retardation in grade placement in the early grades. In the research literature on blind children the use of terms referring to visual experience is often termed verbalism, because the use of such terms is not built on direct sensory experience of the students. A study by Harley (1963), carefully conducted with 40 children blind from birth, led to the conclusion that verbalism is not a significant problem. He found that chronological age, intelligence and experience were inversely related to verbalism, i.e., high occurrence of visual terms, and he found no significant relation between personal adjustment and verbalism.

Fouike, Amster, Nolan and Bixler (1962) measured the listening comprehension of 291 Braille readers of both sexes in the sixth, seventh, and eighth grades of 11 residential schools for the blind. None of these students had previously been exposed to rapid or compressed speech. Materials were presented at rates of 175, 225, 275, and 325 words per minute. A 36-item multiple-choice test was conducted to measure comprehension. It was found that the comprehension level was satisfactory for the compressed speech. In particular, no loss of comprehension of either scientific or literary material was found in listening to compressed speech of up to 225 words per minute. The authors contrasted this to typical recording rates of 175 words per minute and the mean Braille reading rate for high-school blind students of about 90 words per minute. In the case of scientific materials, they found that there was no significant loss of comprehension through 275 words per minute.
Gillon and Young (2002) note that there is little research about the relationship between the development of written and spoken language among students who are blind. The lack of reading progress on children who are visually impaired may have a linguistic basis as well as any other obstacles to gaining access to print that are caused by their visual disability. The purposes of this study were: (1) to describe the phonological-awareness skills of New Zealand children with visual impairments and (2) to compare the phonological-awareness skills of children who are blind and are reading below their age level with those of Sighted children who have a similar level of word-decoding ability. Participants in this study were 19 children from New Zealand, ages 7-15. A speech-language specialist tested the children on a variety of reading and phonological-awareness tasks. The children's reading ability was tested using three measures: the braille standardize version of the Neale Analysis of Reading Ability (NARA), the Burt Word Reading Test-New Zealand Revision, and the Reading Freedom Diagnostic Reading Test. They found that, children who were reading below their age level were also delayed in their ability to understand the sound structure of spoken language. The authors suggested that the findings of this study support the proposition that the major information-processing tasks in reading are basically the same, regardless of whether the input stimulus is visual or tactile.

The results of the study by Gillon and Young (2002) indicated that there is a strong relationship among phonological awareness, braille reading accuracy and reading comprehension. In the special schools for the visually impaired in India, proficiency in Braille reading is an important task that is expected of
Children with Visual Handicap after grade III. Teachers of the special schools visited by the investigator reported that reading proficiency is important for them (Children with Visual Handicap) to understand the statement problems in arithmetic presented to them in Braille. They also mentioned that from Grade V onwards Children with Visual Handicap are expected to write the solution of arithmetic problem in Braille using Nemeth Code, hence, proficiency in Braille is enabler for them to learn arithmetic.

Therefore, some noteworthy studies in relation to learning Braille reading and writing is presented in the below section (2.1.2 Infra).

2.1.2 Studies on Braille reading

Braille has been considered as one of the most important medium of providing literacy to persons with visual handicap. A great debate on using Braille as a medium of instruction and learning continues inspite of evolution of other medium of learning with emerging technologies. Schroeder (1996) points out for some, Braille seems to represent competence, independence, and equality. Braille has been advocated not only persons who are visually handicapped but also to people who have low vision. In this regard, Mangold (Mangold & Jones, 1995) in her address regarding ‘Getting in touch with Literacy’, reminded parents and teachers that people want access to information. It is vital that most efficient methods be available to students with low vision. Making methods available often means providing quality instruction in Braille. Braille is only as difficult to learn as it is believed to be. It may not solve all problems of print access, but it provides a
method of completely legible writing as well as reading which does not result in eye strain or poor posture. Braille does not require special lighting or additional devices to ensure its readability (Mangold & Jones, 1995). Furthermore, it is not a symbol of inferiority but a symbol of literacy and independence. Taking the time to provide quality instruction in Braille will ultimately result in much higher levels of achievement for many students with low vision.

Spungin (1989) proposes eight major reasons for increasing illiteracy of people who are blind or visually impaired. In summary those concerns were

i). the lack of accurate demographic statistics on individuals in the United States who are blind;

ii). the emphasis, during the past 25 years, on teaching children with residual vision to read print;

iii). negative attitudes toward blind people and the communication skills they need;

iv). lack of standardised Braille teaching methods and of quality control to ensure high standards of teaching;

v). the complexity of the Braille code;

vi). technological advances, especially speech output, as a viable substitute for Braille;

vii). the practice of placing visually impaired children in regular classrooms, with support from an itinerant teacher who visits only once a week; and
viii). limitations of the Individualised Education Program (IEP) process, such that the IEP often is based on the school's budget and availability of staff.

In a scholarly article Fellenius (2002) states that 'the theoretical framework for the development of the individual states that every child actively searches for knowledge by interacting with his/her surroundings' (p. 2). The child is simply a constructive learner (Piaget, 1954; Vygotsky, 1978). Research has taught us that the Sighted and the blind child differ in the way they explore and acquire concepts and language (Warren, 1994). We do not yet fully understand, however, how the blind child's surroundings affect his/her efforts to construct the world (Webster & Roe, 1998). That is, how the Braille beginner constructs the concepts of reading and writing in interaction with the family, school, and society. And how does the teacher's knowledge of and experience with the way blind children explore the surroundings affect an individual child's chance to be a constructive learner? (Fellenius, 2002).

According to Smith (1988), the components of the reading process for Sighted people include 1) perception, 2) memory, 3) identification of letters, 4) identification of words, and 5) comprehension. Although the ultimate goal of gaining access to information is the same for both Sighted readers and readers with visual impairments, one must question if the reading process is the same. Is reading a different task for persons with visual impairment than it is for Sighted persons? Does Smith's list of components accurately describe the reading
process for readers with visual impairments, or should the components be different?

One of the earliest attempts to investigate the hand movements in reading Braille was done by Eatman (1942). She made motion pictures of the hands of braille readers as they read. Observation of these motion pictures revealed that her subjects read braille with only their index fingers, and that those who employed two index fingers usually read faster than those who employed only one. When two index fingers were employed, best results were usually obtained by those who divided the reading task between the two fingers by searching for the beginning of the next line with the left index finger while reading to the end of the current line with the right index finger. This strategy permits the reader to eliminate those intervals during which no information is acquired that would be present if the reader read to the end of the current line with both index fingers and then searched for the beginning of the next line.

Foulke (1979) in an investigative approach to the study of Braille reading concludes that our current understanding of the Braille reading process does not warrant final conclusions regarding the manner in which Braille is read, or the manner in which it might be read if it were properly displayed to adequately trained readers. Because of the limited sensing area on the fingertips, not much more than one Braille character can be observed at a time. Even if this were not the case, the mass of the finger, hand and the arm prevents the kind of extremely rapid movement from one fixation to the next that would correspond to the
saccadic movements of the eyes. The efficient perception of Braille requires continuous lateral movement of the fingertip that is actively engaged in reading, and as a result of this movement, Braille characters are encountered and perceived serially. The serial perception of Braille characters has been demonstrated by Nolan and Kederis (1969). They have shown that the time required for the identification of a word written in Braille is frequently greater than the sum of the times required for identification of the characters of which it is composed. This is not always the case, because readers can predict some words and syllables, and the use of contractions sometimes reduces the time required for identification of words, but there is a strong suggestion that Braille readers must first register and accumulate percepts of single characters, and then integrate these stored percepts to achieve the perception of whole words.

Studies have also been done with the legibility of braille characters and the readability of words formed with them. In studying the legibility of dot patterns, one may obtain evidence regarding the speed and accuracy with which subjects can identify them absolutely, discriminate them one from another, or replicate them from memory.

Nolan and Kederis (1969) initiated their investigation of the perceptual basis for braille word recognition by determining the legibilities of 55 of the 63 characters in the braille code. They employed an experimental task in which subjects were required to make absolute identifications of those 55 characters. In order to qualify for an experiment in which absolute identification of braille...
characters is required, subjects must have learned the dot patterns used as stimuli, must have learned the names assigned to those dot patterns, and must have learned the associations between names and dot patterns. The subjects tested by Nolan and Kederis (1969) were blind children in Grades 4 through 12 at residential schools for the blind. Although these subjects exhibited considerable variability in braille reading experience, their least experienced subjects, those in the fourth grade, had four years of experience in reading braille. Nolan and Kederis (op cit) could safely assume that all subjects had extensive over learning, and given this assumption, they judged that the minimum time of exposure required for the identification of a dot pattern would serve as a valid index of legibility. In another study by Nolan and Kederis (1969) reported an experiment in which they found that the time needed by braille readers to identify a word is a function of its length. This is true for both familiar and unfamiliar words and for both slow and fast readers. Furthermore, they found that, in many instances, the time spent by a subject in identifying a word exceeded the sum of that subject’s identification times for the characters of which the word was composed. Of course, the subjects tested by Nolan and Kederis were occasionally able to take advantage of their knowledge of orthography and to identify words without having to identify all of the characters in them, but their results strongly imply that at any given time, the braille reader is acquiring only the information that can be provided by a single character. If this is true, the braille reader must have to identify and remember each of the letters in a word, and then integrate them in order to identify that word. The inevitable
consequence of such a process would have to be a braille reading rate that is much slower than the visual reading rate.

Foulke (1991, p. 227) suggests that a different perceptual focus is required for rapid braille readers: "The stream of dots that flows beneath the reading fingers varies in spatial extension as a function of time. These variations form memorable patterns with which syllables, words and even phrases can be associated."

Other researchers have gone beyond the psychophysics of braille to examine issues related to learning and reading braille (Newman, Hall, Foster, & Gupta, 1984; Newman et al., 1982) and different strategies used by good and poor braille readers (Millar, 1977). Because of a concern about the relative slowness of braille reading, numerous studies have been focused on reading rates. These studies have investigated the implications of manipulating the braille code (Lorimer & Tobin, 1979; Martin & Sheffield, 1976), reading strategies (Crandell & Wallace, 1974; McBride, 1974; Olson, 1976; Olson, Harlow, & J. D. Williams, 1975, 1977), and optimal methods of teaching braille (Birns, 1976; Caton & Bradley, 1978-79). In addition, tactile discrimination and hand movements have received a significant amount of attention (Mangold, 1978; Olson et al., 1975, 1977; Wormsley, 1981). There is a consensus in the field that to lead productive lifestyles, braille readers must develop adequate skills (CEARSVH, 1991; Rex, Sowell, Koenig, Sveen, & Cheadle, 1991).
Mousty and Bertelson (1985) studied how reading speed of braille readers is affected by hand usage and degree of contextual constraint. Twenty-four blind readers read aloud prose, statistical approximations and scrambled words with either hand alone or with the two hands. No overall superiority of one hand was observed in one-handed reading, but there were large and reliable individual differences in pattern of hand superiority, which were not related to general performance level. All subjects read faster with the two hands than with the faster hand alone. The relative gain from two-handed reading was negatively correlated with absolute size of speed difference between the hands in one-handed reading, which suggests that both hands participate in the collection of text information. On the other hand, in each reading condition, reading speed increased with degree of contextual constraint, from scrambled words to prose. The fact that the effect is comparable to the one obtained for visual reading in other studies is inconsistent with the "compensatory processing hypothesis", according to which readers would depend more on context "when the access to textual information is slower. Also inconsistent with the notion are the facts that context effects are not systematically stronger in slow than in fast readers nor for slower than faster hand combinations.

Knowlton and Wetzel (1996) also found similar results on Braille Reading Rates as a function of reading tasks. Their study of the cognitive processes of braille reading compared the reading of 23 adult braille readers in four different reading conditions: oral reading, silent reading, studying, and scanning. The
findings provide support for the idea that braille reading is process driven and that reading rates vary, depending on the purpose of the reading task.

From the above account it is clear that, interest in tactile pattern perception has grown in the past decade and a half as the result of the development of a number of prosthetic devices for the deaf and blind that convert speech or optical patterns into tactile patterns (Loomis, 1981). Even though a few of these devices, most notably the Optacon (Bliss 1969; Linvill & Bliss 1966), have been moderately successful in providing the sensorily handicapped with useful information, research with them has reinforced the conclusions of earlier investigators (DeGowin & Dimmick 1928; Major 1898; Zier & Barrett 1927; Zigler and Northrup 1926) that the cutaneous sense, especially when compared with vision, has rather limited form-sensing capability when the patterns are impressed upon the skin (Apkarian-Stielau & Loomis 1975; Hill 1974; Kirman 1973, 1979; Loomis, 1974; Scadden, 1971). Even tactile reading, where the subject has control over exploration and character sampling rate, falls far short of visual reading, as evidenced by the relatively low reading speeds of Optacon and braille readers (Bliss, 1978; Biirklen, 1932; Lappin & Foulke 1973; Merry, 1937; Nolan & Kederis 1969).

Wittenstein (1993) studied on Braille Training and Teacher Attitudes: Implications for Personnel Preparation. Analysis of completed surveys from 230 teachers of blind and visually impaired learners found that pre-service training programs emphasizing the methodology of teaching Braille and the development
of tactual perception tended to produce teachers who have positive attitudes toward Braille and feel competent in teaching Braille to children.

Amato (2002) explored teachers' competence in braille literacy and the specific role played by the university training programs in achieving such competences. Forty-five instructors representing 34 teacher preparation training programs participated in this study. Respondents were asked to respond a questionnaire, Standards and Criteria for Teacher Competence in Braille Literacy, and answer questions about the course format, course content, course outcomes, and grading and determining level of students' competences among others. Results of this study suggested that there is a widespread diversity and a lack of consistency in university level braille courses. The author recommended future research should be done on the preparation of teachers and teachers' competence in braille literacy.

As with reading, writing for the visual handicap is another challenge. Sighted writers depend heavily on visual feedback during different phases of writing. When they write on paper or enter text into the computer, they look at what they have written to be sure that the words make sense and are spelled correctly. When they compose or revise, they frequently look back at previously written sections and insert, delete or revise text; reorganise paragraphs; or make marginal notes. Lacking visual feedback, writers who are visually impaired frequently use alternative means of displaying text, which rely on other senses. Swenson (1991) provided descriptive information on successful strategies for
helping braille reading primary-age children gain concepts about the functions of writing and skill in the process of writing. She notes that children who are blind often write drafts as easily as their Sighted classmates, but then required additional assistance from a teacher who knows braille during the revision, proofreading and publication stages. She finds the process approach as a highly effective way of teaching writing to children who are blind. Besides the immediate success reflected in students' enthusiasm and confidence, she finds the process "establishes a foundation for the development of future literacy skills, including the use of a talking word processor" (p. 221). Further research is needed to determine how potential braille readers develop concepts about writing.

Mukhopadhyay et.al. (1987) commenting on the teaching of reading and writing braille simultaneously for visually impaired children states "While it is appropriate for these two subjects (reading and writing) to be combined for Sighted children, we cause much difficulty for the visually impaired child by expecting him to be able to WRITE before he has good READING skill. This is a problem in the language arts area, and it is likewise a problem in arithmetic material" (p. 170).

2.1.3 Studies on Mathematics education

Having discussed the importance of language comprehension (vide Section 2.1.1 Supra) and the challenges involved in learning to read and write braille (vide Section 2.1.2 Supra), it is noted that learning of mathematics may be influenced by the proficiency level in language comprehension and braille
reading and writing among Children with Visual Handicap. However, there are some challenges that are Children with Visual Handicap experience in learning mathematics. Thus, Some of the noteworthy studies in the area of mathematics education of the Children with Visual Handicap are detailed below:

Ahlberg and Csocsán (1997) investigated elementary mathematical strategies used by 25 Hungarian children (ages 5 through 9) who are blind. Children dealt with numbers in five ways and experienced numbers in four ways. Although the children went through typical stages of development, their ways of dealing with numbers had some specific characteristics. Some of the characteristics observed by them include:

- There was not a single blind child who claimed that he/she had used mentally tactile models for solving mathematical problems

- The tactile experiences of young blind children have an arbitrary character and the conditions of touch influence the quality of the experience of quantity. This involved the number and size of the elements, qualitative properties of the objects, position of the elements, size of the touching field, etc.

- Many different sorts of spontaneous tactile strategies and those children who developed effective touching methods of their own, e.g. using both hands at the same time, had a higher level of number understanding than those who grasped the elements of a quantity one by one. Thus, the simultaneous sensory experience is one of the most important elements in the development of parts to whole relation – including through touch.
Many observations in the research proved that the blind child did not use his/her fingers to do the calculations. There were many of them who had been taught to show numbers with fingers and they did it well but these children did not actually use this natural tool for counting.

Though the observations from the study by Ahlberg and Csocsán (1997) gives an insight on the many individual ways blind children learn mathematical skills but cannot be generalised on universal blind child. More in-depth research on tactile learning specifically in relation developing mathematical skills is called for.

With the main hypothesis as “most of the blind children experience parts to whole relation through an interiorised simultaneous acoustic number line (Csosćán, 2000, p. 4). Csosćán (op cit) in a pilot study interviewed 25 blind children (age 7 years and 4 months). The 25 children used counting and hearing ways with the various tasks – 8 with everyday problems, 15 with contextual problems and 9 with decomposition problem. It was observed that ‘not only blind children use the counting through listening to solve problems. But blind children prefer this way of handling numbers compared to Sighted ones’ (p. 4). Further in a pilot study aimed to get data about the range of short term memory dealing with structured patterns of rhythms by children aged 7 to 10, Csosćán (2000) employing a structured interview technique and interviewed 40 children in 4 different groups. The children had to identify hidden acoustic patterns, copy the
given patterns and verbalise the given pattern. The three provisional consequences of the pilot study were:

- There are many blind children who have difficulties in haptic perception and have a great interest in acoustic experiences. For these children, the best way is to develop the part to whole relations of number with help of hearable number line.

- Acoustic short-term memory is trainable. This spontaneous training occurs more often with blind children. The effectiveness in number-experience has a very close relation to the capacity of short-term memory.

- Structured auditory inputs connected with movement, verbal impulses and haptic experience are very important elements in elementary mathematics for helping children who are blind. They help them to find the links and relations among objects, persons and events around them.

Yakubu, (1997) investigated into the effectiveness of mathematics teaching to students with visual impairment in Nigeria. The study was conducted in 12 schools with 112 visually impaired students integrated into the mainstream. The sample also consisted of 20 teachers who teach mathematics in classes where students with visual impairments are mainstreamed, and 50 mathematics teachers in special schools. The findings of the study revealed that there are number of factors that undermine the effectiveness of the teaching of mathematics to students, who have visual impairment in Nigeria. Extension of

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1 Gibson (1966) defines the haptic system as "The sensibility of the individual to the world adjacent to his body by use of his body".
traditional beliefs and attitudes to schools had resulted in some teachers and fully Sighted students having low opinions of the status, abilities and needs of learners who are visually impaired. It was also observed that mathematics teachers under-utilise the available local material in the teaching of mathematics to students with visual impairment. This in turn had limited the students’ initiatives in the use of such materials in the learning of mathematics.

Klingenberg (2000) used a phenomenographic\textsuperscript{2} research framework approach to how the congenitally blind children experience numbers and learn Arithmetic Skills to study eight congenitally blind children in the age group of 8-11 years with no apparent additional dysfunction. Subjects were videotaped and recorded while solving different arithmetical problems, also, interview techniques was used to gather information on the following topics:

- What is the focus during the solving process?
- What kind of calculating strategies does the pupil use?

Klingenberg (\emph{op cit}) categorised the results of the study into three major categories depending whether the cardinal-and ordinal aspects are prominent and further on functionality for problem solving:

- \textbf{Group 1: Functional level} – Here we have a simultaneous experience of the cardinal and ordinal aspects. The two experiences are labelled:
  
  Numbers as structures; Numbers as linear structures.

\footnotesize\textsuperscript{2} Phenomography is a content-oriented educational approach aiming to describe and analyse how people experience, understand, and conceptualise different phenomena in the world around them (Marton and Booth, 1997).
• **Group 2:** Less functional level – The pupils show a separate experience of the cardinal and ordinal aspects. The two experiences are labelled: Numbers as countable; Numbers as extents.

• **Group 3:** Not functional level – Certain aspects are held in focus in one case and other in another. These aspects are seldom the cardinality or the ordinality. Two experiences are described and labelled: Numbers as adverbs; Numbers as jingle-words.

Finally, the results of the above research showed that four of the eight pupils in all essential perceive the numbers with a fully integrated cardinal and ordinal aspect. The extraordinary findings compared to Sighted children, is that among 8 pupils, we have a wide range of experiences. As a counterbalance on the theory as "average" delayed development in mathematical among blind pupils compared to Sighted children, it was noticed that half of the group managed the problems in the study quite well. This confirms the theory saying that blind children are an extremely heterogenous group. But it is more appropriate to consider visual impairment a risk factor then a cause of children's difficulties in learning numerical concepts (pp. 3-4).

Chander (1992) on Teaching of mathematics to Children with Visual Handicap at the primary stage remarks that mathematics deals with general and abstract notions as elements belonging to a class, ordering, separating, combining, succession etc. The sensations from the various sense organs can be organised or structures with them. The mathematical relations developed refer
not to the particular characteristics of the objects but to those operations, which are independent of the nature of any particular object or the person doing them. The problems faced by Children with Visual Handicap in learning mathematics, can be attributed partly to visual impairment but to a great extent to the inappropriate teaching methods and non-availability of learning material – needs further literature support. Discuss teaching studies and instructional materials explicitly. An over insistence on the use of conventional material without adequate adaptation and methods which presume too much from the Child with Visual Handicap can only result in inadequate understanding of the mathematical concepts, relations and operations.

Mathematics, for the student who uses Braille, commences in the early grades with an emphasis on the use of a variety of concrete, functional manipulative. Gradually, the Braille writer (also called Perkins), to record data, and the Abacus and talking calculators to compute problems are introduced and become major tools at the elementary level. The introduction of the Nemeth Code starts in Kindergarten and its mastery is a focus right through the graduation.

A thorough review of literature reveals that achievement in mathematics among the blind and severely visually disabled persons is, and always has been, extraordinarily low (Kapperman, 1974). The transdisciplinary approach to instruction in mathematics is an effective way to reduce or eliminate the barriers to achievement in mathematics, thereby improving access to many educational
and vocational opportunities (Sticken & Kapperman, 1998) for the visually impaired.

2.1.4 Studies on assessment of learning

Modifications in assessment procedures, creative placement practices, and Individualised Education Plans (IEP's) specifically designed to meet the unique needs of persons with visual impairment to meet their unique educational needs are essential ingredients of educational programs designed for them (Scholl, 1987). Assessment of learning outcomes among Children with Visual Handicap has attracted few researchers to study the impact of certain variables on the performance of Children with Visual Handicap in the formal testing environment using the traditional modalities.

Erin, Hong, Schoch, Kuo and YaJu (2006) conducted a study on visually impaired high schools students on Relationships among Testing Medium, Test Performance, and Testing Time. Sample consisted of 30 high school students (10 who were blind, 10 who were Sighted, and 10 who had low vision) who agreed to take six tests after reading six chapters of an eighth-grade social studies textbook. This study compared the test scores and time required by high school students who are blind, Sighted, or have low vision to complete tests administered in written and oral formats. The quantitative results showed that the blind students performed better on multiple-choice tests in braille and needed more time while taking tests in braille. The interviews revealed inconsistent relationships between the students' preferred media and performance.
Landan, Russell and Cowan (2003) studied on implementing a new device called Talking Tactile Tablet (TTT) which makes graphic elements from multiple-choice math tests more accessible to students with visual impairments and/or print disabilities. Focus Group testing, interviews, videotapes of participants using TTT methods were used for data collection. Their study examined the usability of the system with 23 students in grades 7 through 12 (aged 15 to 20) who had visual impairments. Eighty-three percent of the students indicated that they use braille and learn mainly through hearing and touching while 13% indicated that they use enlarged print or magnification to read print. After 3 weeks of research time, 91% of the students agreed that they had used the system for between 1 and 5 hours. The study found that 62% of the students found the system very easy to use and most students felt that after 2-3 weeks of exposure they would be able to use the approach in taking an actual test.

Research investigating the use of extended time has yielded little conclusive information about its benefit (Tindal & Fuchs, 1999). However, students with visual impairments will usually require extended time during testing because using braille, large print, and audio formats require more time than does reading regular print with acceptable visual acuity. A study by Wetzel and Knowlton (2000) suggests that experienced adult braille readers may need no more than 50% additional time than the stated duration, with additional time allowed for the manipulation of an audio device or the marking of an answer sheet. In contrast, an earlier researcher found that braille readers with far less braillereading experience than the subjects mentioned in the Wetzel and
Knowlton study may need between 2 and 3 times as much time as their Sighted peers to read the same material (Nolan, 1966, p. 1).

Sellers, Fisher and Duran (2001) evaluated the validity of the Assessment of Motor and Process Skills instrument to evaluate the performance of activities of daily living of 21 students (ages 10 to 21) with severe visual impairments. These preliminary results support the validity of both the motor and process skill scales for use with this population.

Groenveld and Jan (1992) studied intelligence profiles of low vision and visually impaired children. Analysis of scores of 118 visually impaired children on the Wechsler Intelligence Scale for Children (Revised) and the Wechsler Preschool and Primary Scales of Intelligence (Revised) found a consistent response pattern suggesting that the verbal as well as the performance tests provide useful assessment information.

Wilson and Pine (1985) conducted ‘The Word Test’ a language assessment of elementary age visually impaired children. Thirty visually impaired children (6-12 years old) were administered The Word Test which does not use visual stimulus. Correlation between age and scores and comparisons of test results for individuals with test norms indicate that The Word Test can be used in assessing language disorders among visually impaired children.

It is essential to identify the emerging technologies that support the educational needs of the visually handicapped, so that educational process as a
whole can be made flexible to accommodate the available supportive services. Some of the assistive devices used by persons with Visual Handicap include Braille keyboard, Braille conversion software, tactile locators, Braille embossers, speech output software, screen readers, speech synthesisers etc.

Therefore, from the above account it is clear education for the Visually Handicapped encompasses a range of issues including the curriculum, environment and the human resource required to interact with this special group.

It can be observed from the above account that education of Children with Visual Handicap has to be tailored according to their specific needs, specifically speaking primary education of Children with Visual Handicap should be to consolidate their development and help them in making up for their deficiencies and to lay a sound and meaningful foundation for formal education from which the child can effect an optimum learning. The schooling should be both spontaneous and structured in an encouraging and secure environment. It should be remembered that a blind person on leaving school has to face competition with the Sighted in earning his livelihood. He should therefore receive such education as would enable him to stand on his own.

2.2 STUDIES ON DEVELOPMENT OF ARITHMETIC SKILLS

2.2.1 Studies on development of arithmetic abilities

The child acquiring arithmetical skills in our kind of numerate society encounters a variety of number specific cultural tools. The most obvious are the numerical expressions: number words (one, two, twenty-two, million...), numerals
(1, 2, 22, 100000...), roman numerals, patterns on dice, cards and dominoes. Others will be relatively abstract: arithmetical facts (5 \cdot 3 \equiv 15), arithmetical procedures (borrowing, long multiplication), arithmetical laws (a + b \equiv b + a; if a \equiv b \equiv c then a \equiv b + c; and so on). The skills that need to be acquired include reading and writing numbers, counting objects in a set, calculating in the four basic arithmetical operations, reading numerals aloud, writing numerals, applying these skills in money tasks, telling time and dates, finding a page in a book, selecting a TV channel, and so on. All of these skills are much more complex and subtle than they may at first appear to competent adults (Butterworth, 2005).

Various theories have been put forward to explain the process of acquiring these skills. Most of the theories focus on the cognitive processes involved in learning numbers. Although it is widely agreed that possession of something like the concept of numerosity is necessary for normal arithmetical competence, it is by no means agreed how individuals arrive at this concept (Butterworth, op cit).

According to Piaget (1952), necessary preconditions were a grasp of certain logical principles, since arithmetic is really a part of logic: Our hypothesis is that the construction of number goes hand-in-hand with the development of logic, and that a pre-numerical period corresponds to a pre-logical level. Our results do, in fact, show that number is organised, stage after stage, in close connection with gradual elaboration of systems of inclusion (hierarchy of logical classes) and systems of asymmetrical relations (qualitative seriations), the sequence of numbers thus resulting from an operational synthesis of classification and seriation. In our view, logical and arithmetical operations
therefore constitute a single system that is psychologically natural, the second resulting from a generalisation and fusion of the first, under two complementary headings of inclusion of classes and seriation of relations, quality being disregarded (Piaget, 1952, p. viii).

Counting skill is considered as basic to learning of arithmetic. Children make use of their counting skills in the early stages of learning arithmetic (Butterworth, 2005). Fuson and Kwon (1992) point out, “In order for number words to be used for addition and subtraction, they must take on cardinal meanings” (p. 291). Children often represent the numerosity of the addend by using countable objects, especially fingers, to help them think about and solve arithmetical problems. There appear to be three main stages in the development of counting as an addition strategy:

- **Counting all.** For $3 + 5$, children will count ‘one, two, three’ and then ‘one, two, three, four, five’ countables to establish the numerosity of the sets to be added, so that two sets will be made visible – for example, three fingers on one hand and five fingers on the other. The child will then count all the objects.

- **Counting on from first.** Some children come to realise that it is not necessary to count the first addend. They can start with three, and then count on another five to get the solution. Using finger counting, the child will no longer count out the first set, but start with the word ‘Three’, and then use a hand to count on the second addend: ‘Four, five, six, seven, eight’.
• Counting on from larger. It is more efficient, and less prone to error, when the smaller of the two addends is counted. The child now selects the larger number to start with: ‘Five’, and then carries on ‘Six, seven, eight’ (Buttenworth, 1999; Carpenter & Moser, 1982)

The stages are not strictly separate, in that children may shift strategies from one problem to the next. There is a marked shift to Stage 3 in the first six months of school (around 5–6 years in the US, where this study was conducted (Carpenter & Moser, 1982). In one set of experiments, Starkey and Gelman (1982) showed the children two sets one at a time so that there was no opportunity to count all the elements. In these circumstances, most three year-olds could solve 2 + 1, and a few could solve 4 + 2. By 5 years, all could solve the first and 81% the second. Interestingly, only 56% solved 2 + 4, suggesting that some of the children were not counting on from larger, but were still counting on from first.

The skilled adult typically will not need to calculate or count single digit problems such as 3 + 5, 3 • 5, 5 ) 3, or 6 ' 3 and will simply retrieve the solution from memory (Butterworth, 2005). A variety of models of the mental organisation of arithmetical facts has been proposed. One influential view has been that children learn to associate 3 + 5 with several answers, but the association with 8 will end up as the strongest (Siegler & Shrager, 1984). Another view is that facts are typically stored as specifically verbal associations, though subtraction and division require further processes of ‘semantic elaboration’ involving manipulation
of an analogue magnitude representation (Dehaene & Cohen, 1995). In both models, retrieval will depend on the learning history of the individual. Thus, facts that are learned earlier or practised more will show greater accessibility.

The single strongest argument against these views is that retrieval times show a very strong problem-size effect for single-digit problems: the larger the sum or product the longer the problem takes to solve (Ashcraft, Donley, Halas & Vakali, 1992). This factor is much more potent than frequency of occurrence (Butterworth, Girelli, Zorzi, & Jonckheere, 2001). Note also that children who are using a counting strategy to solve arithmetic problems are not using memory retrieval. It is likely that memories are laid down during Stage 3 of counting on from larger. This would mean that the child would work out the result of Larger Addend + Smaller Addend (rather than First Addend + Second Addend) and store it in that form (Butterworth, 2005). Some evidence for this comes from Butterworth et al. (2001), who showed that adults, who presumably retrieve answers, are quicker to solve Larger Addend + Smaller Addend problems than Smaller Addend + Larger Addend problems. The frequency of the problems in textbooks was not a good predictor of solution times. Both this and the problem-size effect suggest that addition facts are organised in terms of number size rather than as orthogonal verbal vectors or a network of associations modulated by practice effects.

Similar results were obtained for children, 6–10 years old, doing multiplication. Larger x Smaller was faster than Smaller x Larger, even though
the (Italian) education system taught Smaller x Larger earlier. For example, 2x6 is in the (Italian) 2x table which is taught before 6 x 2, which is in the 6x table (Butterworth, Marchesini, & Girelli, 2003). In fact, this study showed that children start by privileging the form in which the problem is taught, and later reorganise their memory store to privilege the Larger x Smaller format. Again, this suggests a specifically numerical organisation to arithmetical facts. They are not just rote associations (Butterworth, 2005)

Piaget (1952) has argued, quite reasonably, that a child does not really understand addition or subtraction without understanding the relationship between them. That is, if 5 + 3 equals 8, then 8 − 5 must equal 3, and 8 − 3 must equal 5. This is the Principle of Complementarity. All this should follow from an understanding of sets and numerosities: if set B is added to set A, and then removed, the resulting numerosity will still be A.

Do children understand the Principle of Complementarity, and if so at what age or stage does this understanding begin? Now, of course, it is perfectly possible to arrive at the correct answer without understanding the Principle of Complementarity. Conversely, it is possible to understand the principle, yet sometimes get the answer wrong. This means that the ability or inability to solve these problems is not a sure guide to understanding. Rather, investigators have asked whether 'inversion' problems that can be solved by the principle are solved better than control problems that cannot. Starkey and Gelman (1982) found no convincing evidence for understanding in children of 3 to 5 years of age, while
other researchers have found evidence of understanding in older children (Stern, 1992).

A systematic study of this issue was recently reported by Bryant, Christie, and Rendu (1999). They looked at 5–7-year-olds, and carefully controlled for types of solution strategies that might be used. For example, in a task using a set of objects, if three new objects are added, and then exactly the same are taken away, then the correct answer may be achieved on the basis of a general ‘undoing’ procedure that could apply to non-numerical situations such as splashing paint on a wall and washing it off. Bryant et al. (1999) controlled for this by comparing adding and removing the same objects with adding and removing the same number of different objects. They also looked at equivalent problems with numerals. Children were much more successful with inversion problems, such as $12 + 9 \rightarrow 9$, than control problems matched for sum, such as $10 + 10 \rightarrow 8$. What is more, they could use the Principle in more complex problems that required decomposition of the subtrahend. Thus, they appeared to make use of the Principle in problems such as $7 + 4 \rightarrow 5$ by decomposing 5 into $4 + 1$. Indeed, many of the children revealed by analysis of performance to be using the principle were able to state it in words, but by no means all. Although the children who used the principle to solve inversion problems did better overall than those who calculated the solutions, by no means all the children who did well used the principle. Factor analyses and correlations revealed two separate factors: a calculating factor and an understanding factor. Similar issues arise in connection with complementarity of multiplication and division. If $9 \cdot 3 \frac{1}{4} 27$ is
known, then $27 \cdot 9 \div 3$ and $27 \cdot 3 \div 9$ should both follow without the need for calculation. Butterworth (2005) summarises the principal milestones in the normal development of arithmetic in the following Table 11

Table 11

Milestones in the early development of arithmetic (Butterworth, 2005)

<table>
<thead>
<tr>
<th>Age</th>
<th>Milestones (Typical study)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0:0</td>
<td>Can discriminate on the basis of small numerosities (Antell &amp; Keating, 1983)</td>
</tr>
<tr>
<td>0:4</td>
<td>Can add and subtract one (Wynn, 1992)</td>
</tr>
<tr>
<td>0:11</td>
<td>Discriminates increasing from decreasing sequences of numerosities (Brannon, 2002)</td>
</tr>
<tr>
<td>2:0</td>
<td>Begins to learn sequence of counting words (Fuson, 1992); can do one-to-one correspondence in a sharing task (Potter &amp; Levy, 1968)</td>
</tr>
<tr>
<td>2:6</td>
<td>Recognises that number words mean more than one ('grabber') (Wynn, 1990)</td>
</tr>
<tr>
<td>3:0</td>
<td>Counts out small numbers of objects (Wynn, 1990)</td>
</tr>
<tr>
<td>3:6</td>
<td>Can add and subtract one with objects and number words (Starkey &amp; Gelman, 1982); Can use cardinal principle to establish numerosity of set (Gelman &amp; Gallistel, 1978)</td>
</tr>
<tr>
<td>4:0</td>
<td>Can use fingers to aid adding (Fuson &amp; Kwon, 1992)</td>
</tr>
<tr>
<td>5:0</td>
<td>Can add small numbers without being able to count out sum (Starkey &amp; Gelman, 1982)</td>
</tr>
<tr>
<td>5:6</td>
<td>Understands commutativity of addition and counts on from larger (Carpenter &amp; Moser, 1982); can count correctly to 40 (Fuson, 1988)</td>
</tr>
<tr>
<td>6:0</td>
<td>'Conserves' number (Piaget, 1952)</td>
</tr>
<tr>
<td>6:6</td>
<td>Understands complementarity of addition and subtraction (Bryant et al., 1999); can count correctly to 80 (Fuson, 1988)</td>
</tr>
<tr>
<td>7:0</td>
<td>Retrieves some arithmetical facts from memory</td>
</tr>
</tbody>
</table>

The above Table 11 summarises the principal milestones in the development of arithmetic by age. There are no age norms for the milestones described here, and the ages are those at which most of the children tested (studies are listed against each milestone in Table 11) demonstrate these capacities with reasonable reliability. Bear in mind that the studies described are
not focused on ages, but on stages; different children can reach the milestones at very different ages. The milestones are intended to be culture-free, but the data comes from studies of children raised in European and US contexts. There is evidence that the structure of the number word system can speed or slow the acquisition of arithmetical concepts, so children raised in languages with a very regular system, such as Chinese, acquire some arithmetical concepts earlier (Butterworth, 1999; Nunes & Bryant, 1996).

Broadly, then, the development of arithmetic can be seen in terms of an increasingly sophisticated understanding of numerosity and its implications, and in increasing skill in manipulating numerosities. It is worth noting, however, that there are several major gaps in our knowledge. The relationship between the earliest capacities shown in the infant and later numerical competencies still needs to be described in detail. It is also not yet determined whether there is a critical or sensitive period of acquiring arithmetical concepts, and how this might interact with educational input (Butterworth, 2005).

Studies on cognitive arithmetic seem to explain how the arithmetical concepts are acquired through representation of number and facts. Below some noteworthy studies related to cognitive arithmetic is given to understand the mental processes underlying the arithmetic understanding.

2.2.2 Studies on cognitive arithmetic

Cognitive arithmetic studies the mental representation of numbers and arithmetic facts (counting, addition, subtraction, multiplication, division) and the
processes that create, access, and manipulate them. Arithmetic is one of the fundamental cognitive tasks, which humans have to master. Children go through years of formal schooling to learn first the numbers, and then the facts and skills needed to manipulate them. One can attribute that to humans possessing a complex vision system, which resulted from millions of years of evolution but will require painstaking work to reverse-engineer and replicate in computers. But a task such as arithmetic seems so straightforward and easy to accomplish that it is surprising that it takes years of learning for humans to master. This suggests that human cognition at the most basic level embodies some assumptions about its environment that are at odds with the structure of arithmetic as it is taught. Arithmetic, being a formal mathematical theory, assumes a set of precise and immutable objects (numbers), facts, and procedures. Human cognition, on the other hand, has evolved to deal with approximate concepts, a changing environment, and adaptive procedures. Studying how such a flexible system deals with a formal task such as arithmetic provides an excellent window to its assumptions and mechanisms (Lebiere, 1999).

Lebiere (1999) suggests two main classes of empirical phenomena in the domain of cognitive arithmetic. One concerns the fact that children, and to a certain degree adults, approach answering arithmetic problems with two basic strategies. One strategy is to simply retrieve the answer. The second strategy, referred to hereafter as the backup strategy or backup computation, is to compute the answer. For example, given a problem such as $3 + 4$ children may choose to count (perhaps 4, 5, 6, 7) to provide the answer and given $3 \times 4$ they
may choose to add to get the answer (perhaps $4 + 4 + 4$). This class of empirical phenomena involves how people choose between the computation strategy and the retrieval strategy. The second class of empirical phenomena involves the problem-size (size of the numbers) effect. Children and adults take longer to answer problems involving larger numbers and they also make more errors on these problems. In the case of backup computation the reason for this is fairly obvious – one has to count more to add large numbers and one has to add more things when multiplying by a larger number. However, while much reduced, the problem-size effect also occurs for adults. It has been suggested that this is due to residual use of the backup strategy (LeFevre, Sadesky & Bisanz, 1996), although recent research put those results in doubt (Kirk & Ashcraft 1997). However, a more fundamental argument is that smaller problems also occur more often, offering greater practice and thus better performance. This unequal problem distribution appears in studies of textbooks (Ashcraft 1987; Ashcraft & Christy 1995; Hamman & Ashcraft 1986; Siegler 1988) but also in the world at large, as many (Benford 1938; Newcomb, 1888; Raimi 1976) have noted.

A number of studies have investigated the representations of arithmetic facts in memory and the cognitive processes that underlie aspects of mental arithmetic. Few major determinants of subjects’ performance have been advanced, namely the memory organisation of simple arithmetic facts and working-memory resources (Lemaire, Abdi & Fayol, 1996). According to associative models, the presentation of an arithmetic problem (e.g., $3 + 4$) results in activation of the number nodes specified in the problem (e.g., 3 and 4).
Activation spreads from these nodes along associative links so that related nodes, such as for the sum or product, are also activated (Lemaire et al., op cit).

Subsequent research has tested predictions of associative models in simple cognitive arithmetic with mainly two types of task, that is production (e.g., $8 + 4 = ?$) and verification (e.g., $8 + 4 = 12$. True ? False ?). In both type of tasks, subjects retrieve a set of candidate answers via a spreading activation mechanism (Anderson, 1983; Collins & Loftus, 1975), select the appropriate answer candidate and/or compare the answer selected with the proposed answer. However, it has been shown that the effects of some factors reflect processes of retrieval or computation. For example, the size of the problem determines the difficulty of retrieving the appropriate answer in the memory network (e.g., Ashcraft and Battaglia, 1978; Campbell & Graham, 1985; Groen & Parkman, 1972; Stazyk, Ashcraft and Hamann, 1982). Here, difficult problems (e.g., $9 \times 8 = 72$) produce longer reaction times and more errors than easy problems (e.g., $3 \times 4 = 12$). This problem difficulty effect typically relies on memory retrieval processes and reflects the time to access the solution into the memory network.

With respect to learning of Arithmetic Skills, various cognitive researches indicates the use of long term memory and working memory resources in order to solve a problem. A report by Ashcraft, Donley, Halas and Vakali et al. (1992) addresses directly the issue of the involvement of working memory resources in simple arithmetic. They discussed the possibility that retrieval of basic arithmetic
facts consumes working memory resources. In an exploratory investigation Ashcraft et al. (op cit) asked their subjects to verify simple and complex addition problems such as “8 + 4 = 12. True ? False ? or “24 + 23 = 47. True ? False ?” Each of the problems appeared with its correct answer and also with an incorrect answer in three experimental conditions. Overall, the trends reported by Ashcraft et al. (op cit) suggest that the retrieval of simple arithmetic facts involves attentional resources in working memory.

Logie, Gilhooly and Wynn (1994) suggest regarding the role of each working memory component in arithmetic concerns the possible different involvement of working memory resources in the verification of close false problems (e.g., 8 + 4 = 13) compared with the verification of distant close problems (e.g., 8 + 4 = 17). It is typically observed that subjects take significantly more time to verify close results than distant ones, an effect that has been termed the "split effect" (e.g., Ashcraft & Battaglia, 1978; Dehaene & Cohen, 1991; Zbrodoff & Logan, 1990). This effect is interesting because it suggests that subjects use two different types of strategy – a retrieval strategy used to verify close products and plausibility strategy to verify distant products (Lemaire & Fayol, 1995). Using the retrieval strategy, subjects first retrieve the correct solution, compare the retrieved correct solution, and state their response. Using the plausibility strategy, the whole verification process is not run to completion, but would be short-circuited by a "fast-no" decision, given that the proposed answer is too remote from the correct answer to be plausible. It would be interesting to determine whether these two kinds of strategy are impaired by
overloading the same working memory component and to the same extent in a dual-task situation.

Reviewing the recent progress in developing computational (connectionist) models of numerical cognition Zorzi, Stoianov and Umiltà (2005), focused on three main issues a) number representations; b) basic numerical skills (number comparison, subsidizing, counting); c) simple mental arithmetic.

Theories of number representation place particular emphasis on accounting for the distribution of reaction times (RTs) and errors in the comparison of numerical magnitudes. As with many other stimulus dimensions, it is easier and quicker to select the larger of two numbers when they are numerically dissimilar than when they are similar (the distance effect; Moyer & Landauer, 1967).

Dehaene, 1992; Dehaene, Dupoux and Mehler, (1990) proposed that the number line is held to be compressive, so that larger numbers are closer together on the line than smaller numbers. Accordingly, the subjective difference between two numbers depends on their positions on the line, that is, the subjective difference between \( N \) and \( N+1 \) is smaller as \( N \) increases. A different conception of number line is that of Gallistel and Gelman (1992, 2000). First, they propose that the mapping from the number symbol (word or numeral) is onto a line segment defined from the origin, not a point. Second, the mapping is linear, not compressive, but the variability of the mapping increases in proportion to the magnitude (scalar variability).
Simple arithmetic or basic numerical skill is a fundamental human numerical ability, thought to have a phylogenetic origin (Butterworth, 1999). For example, pigeons can subtract the numerosity of two sets of objects (Brannon, Wusthoff, Gallistel, & Gibbon, 2001) and human infants can sum and subtract small numerosities even before knowing number words (Wynn, 1992). The basic phenomena of single-digit arithmetic performance are robust, widely replicated and well known (for instance, the effect of problem size; see below); yet there has been much controversy as to the psychological processes involved and as to how arithmetic facts are represented and organized in memory. For instance, one major controversy is whether skilled arithmetic performance (e.g., simple addition and multiplication) is built upon abstract semantic representations (e.g., McCloskey, 1992) or on verbally stored facts (e.g., Dehaene & Cohen, 1995). Nonetheless, it is generally agreed that competent adults use some mixture of fact retrieval from memory and procedures for transforming the problem if memory search fails (Cambell & Xue, 2001; Groen & Parkman, 1972; LeFevre, Sadesky, & Bisanz, 1996).

Simple mental arithmetic is systematically affected by the "difficulty" of the problem, indexed by its numerical size. Thus, the problem-size effect indicates that larger problems take longer to solve and are more prone to errors. For skilled adults, correlations from 0.6 to 0.8 are observed between mean RTs for correct responses and the sum of the operands or the square of the sum, with the latter accounting for a larger proportion of variance (Butterworth, Zorzi, Girelli, & Jonckheere, 2001).
All connectionist simulations of mental arithmetic take the associative approach of Ashcraft (1992), among others, that mental arithmetic is a process of stored facts retrieval. Zorzi et al. (2005) distinguish two types of modeling approaches: i) learning models, in which the knowledge about arithmetic facts is acquired through a learning algorithm and stored in a distributed form; ii) performance models, in which the architecture is hard-wired and set up by the modeler(s) according to specific representational and processing assumptions (as it is typical, for instance, in localist interactive-activation models; McClelland & Rumelhart, 1981).

The models of Viscuso, Anderson, and Spoehr (1989; also Anderson, Spoehr, & Bennett, 1994) and McCloskey and Lindemann (1992) are based on associative-memory neural networks that store in their memory a set of patterns representing entire arithmetic facts – the two arguments and the result – and solve arithmetic problems by exploiting the ability of these networks to complete partial or noisy patterns. After training, arithmetic problems are solved by presenting to the network partial arithmetic facts, i.e., their two arguments, which is followed by retrieval of a stored pattern that most closely matches the input. Both models learned single-digit multiplication facts. In the model of Viscuso et al. (1989) numbers were represented both as magnitudes and with a further set of units representing the number name, whereas McCloskey and Lindemann’s (1992) MATHNET model used only the magnitude representation.
The Viscuso et al. (1989) model could only learn about 70% of the problems and its performance was not quantitatively matched against human RT data. MATHNET, which used a more powerful learning algorithm (the Boltzmann Machine; Ackley, Hinton, & Sejnowski, 1985) but had the same basic properties, was shown to account for the problem-size effect: the correlation of the response time with the sum of the arguments was 0.69. However, the authors found that this result could be entirely attributed to the way in which arithmetic facts were presented to the network during training. The network experienced the arithmetic problems with a schedule claimed to be similar to the experience of children learning arithmetic: small facts in the beginning, all facts later. Fact frequency was also manipulated: smaller problems were presented up to seven times as often as larger problems. However, this ratio turns out to be highly implausible when compared to fact frequencies in mathematic textbooks as tabulated by Aschraft and Christy (1995).

The model of Campbell (1995), known as network-interference model, aimed to quantitatively account for skilled performance. Arithmetical knowledge was represented as both a physical code and a magnitude code. The physical code is simply an ordered set of elements (e.g., <{6 3}{+}{9}>), and the magnitude code is specified as a logarithmic function of the numbers. Problems with their solutions are "reactivated" over a series of cycles; variance in RTs is determined by the competing activation of problems similar in terms of their physical and magnitude codes. All parameters in the model were chosen as to provide the best fit to empirical data. The problem-size effect arises because
larger-number problems are more similar in magnitude to their neighbors than are smaller-number problems (i.e., because the number line is compressive). This causes larger problems to activate neighbors more strongly, which turns into more interference by way of inhibition from neighbors.

Another model based on a hard-wired architecture is that of Whalen (1997). This model is a connectionist implementation of the localist associative model of Aschraft (1992), in which number facts are represented by dedicated problem nodes. Both arguments and results are semantically represented, using a compressed (logarithmic) magnitude representation. However, the model was used to simulate an artificial mathematical operation — "diamond arithmetic" — of complexity similar to that of multiplication, but in which the results were not systematically related to the arguments. The network exhibited growing retrieval times as one the argument increased, which was explained with the increasing similarity among the arguments that in turn resulted in a larger number of competing problem nodes.

One important aspect of computational modeling is that theorists are forced to make a number of formal assumptions about the representations on which the computations are carried out. In the context of number processing, numbers need to be turned into patterns of activation over a set of elementary processing units. As previously discussed, most computational models use a representation of numbers that includes magnitude information. Various schemes have been proposed, but they were typically considered as instantiations of the
number-line hypothesis. An alternative proposal, built upon the constraint that magnitude information should encode cardinal meaning, is the numerosity code of Zorzi and Butterworth (1997, 1999).

The analogue "number line" hypothesis has been implemented in most neural network models of arithmetic (Anderson et al., 1994; McCloskey & Lindemann, 1992; Viscuso et al., 1989) as an ordered sequence of input nodes, where each node stands for a particular number. In this scheme, a number is encoded by activating the corresponding node together with the two immediate neighbors: thus 5 was represented as the activation of the node labelled "5" plus activation of "4" and "6". Although this provides some ordering of numbers, "8" and "4", with no overlapping neighbors, would activate orthogonal representations (i.e., nodes 7-8-9 for "8" and nodes 3-4-5 for "4"). This kind of coding scheme has been described as barcode magnitude representation (Anderson, 1998) because number magnitude is coded as a moving "bar" of activation on a topographic scale. It should be pointed out, however, that this scheme does not correspond to either a compressed number line (Dehaene, 2003) or to a linear number line with scalar variability (Gallistel & Gelman, 2000).

In Whalen's (1997) model of arithmetic numerical magnitudes were encoded as patterns of activation over a set of 250 nodes, such that 100 nodes were active for each number. The total activation across nodes for each magnitude summed up to 1. Numerals with similar magnitudes shared nodes with one another and the closer the magnitudes the more similar the
representations. Crucially, the representations were designed so that larger magnitudes shared more nodes than smaller magnitudes. Moreover, the differences between magnitude decreased in a logarithmic fashion as the numbers increased. Therefore, the representation implements the assumption of a compressed number line.

Dehaene (2001) implemented number line representations both in the compressive version (Dehaene, 1992) and in the linear version with scalar variability (Gallistel & Gelman, 1992) to simulate the animal data of Brannon, Wusthoff, Gallistel, and Gibbon (2001). The representation of a number \( n \) was a gaussian, centered according to a logarithmic scale and with fixed variance or to a linear scale with variance proportional to \( n \). The important conclusion was that the two different implementations led to the same metric of number similarity and therefore to the same behavior.

Stoianov, Zorzi, Becker, and Umiltà (2002) contrasted symbolic codes, number-line codes, and the numerosity code in a series of neural network simulations of simple addition. Four different types of representation were contrasted:

i). symbolic code. Symbolic number encoding was abstracted to a simple localist scheme, which was independent from the numerical meaning. Each number was encoded by the activation of a dedicated node. Therefore, this simulation examined a model where simple arithmetic facts are stored in a verbal form (e.g., Dehaene & Cohen, 1995).
ii). **number-line codes.** Following Dehaene (2001), two versions of number-line were implemented. In the compressed number line the representation of a number n was a gaussian, centered according to a logarithmic scale and with fixed variance, whereas in the linear number line with scalar variability the gaussian was centered according to a linear scale with variance proportional to n.

iii). **numerosity code.** Numbers were encoded using the numerosity scheme of Zorzi and Butterworth (1999).

Thus, arithmetic facts are learned and stored as attractor states in the recurrent neural network. After successful learning, if some of the visible units are clamped with a part of a learned pattern (input), the network should iteratively activate the rest of the units according to the data distribution learned (retrieval). In particular, fixing the two arguments (e.g., 7+5=?) the network will retrieve the result of the corresponding arithmetic operation (here, 12) since in the learning data it would have been the only correct completion to this input. The number of cycles to settle is taken as a measure of the network RT for the stimulus. Stoianov et al. (2002) also carried out a formal analysis of the statistical properties of the various representational schemes. This showed that the pattern of network RTs was produced by the joint effects of i) the empirical distribution of active bits (i.e., a bias towards smaller numerosities), and ii) the degree of pattern overlap among arithmetic facts.
Building upon the results of Stoianov et al. (2002), Stoianov, Zorzi, and Umilta (2003) expanded the model with a symbolic component. The aim of the study was to establish how symbolic representations of numbers would interact with the semantic representations. The network could in principle use either type of information, or at least differentially weight them, in learning simple arithmetic. Humans deal with more than one type of symbolic representation of numbers – verbal numerals (both spoken and written), Arabic digits, Roman numbers, etc. However, symbolic number encoding was abstracted to a simple two-digit code, which is independent from the numerical meaning with the exception of the two-digit syntactic structure (the right digit stands for the units and the left one stands for the decades). Thus, each number was encoded by the activation of a dedicated node and an additional "ten" unit allowed the representation of two-digit numbers (for encoding sums up to 18).

Concluding remarks on the computational modeling attempt to decipher the numerical cognition Zorzi et al. (2005) states that much modeling work remains to be done. Further, extending the models of mental arithmetic (e.g., Stoianov et al., 2003) to deal with multiple operations (addition, subtraction, multiplication) might be crucial for understanding the neuropsychological and neuroimaging patterns of association and dissociation among different arithmetic operations (see Dehaene et al., 2003).
2.2.3 Studies on gender differences in relation to achievement of arithmetic skills

Aunio (2006) attempted to study the development of number sense in early childhood. One of the aim of the study was to compare the development of number sense in children (3–8 years) with different backgrounds (age, gender, location/nation, language, low performance). A total of 5 studies were conducted to ascertain the number sense development at early childhood stage. The effects of children's age and gender on number-sense performance were considered in studies I to IV. A total of 1,995 children (1,079 boys and 916 girls) participated in these studies: 1,489 were from Finland (827 boys/662 girls), 376 from China (186 boys and 190 girls), and 130 from Singapore (66 boys and 64 girls). Overall the age of the children varied between 34 and 130 months (two years and ten months and 10 years and 10 months), the highest age being a result of the fact that some SEN children were exceptionally old: in general, the age varied between three and eight years. One of the theoretically interesting outcome was the girls outperforming the boys in Study I and IV. Aunio (op cit) reports that the results were inline with the research focusing on the same age children and little different mathematical skills (Demie, 2001; Gorard et al., 2001; Strand, 1997, 1999), but contradict the research results from basic numerical skills (Dehaene, 1997; Nunes & Bryant, 1996).

Wakefield (2006) in an investigation regarding wether strategies for solving double column addition reveal about math aptitude and gender notes that when children are tested for math the accuracy of their answer is often of most
interest to teachers and administrators. Wakefield (op cit) conducted a study on 87 third grade students studying in schools of Qatar. Sample was categorised into three groups – Group 1 consisted of 41 students studying two separate single sex elementary schools. Group 2 of subjects consisted of 22 third graders attending a co-educational elementary school. The third group of subjects consisted of 24 third-grade students in co-educational, private school. The investigation reported the observed responses of third grade boys and girls when confronted with the same mathematical challenge. It focussed on the strategies used by three very different groups. Each student, one by one and privately, was asked to add 16 plus 17 mentally without paper and pencil. The problem was not meant to be difficult but rather to identify the strategy or procedure used to solve the problem. Results were analysed for the strategies used to solve the tasks given to the subjects. The three major strategies that were a focus of the study were – One-by-One counting, Hierarchical Counting Strategy and Learned Algorithm. From analysis of the data regarding these strategies used, the study found an unexpected layer of data with more thought-provoking implications. Does the freely chosen strategy to solve a simple double-column addition problem in third grade indicate an aptitude for number sense? If so, are there long-term effects of this aptitude? For example, is the choice of a higher-order strategy in third grade predictive of participation and success in the more demanding math classes of middle, high school and beyond? Are there any implications for teaching math to girls? For example, can girls be encouraged to
move from simple counting strategies to strategies requiring more mature, hierarchical thinking?

Van Den Heuvel-Panhuizen (2004) reports on a study conducted in Netherlands to inquire into the gender differences in solving problems in primary school mathematics. The study was called MOOJ study (The letter M stands for the Dutch word "meisje" that means girl and the letter J stands for the Dutch word "jongen" that means boy. In the logo of the MOOJ study, the two O's were used to make the two gender signs. Moreover, the word "MOOJ" is pronounced in Dutch as "mooi" which means "beautiful".) and consisted of three phases, with Stage I (conducted during 1993-1995) starting with around 5000 schools, while Stage IIa zoomed in on 14 schools and Stage IIb zoomed in even further on only 4 schools. Results from the MOOJ Stage I revealed confirmation of gender differences in mathematics scores and discovery of gender-specific problem characteristics in line with the results of the previous two PPON (National Assessment of Educational Achievement) studies (Bokhove, Van der Schoot, & Eggen, 1996; Wijnstra, 1988). Further, from a qualitative analysis was carried out on these "extreme" items, focusing on the mathematical content of the problems. For this analysis no particular criteria were formulated in advance. It was a very open analysis in which the following procedure was followed: (a) repeated studying of the problems (reading and rereading the problems and imagining for oneself again and again what the cognitive and mathematical demand of the problem is when a student is solving it), until particular characteristics emerge; and (b) trying to identify these characteristics in other problems; (c) possibly
followed by a revision of the characteristics. The analysis of the two groups of extreme items made it clear that there were very distinct differences between the girls' problems and the boys' problems. The girls performed equally as well as the boys or a little bit better on:

- problems which ask for accuracy,
- problems of which the text is complex,
- problems which ask for (reflection on) strategies and not for calculations,
- well-known problems which refer to standard procedures,
- straight-away problems (in which the operation and the numbers are given and no re-organisation is necessary), and
- problems which refer to shopping situations.

On the other hand, the boys perform clearly better than the girls on:

- problems which ask for daily-life knowledge on numbers and measures,
- problems in which large numbers with many zeros are used,
- problems in which different numbers or different units of measurement are used,
- problems which have possibilities for "tinkering" with numbers 3, and
- problems which ask for reasoning backwards.
One of interesting finding of the study was 'that contrary to what is often assumed, the study revealed that it is not the nature of the context – whether it is referring to the world of girls or to the world of boys – that appeared to be the leading factor for causing the difference in scores. Instead, the mathematical nature of the problems and the connected cognitive demand proved to be more important for reaching higher scores by boys or girls' (p. 242).

Results from MOOJ Stage IIa further provided evidence for gender-specific problems and strategies. A confirmation of the discovery that certain problems were completed better by boys and others were solved relatively well by girls, came from the mathematics test that was administered in seven extreme boys’ schools and seven extreme girls’ schools. The girls chose to solve a problem that could easily be solved in a smart way by calculating it on paper significantly more often. Also, girls used smart calculations less often when calculating something by heart.

Van Den Heuvel-Panhuizen (2004) reports international findings about gender-specific strategies. For instance, Fennema, Carpenter, Jacobs, Franke, & Levi (1998) found clear strategy differences in a longitudinal study where 44 boys and 38 girls were interviewed a total of five times over a period of three years (grade 1 to grade 3). Compared to the boys, the girls counted more often and used materials more often. The boys on the other hand used abstract strategies that showed a higher level of understanding more often. At the end of grade 3, the girls used standard algorithms more often. A positive connection between
being able to create their own approaches in the lower grades and being able to solve complex problems in grade 3 existed for both boys and girls. Carr and Jessup (1997). In their research, 58 first graders were tested several times and were asked about the used strategy. It emerged that there was not much of a difference between boys and girls as far as the number of mistakes was concerned, but there were differences in the way they worked. The girls made much more use of tools like the abacus or counted on their fingers, while the boys made more use of the problems they already knew or thought they knew. In fact, the boys were no more aware of the way they solved the problems than the girls. Compared to the boys, the girls worried more about finding the right answer. Finally, it was remarkable that the girls used tools less when working in a group. The researchers eliminated social pressure as a cause for this.

Reporting on the international findings about gender-specific problems, Van Den Heuvel-Panhuizen (2004) notes that although the issue of gendered problems had not been researched in the Netherlands before and had not been connected to girls’ relatively poorer results in primary school mathematics, the existence of gender-specific problems had already emerged from international research. For instance, Fennema, Peterson, Carpenter, & Lubinski (1990), when giving a whole-class test in grade 1, found that girls did score as high as boys on some problems, yet not on others. The problems where the scores were the same were the standard problems one regularly finds in the grade 1 curriculum. The problems where the boys’ scores were significantly higher were complex problems, like problems combining several operations and with irrelevant
information. These problems usually are not explicitly taught in school. According to Fennema et al. (1990), the fact that boys outperform girls on non-standard problems could mean that boys take a much more autonomous approach in learning mathematics. However, in their later longitudinal study, Fennema, Carpenter, Jacobs, Franke, & Levi (1998) found hardly any differences in performance between boys and girls, including results on non-standard problems. Boys only did better on one particular kind of problem in grade 3. This was a kind of problem that lends itself to operating flexibly with large numbers.

From the TIMSS 2003 study of trends in achievement of Mathematics, specifically in relation to gender differences, data collected from the participant countries at grade 4 and grade 8, revealed significant differences in achievement among boys and girls in the reasoning cognitive domain. Below Table 12 gives an overview of the international average of IV grade student performance in TIMSS 2003 study in relation to gender across three major cognitive domains.

### Table 12

**TIMSS 2003 - Average Mathematics Achievement by Gender for Knowing Cognitive Domain; Applying Cognitive Domain; Reasoning Cognitive Domain**

<table>
<thead>
<tr>
<th>International Average</th>
<th>Knowing Cognitive Domain</th>
<th>Applying Cognitive Domain</th>
<th>Reasoning Cognitive Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Girls</td>
<td>Boys</td>
<td>Girls</td>
</tr>
<tr>
<td>Average Scale Score</td>
<td>496 (0.9)</td>
<td>495 (0.9)</td>
<td>494 (0.8)</td>
</tr>
</tbody>
</table>

TIMSS (2003) reports that at the fourth grade, while performance was about the same internationally for boys and girls in the knowing domain, there was a significant difference, on average, favoring boys in the applying domain.
Also, boys had significantly higher achievement in considerably more countries than did girls. In the reasoning domain, there was essentially no difference internationally between boys and girls, but in the few countries where significant differences were found, girls had higher performance.

Gender differences have been found in young children's arithmetic strategy selection – the most appropriate strategy to solve a problem (Carr & Jessup, 1997; Carr, Jessup, & Fuller, 1999; Fennema, Carpenter, Jacobs, Franke, & Levi, 1998). More specifically, girls are more likely than boys to use procedural strategies (e.g., counting) whereas boys are more likely than girls to use retrieval strategies. These gender differences in simple-arithmetic strategy selection have shown to be driven not only by skill differences, but also by girls' and boys' strategy preferences (Carr & Davis, 2001). Furthermore, gender differences have been found in retrieval efficiency as well, with boys being faster than girls from fifth grade on (Royer, Tronsky, Chan, Jackson, & Marchant III, 1999). Carr and Davis (2001), however, observed no differences between boys and girls in retrieval efficiency. Although no study explicitly investigated whether gender differences in strategy selection and strategy efficiency exist in adults, Geary, Saults, Liu and Hoard (2000) re-analyzed simple-arithmetic performance data obtained in an earlier study by Geary, Frensch & Wiley. (1993). They observed a trend to more frequent retrieval use by men than by women (86% vs. 66%, p < .07) but no gender differences in retrieval efficiency.
The source of the male advantage in solving mathematical word problems is currently debated. Alternative explanations include sex differences, favoring males, in spatial cognition, computational fluency, and attitudes toward mathematics (Casey, Nuttall & Pezaris., 1997; Geary, 1996; Johnson, 1984; Royer, Tronsky, Chan, Jackson, & Marchant, 1999). Casey et al., (1997) found that the male advantage on the mathematics section of the Scholastic Achievement Test (SAT-M) was mediated by a male advantage in three-dimensional spatial abilities and by more positive attitudes toward mathematics. In a series of experiments, Johnson (1984) found that male college students consistently outperformed their female peers on tests that involved the solving of algebraic word problems, but not on tests of nonmathematical problem solving. Johnson (op cit) also found that individual differences on the tests of algebraic problem solving were correlated with spatial abilities, IQ, and attitudes toward mathematics (e.g., rated importance of mathematics). In Johnson's studies and those of Casey et al. (1997), the sex difference in mathematical problem solving was more strongly related to spatial abilities than to attitudes toward mathematics (see also Benbow, 1988). In an analysis of the academic achievement of children in Japan, Taiwan, and the United States, Lummis and Stevenson (1990) found no overall sex differences in attitudes toward mathematics or on a general test of mathematics achievement. However, in both first and fifth grade and in all three nations, boys outscored girls on three mathematics subtests, word problems, visualization, and estimation.
Royer and his colleagues argued that the male advantage in mathematics is not related to spatial cognition at all, but rather to a male advantage in computational arithmetic, specifically, in the speed of retrieving arithmetic facts from long-term memory (Royer et al., 1999).

Across a series of nine experiments, Royer et al. (1999) showed that speed of solving simple (e.g., 4 + 6) and complex (e.g., 4 x 7 x 3) arithmetic problems was correlated with performance on paper-and-pencil arithmetical computational and arithmetical reasoning (i.e., word problem) tests (Royer et al., 1999; Geary & Widaman, 1987, 1992). In fourth through eighth grades there were no sex differences in speed of computational problem solving, but as a group boys were more variable in their performance. At these grade levels, the fastest boys were faster than the fastest girls at basic computations, whereas the slowest boys were slower than the slowest girls. For college students, males were faster, on average, at solving arithmetic problems than were their same-age female peers, and this sex difference appeared to contribute to a male advantage in mathematical problem solving, that is, on tests that required the solving of word problems.

Other studies of elementary-school children suggest no sex difference in overall arithmetical performance, but sex differences in problem-solving approaches are often found (Carr & Jessup, 1997; Carr, Jessup, & Fuller, 1999; Fennema, Carpenter, Jacobs, Franke, & Levi, 1998; Siegler, 1988). In these studies, boys used direct retrieval and covert strategies (e.g., mental counting)
more often than girls did and girls used finger counting and overt strategies (e.g., blocks) more often than boys did. To assess these differences further, Chinese and American kindergarten through third-grade children’s use of finger counting and direct retrieval to solve simple addition problems (e.g., 4 + 3) was analyzed using data from several cross-national studies (Geary, Bow-Thomas, Liu, & Siegler, 1996; Geary, Fan, & Bow-Thomas, 1992; Geary, 1994). Although the differences were not always statistically significant, in all five samples American girls used finger counting more frequently than did American boys. In third grade, U.S. boys correctly retrieved significantly more addition facts than did U.S. girls (p < .05). In kindergarten, Chinese boys used finger counting more frequently than Chinese girls did (p < .10), and in first grade boys correctly retrieved more addition facts than did girls (p < .05), but there were no other sex differences in the Chinese samples.

Speelke (2005) in a critical review on Sex differences in intrinsic aptitude for Mathematics and Science, concludes that ‘Research on the cognitive abilities of males and females, from birth to maturity, does not support the claim that men have greater intrinsic aptitude for mathematics and science. Male and female infants do not differ in the cognitive abilities at the foundations of mathematical and scientific thinking; they have common abilities to represent and learn about objects, numbers, language, and space. Male and female children harness these abilities in the same ways, at the same times, to master the concepts and operations of elementary mathematics. Although older boys and girls show somewhat different cognitive profiles, the differences are complex and subtle (it
is not the case, e.g., that women are verbal and men are spatial). These differences tend to be small, and they stem primarily from differing strategy choices. Above all, these differing profiles do not add up to a male or female advantage in learning advanced mathematics. High school boys show both higher mean scores and greater variability on the SAT-M, but high school and college men and women are equally proficient in mathematics classes, both on average and within the pool of the most talented students (p. 256).

Thus, from the above reviewed studies it is evident that the achievement of Boys and Girls may not significantly differ however, there may be difference in the strategies that they adopt in solving the mathematical problems.

**2.2.4 Studies on Teaching-Learning in Mathematics**

Teachers shape the learning of their students through the selection (of curricular topics, teaching strategies and assessment procedures) they make, the emphases they place, and their delivery of the range of topics included in the intended curriculum. Recognition of the important role of teachers in promoting student learning is represented in publications, which link a range of teaching models to different aspects of student learning. Teachers, however, for reasons of expediency, often restrict both their curricular selections and the range of teaching strategies they adopt in order to maximise students’ scores on the pen-and-paper tests, which generally form the means to describe what students have learnt (Leder, & Forgasz, 1992).
Carpenter, Fennema, Peterson, Chiang and Loef (1989) studied in the area of basic addition and subtraction in first-grade classrooms has shown that when teachers are sensitised to how children learn, are provided with more information about student learning in specific content area, are encouraged continually to assess student knowledge and solution methods that students construct, then positive changes occur in their teaching practices and significant mathematical learning occurs.

Studies by Yackel Cobb, and Wood (1991) in primary classrooms have shown that when teachers come to regard students learning as social construction of knowledge, their teaching practices change and student interaction increases. From this interaction teachers gain insight into the depth of learning occurring and the misconceptions that children develop and resolve in the process of learning.

If mathematics education is going to live up to the expectations set forth in the current era of educational reform add an example reference and begin implementing new curricula and teaching methods, nothing is going to be more important than having trained, effective mathematics teachers. Researchers studying effective teaching during the past two decades understood this well. Effective teaching research sought to develop an understanding of what successful mathematics teaching was and how to foster it. Its goal was ‘to build conceptual networks that will help us understand and improve the complex process of teaching’ (Grouws, 1988, p.1).
In recent years there has been an interesting shift away from the notion that teachers can best help their students to learn mathematics by deciding in what order and through what steps new material should be presented to the students. Instead, it is believed, learning is the result of children actively constructing their own mathematical meaning in unique ways as they interact with their environment, process different experiences and build on their existing knowledge. This perspective is not entirely new. Leder and Gunstone (1990) pointed out that Bartlett (1932), Brownell and Hendrickson (1950) and Piaget (1952) had been among earlier proponents of this view. Kelly’s (1955) work on the psychology of constructs is also highly relevant. 1980’s saw von Glasersfeld, Steffe and Cobb (1988) particularly thoughtful and fruitful advocates of the constructivist theory of mathematics learning.

Silver (1990) acknowledged that ‘students actively and personally construct their own knowledge rather than making mental copies of knowledge possessed and transmitted by teachers or textbooks’ (p. 7). Kilpatrick (1987) has identified two underlying principles of ‘constructivism’. The first is embodied in the quotation from Silver (1990) above; that is, that “knowledge is actively constructed by the cognising subject, not passively received from the environment” (Kilpatrick, 1987, p. 7). The second principle, that ‘coming to know is an adaptive process that organises one’s experiential world; it does not discover an independent, pre-existing world outside the mind of the knower’ (p. 7) is more contentious. There is debate in the literature about the implications of
this principle and its direct application to mathematics and mathematics education (Leder, 1992).

Even very young children have been shown to have a rich store of 'spontaneous' mathematical knowledge (Leder, 1990; Carraher, Carraher & Schleimann, 1987). Traditional mathematics teaching however, has tended to ignore this and has assumed that the pupil is more akin to an empty vessel into which 'formal' knowledge is to be poured. Children often abandon or ignore their own spontaneous strategies for problem solving to conform to that of the prior knowledge and what they are taught (Hiebert, 1984).

Teaching mathematics from a constructivist perspective encourages children to talk about mathematics as they grapple with concepts and problem solving. The process enables children to confront their knowledge and beliefs about mathematical concepts and to reconstruct, restructure and build on their previous knowledge.

A socioconstructivist approach to teaching-learning of mathematics emphasise learning of mathematics through the process of social interaction. von Glasersfeld (1983), Bauersfeld, Krummheur and Voigt (1988) and Cobb and Wheatley (1988) provided the theoretical basis for the development of instructional activities and for the analyses of individual children's mathematical conceptual development. Steffe (1988) describes the process of mathematical learning as: "[it] is viewed as consisting in the adaptations children make in their functioning schemes as a result of their experiences to neutralise perturbations
that can arise in one of several ways...problem solving conceived as a goal
directed activity is a crucial aspect of learning mathematical knowledge' (p.5).

There are multiple links between teaching, testing and learning, but the
exact nature of these connections is still unclear (Cooney, Grouws & Jones,
1988). As research continues in search of a deeper understanding of successful
mathematics teaching, the understanding of relationships becomes more
important. Understanding what constitutes effective mathematics teaching and
how to foster it is an essential goal for mathematics education research. As
research proceeds, the classroom context and the nature and means of
assessment will be central issue. (Grouws & Meier, 1992)

Increasingly, meaningful learning is being recognised as an active,
constructive process through which students develop their own interpretations,
approaches and ways of viewing phenomena, and through which learners relate
new information to their existing knowledge and understandings (Masters &

Emphasising the prenumber knowledge that a child possesses before
coming in contact with the formal environment of knowledge impartation,
Romberg and Carpenter (1986) states: 'Children are not passive learners who
simply absorb knowledge. Children come to school with rich and informal
systems of mathematics. They actively structure incoming information and
attempt to fit into their established cognitive framework' (p. 858).
2.2.5 Studies on Assessment in Mathematics

2.2.5.1 Studies on diagnostic evaluation of performance in Arithmetic Skills

What has been learnt? Is as much important as what to teach and how to teach? Various researches (Fiske, 1997; McLean, 1982; National Council of Research, 1989; Resnick & Resnick, 1992; Romberg, Zarinnia and S. R. Williams, 1989; Stephens, Clarke, & Pavlou, 1994; Wiggins, 1989) support the fact that any reformation in curriculum must be 'assessment-led'. Homework assignments, teacher made summative tests, classroom observations and teacher made formative test indicates the achievement of what has been learnt by the student of what has been taught by the teacher. In this regard, Pandey (1990) aptly points that traditionally, instruction has been driven by the curriculum, but assessment has not been an effective part of a feedback loop linked to instruction. Assessment is most valuable when it is an integral part of teaching, not merely a tool for ranking students, but a mechanism for influencing instruction. To realize the full potential of the assessment process requires that the profession develop and implement assessment tasks to measure student productivity and performance on tests that require mathematical thinking in pursuit of a result that has meaning to the student. Because these tasks have essentially the same character as instructional tasks, they also have meaning for teachers and, therefore, are useful for improving instruction.

Romberg, Zarinnia and Collis (1990) describe the needed changes in mathematics assessment as a need for a 'new world view' where an integrative approach is taken to teaching and assessment and one in which assessment
becomes qualitative rather than quantitative. Reinforcing the view of Romberg et al. (1990), National Council of Teachers of Mathematics (NCTM 2000) states: Assessment should support the learning of important mathematics and furnish useful information to both teachers and students. Assessment should be more than merely a test at the end of instruction to gauge learning. It should be an integral part of instruction that guides teachers and enhances students' learning. Teachers should be continually gathering information about their students through questions, interviews, writing tasks, and other means. They can then make appropriate decisions about such matters as reviewing material, reteaching a difficult concept, or providing something more or different for students who are struggling or need enrichment. To be consistent with the Learning Principle, assessments should focus on understanding as well as procedural skills. Because different students show what they know and can do in different ways, assessments should also be done in multiple ways, and teachers should look for a convergence of evidence from different sources. Teachers must ensure that all students are given an opportunity to demonstrate their mathematics learning (NCTM, 2000).

It has been recognised through various research projects, that lot of work is being done on developing various forms of assessment, examples are portfolio assessment (Herman & Winters, 1994; Mumme, 1990; Wolf, 1989), performance assessment (Baker, O'Neil, & Linn, 1993; Collison, 1992; Linn, Baker, & Dunbar, 1991), authentic assessment (Lajoie, 1995; Wiggins, 1989b) informal assessment(Watson, 1999; 2000), ‘instructionally embedded assessment’
(Webb, 2001) or 'didactical assessment' (Van den Heuvel-Panhuizen, 1996), ‘formative assessment’ (Wiliam & Black, 1996) and ‘classroom assessment’ (De Lange, 1999) All of these descriptive phrases refer to assessment which is intended to support the teaching and learning process. It is closely linked to the instruction and to the subject matter – in this case school mathematics – and, in principle, is part of the teacher's daily educational practice (Van den Heuvel-Panhuizen, Becker, 2003).

Classroom assessment, as a form of assessment includes everything from informal to formal assessment and also includes an ongoing collection of evidence. This term can be used with respect to the teacher or students as the assessor. Its purpose is clear, namely to support learning and teaching, and it is strongly connected to the domain of the school subject and the way it is taught (Van den Heuvel-Panhuizen, Becker, 2003).

Though lot work is being done to identify the best form of assessment that will help the teacher to enhance the learners' achievement level is being done, a theory and a model of mathematical assessment must be developed to unify these efforts and to provide a base on which to build. During the past decade, 'a general outline of theoretical grounds for forms of assessment that assist educators in monitoring the characteristics of new learning and attained levels of ability has begun to develop. A complete theory of mathematical assessment is needed to:

• provide a structure from which to evaluate work already done;
• provide direction for further research;
• provide a common language of assessment in mathematics that will facilitate discussion; and
• establish relationship between teaching and learning. (Glaser, 1986)

Much past practice in the assessment of mathematics achievement reflects a traditional view of mathematics teaching and learning as the presentation and mastery of pre-specified bodies of facts and skills (Schoenfeld, 1988). In conventional tests and examinations, mathematics is treated as a relatively fixed body of knowledge (for example, facts, algorithms, proofs). During instruction these facts and skills are organised and presented in logical sequences, and subsets of ‘behavioural objectives’ are specified for each instructional module. Romberg and Carpenter (1986) summarise this traditional approach to mathematics teaching:

‘Mathematics is assumed to be a static bounded discipline within each subject, ideas are selected, separated, and reformulated into a rational order. This is followed by subdividing each subject into topics, each topic into studies, each study into lessons, and each lesson into specific facts and skills’ (Romberg & Carpenter, 1986, p. 851). Such an approach to mathematics teaching can be characterised as ‘a top-down’ approach in the sense that its focus is on the delivery of knowledge (Masters & Doig, 1992). Then, assessment is relatively straightforward in a top-down approach. The starting point is a list of objectives which according to Bloom, Hastings and Madaus (1971) should be stated as
directly observable student behaviours, which can be reliably recorded as either present or absent. These objectives should be ‘stated in terms which are operational, involving reliable observation and allowing no leeway in interpretation’ (p. 28).

It has long been ‘known’ that children’s errors and misconceptions can be the starting point for effective diagnostically-designed mathematics teaching. Summative and formative assessment has been widely used to assess what the child has learnt. Summative assessment is usually motivated by the need to sum up what has been learnt over a period of time, by a need for accountability to the wider community. Formative assessment is motivated by the need to identify children’s strengths and weaknesses so as to inform the next steps in teaching. Alternatively, the need for diagnostic assessment of learning has been of interest to many educators and recommends that diagnostic assessment lead to diagnostically designed teaching thus, improving the learning experience of the child and escalating the achievement level. Diagnostic tests aims at interpreting the error response. As Sebatane (1998) argues: the provision of diagnostic information based on performance on standardised tests of pupils in primary schools, compared to the provision of norm-referenced information only, has been found to improve pupils’ achievement as measured by standardised tests.

J. Williams, and Ryan (2000) study on National testing and the improvement of classroom teaching: can they coexist? describes an analysis of children’s performance in the 1997 UK mathematics tests by seven and fourteen
year olds. The children’s responses and errors were scaled against their ability using Rasch methodology. These were then interpreted in terms of the literature on the psychology of mathematics education, especially that related to the misconceptions, and attempted to describe children’s progression in thinking as it related to their test performance. In conclusion, they emphasise that research literature needs to be considered while pooling items for designing the diagnostic tests.

Bell, Swan, Onslow, Pratt, and Purdy (1985) stressing on the need of diagnostic testing states the aim of our research on ‘Diagnostic teaching’ has been to develop a way of teaching which contributes clearly to long term learning and which promotes transfer. The key aspects of this method are the identification and exposure of pupils’ misconceptions and their resolution through ‘conflict discussion’. Conceptual diagnostic tests also play a part both in helping pupils to become aware of their misconceptions and enabling the teacher to observe progress.

Thus, it is essential to probe into the learning models of arithmetic skills and further enquire into the categorisation of errors which will be helpful in planning diagnostic tests to assess the achievement of arithmetic skills among children studying through grade I – IV. Below a review on learning models and error categorisation models is presented. Some of the models though have been studied on adult population has greater implication for arithmetic instruction for children.
A. Learning Models

Various approaches have been taken to modelling in definition of Arithmetic Skills. Examples included: counting models, sometimes called digital models; analogue models; network models; and procedural models, based around rules for arithmetic.

Counting models assume that adults have internalised children's counting strategies. For example, one of the most successful counting models is the MIN model (Groen & Parkman, 1972). To add two numbers, n and m, two counters are postulated: the first is set equal to the larger of the two numbers (say, m); the second is used to increment the first counter n times. Hence, addition is achieved by repeated incrementing, and the RT will be a function of the smaller (MIN) number. Although this model is a good predictor of addition RT, there are problems when the system is applied to multiplication.

No counting account has been proposed for multiplication, but as Stazyk et al. (1982) note, there are some straightforward ways to extend the model to multiplication. The simplest scheme assumes that the first counter is set to zero, and then incremented by the larger number, m, for a total of n times. However, this proposal, requires that the subject be able to count not only in 1s, but also 2s, 3s, etc, and that the size of the increment should not effect the speed of counting. Ashcraft and Stazyk (1981) note:
“Since the counting model proposed for addition asserts that subjects count in units of one, the suggestion that adults can count in units of varying size when multiplying, but not when adding, lacks internal consistency” (p. 47).

In addition, confusions between addition and multiplication, namely operand errors, also suggests a single system for both skills (Miller et al., 1984; Winkeriman & Schmidt, 1974), and this is something difficult to capture in a counting system. One of the reasons for postulating the MIN model was to capture the problem-size effect. However, the counting model has difficulties with the 5s and ties exceptions to the problem-size effect.

Little progress has since been made with counting models, and no detailed models of errors have been proposed. Attention has been focused on network models, and are discussed below.

The set of network models include the most explicit models that exist. Unlike the reconstruction of answers performed by counting models, network models assume an associative memory structure. Activation is assumed to spreading between problems and answers nodes. A review of network models is given by McCloskey, Harley and Sokol (1991). Two of the major network models reported by McCloskey et al. (1991) are:
1. Distribution of associations

One of the more interesting network models is the distribution of associations (DOA) model proposed by Siegler (Siegler 1988b, 1987; Siegler & Shrager, 1984). The model is an account of strategy choice—why retrieval is used sometimes, and other methods, such as repeated addition, are used on other occasions. Each problem is connected to a set of candidate answers consisting of the correct product plus some incorrect answers. For adults it is assumed that the strongest associations are to correct products. For children, however, the distribution of associations will be "flatter", representing limited experience with multiplication. The recall process begins by randomly selecting a confidence criterion. Siegler (1988b) notes that "even 1-year-olds often will not state an answer that is suggested to them if they do not believe the answer is correct. Such resistance suggests that they possess a standard, akin to a confidence criterion, for stating an answer" (p. 261). An answer for a presented problem is retrieved according to the strength of the association between the problem and the answer: the stronger the association, the more chance the answer has of being selected. If the association to the proposed answer is stronger than the confidence criterion, the answer is stated. In the cases where the strength falls below the confidence criterion, another answer is selected. This loop continues until a maximum number of attempts has been made (1, 2 or 3—a number randomly selected at the start of the retrieval process). If no answer has been stated after the maximum number of attempts, one of two processes can, probabilistically, follow: sophisticated guessing, or problem elaboration.
Errors can arise from either: retrieving an incorrect answer that happens to exceed the confidence criterion; or from miscounting when using a back-up strategy. For miscounting, there is a fixed probability that a number is added twice or skipped over. Also, two numbers could be added incorrectly. For example, on $2 \times 8$, the model could produce $8 + 8 = 155$. The frequency of these errors is proportional to the size of the number being added, except for 5s problems which are added as accurately as 2s. Siegler (1988b) reports that children are particularly accurate at adding 5s, hence the exception.

Commenting on the DOA model, Dallaway (1994) states that although more explicit than many models, there are some details missing from the DOA account. For example, answers are retrieved from the associative network with a probability proportional to associative strength. This aspect of the model is not fully elaborated: is it a spreading activation system? If so, how does activation spread? By what mechanism are responses selected? As will be shown later, many models have focused on just this aspect of retrieval. The model contains a number of separate mechanisms: recall, elaboration, sophisticated guessing, and back-up. No details were given about the nature of the overall control mechanisms: what factors determine why sophisticated guessing is used sometimes (20 per cent of trials), but elaboration used at other times? How and why is the associative strength temporarily boosted for the elaboration stage? The simulation results reported by Siegler (1988b) were for just 20 problems due to limited resources, and this should be extended. Finally, it is assumed that back-up strategies behave in the ways specified. It would be interesting to see a
more detailed account of the structure of these strategies, explaining why they err in the ways that they do (p. 16).

2. Network interference

Campbell and Graham's (1985) “network interference” (NI) model does not concern itself with multiple strategies as the Siegler (1988b) model does. Rather, just the associative retrieval component is considered. The NI model suggests that problem presentation frequency and order determine the strengths of the associations in the network. Campbell and Graham (1985) assume that smaller problems are learned first, and are presented more frequently, than larger problems. These assumptions are not unreasonable, and chapter 3 looks in detail at presentation frequencies. However, for problem frequency there is an inherent difficulty in discovering the actual frequency experienced by subjects. Operand errors may occur because each operand unit is connected to a set of product units. On some occasions, the wrong product may be selected, giving an operand error. Close operand errors are produced because the general magnitude units activate products that are approximately the correct magnitude for the presented problem. Errors also occur because of false associations. During learning, inputs are associated with the correct product. However, due to the fact that activation spreads, other product units will be active. Associations will also be made to these, slightly active, false products. Learning usually occurs in lessons, where a number of problems will be presented and solved. Campbell and Graham (1985) argue that residual activation of problem units and product
units other than the one being solved will be associated with the current problem. In this way, false associations are formed, which may be produced as errors. In particular, it is assumed that problems close in magnitude (e.g., 7_8 and 7_9) are more likely to be learned in the same lesson than more distant problems (such as 7_8 and 7_2). As McCloskey, Harley and Sokol (1991) note, this suggestion is not backed by any empirical evidence. If true, it would further contribute towards the production of close operand errors.

However, Dallaway (1994) notes that this system is a conceptual analysis of arithmetic and raises a number of questions:

- Why so many kinds of nodes? What does each knowledge source contribute to the model? Are they all necessary?
- How do these knowledge sources develop? In particular, how are the magnitude units formed?
- How are problem-size exceptions handled? Although the system can accommodate exceptions, it would presumably have to be via problem frequency or order. Campbell and Graham (1985) briefly comment on the possibility of a rule-based system for 5s (the product should end in zero or five), but do not pursue it in any depth.
- By what means are the associations strengthened? A brief description was given, but without more specific learning mechanisms it is not possible to determine the relative importance of each kind of link.
Dallaway (1994) suggests that ‘symbolic models offer a very plausible interpretation of children’s arithmetic’ (p. 66). Solving problems like 320027 or 49_12 involves a host of skills: not only do you need to know the arithmetic facts, you also need to know how to borrow and carry, which column to process next, what to do when a number does not have a number below it (as in 12+5), and so on. It appears that students are following rules when solving multil-column problems. Perhaps the main evidence for this comes from the observation that children discover faulty rules (malrules) when learning arithmetic. Hence, it seems appropriate to use something like a production system to model multil-column arithmetic. Indeed, to date the only systems used to investigate arithmetic have been rule-based systems (Brown & Burton, 1978; Brown & VanLehn, 1980; VanLehn, 1983, 1990; Young & O’Shea, 1981).

At school children are introduced to multil-column arithmetic over a number of years in lessons of ever-increasing difficulty. Maths textbooks (Howell, Walker & Fletcher, 1979) suggests the following problem ordering for addition:

1. One column addition, sum < 10.

2. Two digit numbers, no carrying.

3. Addition of three rows.

4. Addition with gaps.

5. Two digits with carry.
6. Three-column addition, without carry.

7. Three-column with carry.

There is a similar sequence for multiplication by Howell, Walker and Fletcher (1979), and VanLehn (1990) identifies one for subtraction. By working through these stages students build up a hierarchy of maths skills, enabling them to tackle the next level of problems. Interestingly, Resnick and Ford (1982) discuss studies involving a "deep end" approach to arithmetic teaching, in which students are just taught the higher level skills. The reasoning behind his idea is that the student should be given meaningful problems, rather than many boring, relatively simple problems. Of course, students taught this way still end up learning all the prerequisite skills. However, the majority of students seem to learn best when led through a hierarchy of skills, and this is the way arithmetic is taught in most schools.

Explaining the bug phenomenon Dallaway (1994) notes that each lesson typically begins with the teacher working an example, and then the children solve similar problems on their own. Textbooks work in much the same way. An example is worked, by printing a snapshot of the various stages that need to be completed, and then there is a list of exercises. There are at least two multiplication algorithms that are taught in schools. The one considered here involves building up the product of a multiplier on one row. For example:
Studies of errors in arithmetic (Brown & Burton, 1978; Brown & VanLehn, 1980; Burton, 1982;) have characterised students as having buggy procedures – perturbations to the correct procedure. For example, one error for multiplication (zero-in-first-row) begins like this:

\[
\begin{array}{c}
3 \\
5 \\
4 \\
\end{array}
\times
\begin{array}{c}
6 \\
3 \\
2 \\
\end{array}
= 
\begin{array}{c}
0 \\
6 \\
3 \\
\end{array}
\]

The student has incorrectly inserted zero into the first column.

VanLehn’s (1990) Sierra theory of learning arithmetic is the most detailed account of multicolumn arithmetic to date. Like Young and O’Shea (1981), he focuses on long subtraction, but asserts that the principles should apply not only across the other three arithmetic procedures, but also to other domains. The account, a continuation of repair theory (Brown & VanLehn, 1980), tackles two problems of bug migration and learning. Three elements of his theory stand out:

- Children’s mistakes are the result of various kinds of syntactic changes to rules. This not only includes the faulty rules from the Young and O’Shea account, but also run time changes.

- Faulty rules are due to the skew in the school curriculum.

- Learning is from examples, not verbal recipes.
Some important observations that support the above points are:

- It seems that students do not understand the operations they are performing: they are symbol-pushing (VanLehn, 1990). There is now work considering the role of the semantics of arithmetic (Hennessy, 1990; Payne & Squibb, 1990; Ohlsson & Rees, 1991) but there are a number of reasons why it seems appropriate to just model syntax. To support his “aetiological assumption”, VanLehn points out that students who understand the arithmetic procedure they are trying to perform should be able to see if their procedure is wrong, and also be able to fix it. Intuitively it is clear that students can learn procedures without the slightest understanding of the underlying principles, and this point appears uncontroversial (VanLehn, 1990).

- VanLehn (1983, p. 82) points out that arithmetic texts are nothing like cookbooks or other kinds of manuals: they consist mostly of worked examples and exercises. In contrast to examples are “explanations” — some form of natural language information, perhaps given by the teacher in class. VanLehn (1990, pp. 96-103) argues that arithmetic is primarily learned from examples, and not explanation. His justification of this assumption first notes that AI has experienced a number of problems in translating from natural language to programs. However, there has been much more success in learning from examples. Second, VanLehn supposes that if children learned mostly via natural language, there should be language fragments in their bugs, such as references to the “tens place” or the “multiplicand”. However, VanLehn found that “85 per cent of all the observed bugs can be described with a small set
of visual/spatial features. No bug requires linguistic features for its description" (p. 102).

Understanding of learning models are useful in informing the teacher to understand the process involved in arithmetic performance specifically in diagnosing the algorithms using fact recall procedures. Such an understanding may not only help to diagnose the specific errors committed by the children but also design specific remedial programmes to address the deficits.

B. Error categorization

A number of professionals have developed error categorisation systems, models and 'kinds of bugs' (meaning errors) for arithmetic skill diagnosis (Attisha, 1982, 1983; Attisha & Yazdani, 1984; Buswell, 1926; Reisman, 1982) wherein they have tried to analyse the error frequencies made by subjects studying in primary grades and many battery of diagnostic test were also developed based on these categories to identify the errors and cumulative deficiencies in learning Arithmetic Skills. Some noteworthy studies on error categorisation for diagnosis of errors in acquisition of arithmetic skills are:

Note: Though the studies reviewed below dates back to a range of 25 – 100 years old. Studies have been reported as an attempt to trace historical source of identifying bugs in performing arithmetic tasks. In the current study errors committed by Children with Visual Handicap on arithmetical tasks have been analysed based on some of the below reported sources of bugs (e.g., Dallaway, Ainsworth, Cox and Buswell).
Buswell and John (1925) have produced checklist for ‘Information Assessment of Arithmetic Abilities’ for four fundamental process of arithmetic – addition, subtraction, multiplication and division. Buswell frequencies are based on total frequencies made by 263 subjects spread over Grades 3-6 (1926, tables XXXV to XXXVII, pp. 136–139) and the comprehensive checklist is useful even today not only for classroom teachers but also for researchers. Dallaway (1994) discussing the sources for bugs in addition and multiplication highlights contribution of Buswell (1926) and remarks that “The study lists the number of occurrences of various “habits”—consistent behaviours, but not necessarily behaviours that could be classed as “buggy”. For example, one particular habit, “added carried number last”, describes a subject who always added the carry digit after adding up a column, rather than before starting on the column. Buswell (op cit) notes that occasionally the subject would forget to add the carry, and this could be avoided if the subject added the carry first (p. 160). This kind of behaviour does not fit with the modern notion of a bug. However, many of the habits described do appear to be bugs, and are included here. Whereas Cox’s descriptions may be over specific, Buswell suffers the opposite problem. For example, Buswell describes the general bug wrong operator, when other authors give more specific bugs like multiplies-instead-of-adding or subtracts-instead-of-adding” (Dallaway, 1994, p. 137).

Cox (1974) tested children on four-multi colour task by having them complete tasks which were based on a number of levels. For addition, there were 8 levels each level becomes increasing difficult starting from 1 digit to 3-2 digit
numbers with renaming. For multiplication, there were 10 levels. She listed the bugs found at each level and then over the levels produced a categorisation of the bugs. She classified the bugs according to the kind of faulty knowledge that caused the error.

Reisman's (1982) error analysis system provides types of errors in not only in addition, subtraction, multiplication and division facts but also the details about the misconceptions or faulty learning among children in solving arithmetic tasks. He has identified 27 categories of errors.


Dallaway (1994) the time course and mistakes of adults and children solving arithmetic problems. Two models are presented, both of which are built from connectionist components. First, a model of memory for multiplication facts is described. A system is built to capture the response time and slips of adults recalling two digit multiplication facts. The phenomenon is thought of as spreading activation between problem nodes (such as “7” and “8”) and product nodes (“56”). The model is a multilayer perceptron trained with backpropagation, and McClelland’s (1988) cascade equations are used to simulate the spread of activation. The second model is of children’s errors in multicolumn multiplication. The system is analysed in terms of hidden unit activation trajectories, and the errors are characterized as “capture errors”. As the list of bugs (captured errors)
were very useful for diagnosis of Arithmetic Skills among Children with Visual Handicap for the current study, it is included in the Appendix - IV.

According to the connectionist view of Arithmetic Skills by Dallaway (1994), there are two major components in solving multiplication task – (i) mental arithmetic and (ii) multicolumn multiplication.

The basic idea is a simple one: memory for multiplication facts consists of a set of associations between operands and products; recall is the process of spreading activation, resulting in a product's activation exceeding a threshold. The activation spreads at different rates for different problems, giving different reaction times. Occasionally, when under some kind of time pressure, a false product exceeds the threshold. Most of these errors are operand errors, and the reasons for this are explored.

Regarding children's errors on multicolumn multiplication problems, it was noted by Dallaway (1994) that behaviour on these problems seemed to be rule governed. Children pick up a collection of "bugs" – systematic perturbations to the correct rules of arithmetic – and apply the rules producing all sorts of errors. For example:

\[
\begin{array}{ccc}
5 & 2 & 4 \\
\times & 7 & 3 \\
\hline
3 & 5 & 6 \\
\end{array}
\quad
\begin{array}{ccc}
7 & 6 \\
\times & 4 \\
\hline
14 & 2 & 4 \\
\end{array}
\]

In the first example, the child multiplies using the pattern for addition: \(1 \times 4 = 4\), \(3 \times 2 = 6\), \(7 \times 5 = 35\). The second example shows a child getting the first
multiplication, $4 \times 6 = 24$, correct. Then, when there is no second multiplier, the child uses the carry in the next multiplication: $2 \times 7 = 14$.

The splitting of Arithmetic Skills into two models – fact recall and procedural skills – is supported by studies of brain-damaged subjects (McCloskey, Aliminosa & Sokol, 1991; McCloskey & Caramazza, 1985). A structure on number-processing system was devised by noting that particular components of the system can be selectively damaged. For example, one subject (RR) was asked to read Arabic numbers aloud. For 37,000 he said “Fifty-five thousand”, for 2 he said “one” (McCloskey & Caramazza, 1985). Yet RR could determine which of two presented numbers was larger, and had no trouble selecting a pile of tokens that corresponded to a presented Arabic number. It seems that RR had no difficulty in comprehending and representing number, but was impaired in production alone. The model of arithmetic memory deals with the “arithmetic facts” and “abstract representation” parts of the figure. The multicolumn model is concerned with the “calculation procedures” part of the structure.

Campbell and Graham (1985) found that adults under mild time pressure make errors in multiplication at the rate of 7.65 per cent. Errors tend to be clustered around the correct product. More specifically, the errors can be classified as follows (McCloskey, Harley & Sokol, 1991):

- **Operand errors**, for which the erroneous product is correct for a problem that shares a digit (operand) with the presented problem. For example, $8 \times 8 = 40$
is an operand error because the problem shares an operand, 8, with $5 \times 8 = 40$.

- Close operand errors, a subclass of operand errors, where the erroneous product is also close in magnitude to the correct product. That is, for the problem $a \times b$, the error will often be correct for the problem $(a \pm 2) \times b$ or $a \times (b \pm 2)$. An example is $5 \times 4 = 24$. This phenomenon is referred to as the "operand distance effect".

- Frequent product errors, where the error is one of the five products 12, 16, 18, 24 or 36. These products happen to occur more frequently than most in the multiplication table defined over integer operands from 2 to 9.

- Table errors, where the erroneous product is the correct answer to some problem in the range $2 \times 2$ to $9 \times 9$, but the problem does not share any digits with the presented problem (e.g., $4 \times 5 = 12$).

- Operation errors, where the error to $a \times b$ is correct for $a + b$. These errors occur even in blocks of problems, which only contain multiplication problems. Hence, it is unlikely that they are only the result of a perceptual slip (Winkerlman & Schmidt, 1974).

- Non-table errors, when the answer is not a product found in the multiplication table. For example, $4 \times 3 = 13$, is a non-table error because 13 is not a number found in the multiplication table defined over $0 \times 0$ to $9 \times 9$. In the Campbell and Graham study, only 7.4 per cent of errors were in this category, and they are not shown in Table 13.
Each element represents the number of times a product occurred as an incorrect answer to the corresponding problem (c = correct). Given a total of 8640 trials (each subject tested on 4 blocks of 36 problems), and an error rate of 7.65 percent, it appears that each "c" in the table corresponds to an average of 125 correct recalls. In addition to the errors shown here, Campbell and Graham labelled 17 errors above 5 x 4 as involving products less than 20 (Campbell & Graham, 1985).
Table 14
Percentage breakdown of errors

<table>
<thead>
<tr>
<th>Category</th>
<th>Campbell and Graham</th>
<th>Harley</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operand errors</td>
<td>79.1</td>
<td>86.2</td>
</tr>
<tr>
<td>Close operand errors</td>
<td>76.8</td>
<td>76.74</td>
</tr>
<tr>
<td>Frequent product errors</td>
<td>24.2</td>
<td>23.26</td>
</tr>
<tr>
<td>Table errors</td>
<td>13.5</td>
<td>13.8</td>
</tr>
<tr>
<td>Operation error</td>
<td>1.7</td>
<td>13.72</td>
</tr>
<tr>
<td>Error frequency</td>
<td>7.65</td>
<td>6.3</td>
</tr>
</tbody>
</table>

(Figures in Table 14 are mean values for sixty adults tested on 2 x 2 to 9 x 9 from Campbell and Graham (1985), and 42 adults tested on 0 x 0 to 9 x 9 from Harley (1991)).

As can be seen from the table, most errors are operand errors, and more specifically, close operand errors. Both studies agree on this, but note the difference in the operation errors row. Harley's study included the 0 x 0 to 9 x 9 problems for which there are numerous errors of the form 0 x N=N. Approximately 11 per cent of the errors were 0 x N=N errors, and these have been classified as operation confusion errors in Table 14.

Brueckner and Elwell (1933) studied faulty procedure in multiplication of fractions. They sought to determine the extent to which incorrect solutions of examples involving identically the same skills are due to the same kinds of error or faulty procedure. This problem has important bearing on the reliability of diagnosis since undoubtedly many errors are chance accidental slips rather than definite indications of basic difficulties. This is likely to be the case when the pupil misses only one of a group of four similar examples.
In this study Brueckner and Elwell (op cit) had 327 sixth-grade pupils work four examples of each of six different types in multiplication of fractions – 24 examples in all, arranged in a random order. In their study Brueckner and Elwell (op cit) located 3458 specific errors, which were classified into ten major groups. Each of these major groups was divided into sub-types; in all there were 52 different sub-types, each one representing a peculiar variation of procedure. In the work of particular pupils these types of errors appeared in many combinations and variations. There was no particular combination of errors that appeared to be symptomatic of general disability.

The investigation by Brueckner and Elwell (1933) shows conclusively that errors in arithmetic processes made by superior as well as inferior workers/students check are highly variable and that the mental processes involved in arithmetic cannot be readily explained on a simple mechanical basis. If errors persisted steadily, or appeared in definite systems or patterns, the nature of the mental reactions of the learner might be quite readily analysed.

2.2.5.2 Studies on diagnostic evaluation in Indian context

Singh and Prakash (2001) reviewed the usage of diagnostic tests in the Indian settings and states that very little work has been done in the area of diagnostic testing. Their review of diagnostic tests in the area of Mathematics reveals that very few studies have been done namely: a diagnostic test related to the use of four fundamental rules of arithmetic was constructed by Patel (1976) Mehta (1966) developed a battery of diagnostic tests in arithmetic for Gujarati-
medium students studying in grades V, VI and VII in greater Bombay; Thakore (1980) constructed diagnostic tests on fractions and decimal fractions for students of V grade. H. L. Sharma (1969) constructed a diagnostic test in algebra for students of grade VIII of Uttar Pradesh. Ashar (1972) standardised a diagnostic test in basic algebraic skills for Gujarati-medium students of grade VIII, IX and X from Greater Bombay. The various studies reported above underlines the need for paying more attention to diagnostic testing in mathematics and developing remedial programmes.

Operating from the point of view that instruction and assessment are closely linked, that good teachers constantly assess students informally, that student self-evaluation is a vital part of learning, that formal assessments are stronger if they relate closely to the content and form of classroom instruction, and that documentation of assessment is important in connecting classroom work to external evaluation, mathematics educators may indeed be able to formulate assessment practices that will elicit improved mathematical achievement (Stenmark, 1991).

2.3 Studies on Piaget's Stages for Cognitive Development

Sensation, perception and cognition are essential for any learning to take place. Cognitive development can be considered as a gradual progression from overt actions, to internalised representation of actions and their consequences, to fully representational though that does not depend on immediate perception or action. Thus, internalisation is a key process in cognitive development. Gradually
the child acquires a more refined and appropriate set of classification skills and moves toward a refined understanding of the important properties of the world and objects in it, and toward the ability to deal with the world in representational terms. Perception is very important to cognition. This, then raises the question, how do Children with Visual Handicap develop cognitive processes in the absence of perceptual limitation – vision. A lot of research work has been done in the area of cognition of Children with Visual Handicap in general and in relation to the spatial perception and cognitive processes. However, very limited research is reported in relation to cognitive processes involved in learning a specific subject area for e.g., mathematics.

Below an overview of certain studies done in the area of development of cognitive processes in general among Children with Visual Handicap are given.

2.3.1 Studies on cognitive capabilities – General

Focusing on different types of conservation – number, length, area, quantity etc. – there are good evidences to prove that the children acquire the contents associated with Piaget’s three stages – No conservation, Transition, and Conservation in an invariant sequence.

A correlational study carried out by Goldschmid (1968) and McNally (1971) on conservation of different concepts shows that the tests on substances, continuous quantity and discontinuous quantity conservation score based upon a much larger battery of conservation tests.
Evaluation of the training corollary of ordinality is provided by a number of training studies that demonstrate that success depends critically on what the child currently knows. For example, it had been suspected that compensation (the realisation that an increase in one physical dimension was compensated for by a decrease in another dimension) was a pre-requisite to learning about conservation. This was supported in an experiment in which trained compensators learned more about conservation than did either untrained compensators or trained non-compensators (Curcio, Kattef, Levine & Robbins, 1972).

Flavell (1971) has analysed the essentials of what Piaget seemed to mean by the concept of stages and reviewed the literature to determine whether these essentials are supported. He noted four principal implications of the stage concept. First, the change from one stage to another was to be qualitative rather than quantitative. It was not simply a matter of adding more information, but rather emergence of a substantially different way of processing information. Second, stage changes were supposed to be concurrent. That is, there were simultaneous and similar changes across many domains at once. Third, the transition between stages was to be abrupt rather than gradual. Fourth, stages were highly organised, rather than containing independent, unrelated ideas. That is, cognitive items are interrelated so as to constitute an organised whole.

Ghatala (1973) study on cognitive operations specified in the model revealed that in conceptual learning and development, there are four levels of
mastery. The internal conditions of concept learning comprise the level of operation as well as the acquisition of the concept preceding this level.

There is considerable agreement that, where stages of cognitive development occur, there is a strong tendency for children to proceed through them in a uniform order. However, there is typically some degree of skipping of stages and regression to earlier stages. An example of ordinality occurs in the 6-stage development of the concept of physical objects. Describe this development (Kramer, Hill, & Cohen, 1975)

Shayer and Adey (1978) used Science Reasoning Tasks to assessing stages of Cognitive Development of pupils (N = 12,000) from the schools of England and Wales. Study reveals that 40% of the ten year olds are at or above the stage of Late Concrete Operational Thinking, while nearly 90% of children have reached this stage by age of 15. In practical terms this means that in the last year of an average primary school, most material would have to be pitched at the early and midconcrete level for some 60% of the pupils.

In Vasundhara's (1992) study on 11-13 year old upper primary pupils, the percentage of children in the early formal stage was 1 to 37 percent in 11 year olds, 5 to 40 percent in 12 year olds and 6 to 45 percent in 13 year olds. Most of the children were found to be in the transition stage with respect to geometrical sections, conservation of weight, combinatorial thinking and proportional thinking.
Schultz (1991) investigated into the Simulating Stages of Human Cognitive Development with Connectionist Models, observes that investigation into the issue of connectionist simulations led to new insight about stages — the idea that stages result from solving part of a problem before solving all of the problem. Because there are several different ways that partial solutions can occur, this basic insight then unravelled into hypotheses about different sources of stage: hidden unit herding, over-generalisation, training pattern bias, and hidden unit recruitment.

2.3.2 Studies on cognitive capabilities – Children with Visual Handicap

Zweibelson and Barg (1967) reviewed some of the earlier literature and studied the concrete, functional, and abstract levels of concept formation in blind children in comparison with Sighted children. The sample was small (eight in each group), but carefully selected. The primary instrument of measurement was the Wechsler Intelligence Scale for Children. Using nonparametric tests because of the smallness of the sample, the investigators tested the hypothesis that the blind children would use as many abstract concepts as the Sighted children, and they rejected it at the .05 level. The authors point out that their findings are in agreement with those of Hayes (1941, 1950), who found that blind children tend to obtain lower scores than Sighted children on reasoning tasks, and those of Rubin (1964), who found that deficiencies in concept formation in the congenitally blind tend to persist into adulthood. A detailed explanation of the source of these related deficits is not to be found in the literature and is not obvious.
Warren (1984) states that there are clear areas of cognitive development in which Children with Visual Handicap lag developmentally behind Sighted children. Two major hypotheses are offered to account for these lags. One possibility is that lags in the development of the cognitive capabilities of early and middle childhood, such as classification and conservation, are the result of carryover lags from the sensorimotor period.

A second view was offered by Gottesman (1976), who suggested that the observed delays in cognitive development in early school-age blind children are partly the result of the child’s reliance on ‘less sophisticated sensory discrimination abilities’. He found significant lags in the acquisition of various conservation concepts, but these lags tended to decrease with increasing age.

Gottesman (op cit) suggested that although lags occur initially, Children with Visual Handicap gradually become able to perform conservation tasks because of their ‘increased reliance in integrative processes of cognitive functioning, rather than a reliance on the less sophisticated sensory discrimination abilities.

Investigation with Piagetian tasks on conservation of mass, weight and volume was conducted by Gottesman (1976) and he found the differences on these tasks between blind and Sighted children to disappear at the age level of 8-11 years. Of significance of his observation was that (their) “increased reliance on integrative processes of cognitive functioning rather than a reliance on the less sophisticated sensory-discrimination abilities” (pp. 94 –100).
Identifying the learner needs of the Children with Visual Handicap, Mukhopadhyay et al. (1987) states that a Sighted child attains his conceptual level by learning the attributes of an object in terms of its existence, having permanence and differing from other objects. He learns to define the characteristics of the object. When he passes from concrete to abstract stage of learning, he learns to abstract common elements from several sensory experiences and acquires the skills of generalisation. It is not necessary for a Sighted child to be taught about every object that exists around him. He learns about them because of his multi-sensory capacities to approach them. The visually impaired child, however, does not follow the same path. He cannot see the objects in their wholeness but moves from parts to whole. His experience is limited because information through tactual manipulation, auditory reception and olfactory sensation cannot be a substitute for visual perception. It will be very difficult for him to learn all the attributes of an object in a single experience. In order to know how a cow differs from a horse, he does not have the same ‘global’ experience that a Sighted child will have in associating the message of all the senses simultaneously – seeing the shape, perceiving the size, colour and associating the sound that is produced by each. He has to go for step-by-step assimilation of attributes and that too when it is brought nearer to his reach. He does not ‘spot’ it. Sometimes, he may take longer time to develop a sense of object permanency. Naturally, his association of objects to their functions is also restricted. The learning at abstract level causes all the more problems (pp. 179 - 180).
2.3.3 Studies on Piaget's cognitive development stages in relation to
development of mathematical skills

Does visual handicap mean mathematical inability? Various treatises and
theses are available to discuss the nature of mathematics per se; few attended to
the experience of mathematics. The researchers in the field of cognitive
development of children have revealed that in his attempts at assimilation of the
environment, the child makes a succession of adaptations achieving various
degrees of ‘equiliberations’, which is dynamical in nature. In passing through the
stages of sensori-motor, pre-concrete operation, concrete operations to formal
operations there is a tendency of development to progress from relatively simple
to successively complex, from relatively unstable to progressively stabler forms
of equiliberations within the physical capacities of the child. Individuals may differ
as to the pace and extent of this equilibration with the environment but on this
view the sequence of development can’t be altered skipped or hastened to any
large extent. (Gupta, 1992).

Duval (1995) in relation to learning of mathematics probes: Is the way of
thinking the same in mathematics as in the other areas of knowledge? In other
words, does mathematical activity require only the common cognitive processes
or, indeed, certain very specific cognitive structures whose development must be
taken care of in teaching? Issues about the learning of mathematics has a great
significance if the goal of teaching mathematics, at the primary and secondary
level, is neither to train future mathematicians nor to give students tools, which
can only possibly be useful to them many years later, but rather to contribute to
the general development of their capacities of reasoning, analysis and visualization. In any case, it makes it necessary to consider semiotic representations at the level of mind’s structure and not only with regard to the epistemological requirement for getting access to knowledge objects (Duval, 1995a, pp. 3-8, 15-35).

The steps in developing mathematical concepts substantively are not different than those in other area. Van Engen (1953), in the 21st Yearbook of the NCTM, listed the steps as: Sense-perception – abstraction – generalisation. That is, mathematical abstractions can be derived substantively only through sense-experiences. Children, in general, find difficulty in the formation and use of abstract concepts and symbols. Mostly, they get blocked because they have no experiences to relate the new concepts substantively to their existing cognitive structures. Instead, exclusively symbolic material is fed to them directly through the textbook or the chalkboard or the teacher talk. As a consequence, concepts are not properly formed. Reinforcing Van Engen’s view point on the steps for learning mathematics, Duval (1995) discusses the semiotic representations in learning mathematics. Duval considers that Mathematical objects, in contrast to

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3 It is the general task of sign theory (semiotics) to explore possible structures and functions of sign systems (theoretical semiotics), to describe and compare actual sign systems (descriptive semiotics), and to recommend particular sign systems for particular purposes or, if necessary, to devise new sign systems (applied semiotics). (Roland Posner, 1996, p. 10)

4 Mathematical objects (numbers, functions, vectors, etc.) are knowledge objects, and semiotic representations which can support two quite opposite foci of attention (either the visual data given or some represented object which can be a concrete one or some invariant) are transient phenomenological objects. If we consider an algebraic equation and the graph of a line, they are first different semiotic representations. They are "mathematical objects" under the condition that attention can focus on some invariant (the assumed represented relations) and not only on their visual data and their perceptual organization (Duval, 1995b, pp. 53-54; 2002). It is only from a strict formal point of view that semiotic representations can be taken as concrete objects (Duval, 1998a, pp. 160-163).
phenomena of astronomy, physics, chemistry, biology, etc., are never accessible by perception or by instruments (microscopes, telescopes, measurement apparatus). The only way to have access to them and deal with them is using signs and semiotic representations. The crucial problem of mathematics comprehension for learners, at each stage of the curriculum, arises from the cognitive conflict between these two opposite requirements: how can they distinguish the represented object from the semiotic representation used if they cannot get access to the mathematical object apart from the semiotic representations? And that manifests itself in the fact that the ability to change from one representation system to another is very often the critical threshold for progress in learning and for problem solving (p. 107). Given this viewpoint, perception in the form of signs is essential for cognition of the mathematical symbols and learning mathematical abilities. Children with Visual Handicap are deprived at the sense-perception level by vision – the gateway to meanings (abstraction) and knowledge (generalization). The alternate way of sense-perception through touch poses further challenges in the representation of the symbols necessary for mathematics learning.

Keeping in mind the challenges of sense-perception by vision as a limitation in learning mathematics, some noteworthy studies to reinforce the assumption that Children with Visual Handicap lag behind in acquisition of number concepts and skills are given below:
Beard (1961) found that the order of difficulty of various number concepts and the order of the stages at which they are grasped is as Piaget claimed. However, she found that both the speed with which concepts are attained and child's approach to experimental situations are affected by intelligence.

An examination of children's style of learning certain mathematical structures led Dienes (1965) to relate various styles of thinking to such characteristics as anxiety, introversion-extraversion, etc., which are usually considered to be personality traits. He suggested an analysis of conceptual style in terms of three dimensions – The open-closed dimension: the degree to which an individual who has grasped one concept goes on to further stages of conceptualising; The explicit-implicit dimension: the degree to which an individual finds it necessary to formulate a concept before he can use it; The constructive-analytic dimension: the degree to which an individual thinks intuitively rather than in terms of logical relationships.

Juurma (1967) investigated the ability structure of children in relation to loss of vision and his factor-analysis revealed that blindness did not hamper differentiation of abilities. No factor was found for any retardation rather at an early stage the blind excelled in mental arithmetic.

Nash (1969) in his detailed study on number and spatial concepts of Children with Visual Handicap demonstrated that the children with visual perceptual difficulties functioned at a lower level of spatial reasoning.
Jordan and Jensen (1979) in a review of correlational studies of measures of Piagetian Cognitive Development and Mathematics Achievement has indicated that correlations between Piagetian and psychometric tests and chronological age are influenced by the size of the age range of the sample groups. To examine whether the size of the age range and mean age are related to the size of the correlations between Piagetian and arithmetic tests, Spearman rank-order correlations were obtained between these age-related variables and the Piagetian and arithmetic test correlations, however, an inverse and the size of the correlation between Piagetian and arithmetic tests, indicating that groups with younger mean ages tended to have larger correlations than groups with older mean ages. The data suggest that a moderate relationship exists between Piagetian and arithmetic tests. Many conserving children perform higher on arithmetic tests than do their non-conserving peers, although some non-conserving children nevertheless attain comparable levels of arithmetic performance. These results suggest that Piagetian tests might be used as a diagnostic aid for children who are experiencing difficulties in arithmetic (Elliot, 1982; Farrell, 1980; Kenny, 1978; Smith, 1973).

In relation to learning of arithmetic skills, Zelazo and Shultz (1989) found that a subtraction rule is learned first, and before a division rule is learned, children over-generalise the subtraction rule to division problems. Cascade-Correlation networks simulated these and other psychological results, further clarified that networks learned subtraction first, and then temporarily over-generalised subtraction to still unlearned division problems. The stage of over-
generalisation was eventually replaced by a more mature stage in which subtraction was restricted to subtraction phenomena and division applied to division phenomena.

In a study conducted by DeCorte and Verschaffel (1981), the investigators used error analysis and individual interviews to determine the problem-solving activities of first- and second-grade students completing addition and subtraction problems. The errors made by the sample of children were principally of two types, which the investigators categorised as thinking (or algorithmic) and technical (or computational). Seventy-eight percent of the errors were judged to be algorithmic, while 13% were computational. The data, which were collected by having the students think aloud or report retrospectively their problem solving approach, reveal that a number of students did not appear to engage in any thinking whatsoever in their solutions but, rather, performed by rote. More interesting were the thoughts of those students who did engage in mental activity when confronted with an unfamiliar problem type. The investigators observed two different approaches in these students’ problem solving. The first approach was categorised as semantic. Students using this approach transformed the task into a cognitive scheme available to them. The second approach was characterised as intelligent trial and error, a circuitous and less efficient but nevertheless inventing strategy. Then conclusions are evident. Cognitive structures of children are not affected by visual loss. What they grow deficient in are primarily their perceptions and secondarily their symbolic abstractions gained through visual perception. Given alternative perceptual inputs through touch or otherwise and
the time required for their effective understandings substantiated by teacher-gu­
guidance, the visually handicapped children should not find mathematical
concepts an impossibility or a distant possibility for themselves (Gupta, 1992).

The study of cognitive processes in handicapped children is an
opportunity both for important theoretical work and for direct application of
significant theoretical results to practical problems of education (Suppes, 1974, p.
170).

2.3.3.1 Studies of Piagetian tasks in relation to development of number
concept - General

Some of the key experiments conducted by Piaget as described in the
book ‘Child’s Conception of Numbers’ (1952) has been comprehensively
reviewed by Jones (2005). Experiments conducted by Piaget (1952) narrated by
Jones (op cit) is given below:

Experiment on Conservation of Continuous Quantities

The child is shown two identical containers holding an identical amount of
liquid. One container is poured into two smaller containers of equal dimensions
and the child is asked whether the quantity is still the same. At stage 1 the child
has no conception of conservation of volume which Piaget explains in terms of
perceptual intuition. The child says there is more or less depending whether he
attends to the height or width of the second container. At stage 2 the child
qualitatively attends to logically multiplying the height and width but with limited

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success, because perceptual intuition is still a strong factor and so "when he is concerned with the unequal levels, he forgets the width [and vice versa]" (p. 16).

At stage 3 the child answers correctly without difficulty. He conceives the liquid as an intensive quantity, is capable of reversible operational thinking and, as such, the law of conservation is obeyed in his answers. Stage 3 is significant in the child’s construction of the concept of number because it demonstrates "the process of quantification as revealed in the child’s discovery of the conservation of quantities" (p. 23).

**Conservation of Discontinuous Quantities**

The first experiment is repeated, this time using beads arranged in two lines and asking the child to consider which line of beads would make the longest necklace. The same stages are discovered again, although this time the logical multiplicative relationships are length and density (rather than width and height). At the 2nd stage the child is able to test the equivalence of two lines of beads using one-to-one correspondence, which conflicts with perceptual intuition: "When the child considers the sets of beads he thinks that there is non-equivalence, and when he remembers the correspondence he again thinks that the sets are equivalent" (p. 32).

At the 3rd stage operational mechanisms dominate perceptual intuition: "the child is already conscious of the invariance owing to the one-to-one correspondence, and only has to explain the apparent variations, [and] he resorts
to multiplication of the relations, thereby co-ordinating all the relationships in question in an intensive quantifications” (p. 36).

**Provoked Correspondence**

Next Piaget investigated provoked correspondence – in which two sets have an obvious link, such as eggs with egg-cups and swapping pennies for sweets. Stages similar to those of experiments 1 and 2 are discovered again. Piaget concludes that one-to-one correspondence does not “necessarily entail[s] in his mind the idea of lasting equivalence between the corresponding sets (p. 42) although activities such as counting aloud may, no doubt, hasten the process of evolution. Our only contention is that the process is not began by numerals as such” (p. 64).

**Spontaneous Correspondence**

In this experiment Piaget attempts to analyse how children spontaneously estimate the cardinal number of a set using one-to-one correspondence. This time the sets used lacked an obvious link and the child was asked to “pick out the same number” (p. 65) to observe what method was used. Sets were presented in a variety of formats from randomly distributed counters through to closed figures dependent on the number of counters used. The three stages emerged again. Children operating at the 1st stage (who constructed a replica set based on global, qualitative properties) were more accurate with familiar figures dependent on the number of counters used than with unfamiliar figures, figures not
dependent on the number of counters used, or random distributions. Children operating at the 3rd stage were able to conceive the reversibility of operations – i.e. permanence of equivalence arises from (i) the initial one-to-one correspondence plus (ii) knowing that sets made asymmetrical can be made symmetrical again thereby restoring global, qualitative equivalence.

**Seriation**

The first four experiments considered cardinal correspondences in which elements were identical and order unimportant. Piaget calls this order ‘vicariant’ “which means that either of two elements can be the first or second, provided there is a first and a second” (p. 96).

The experiment is repeated using elements that have a natural ordinance (such as sticks of increasing length) to see if it has an effect on the child’s sense of “lasting cardinal equivalences” (p. 97). It is discovered that a “co-ordination between ordination and cardination” (p. 120) has its beginnings at stage 2, but is not complete until stage 3: “Ordinal correspondence is therefore acquired on the operational level, owing to its fusion with cardination” (p. 121).

**Ordination and Cardination**

Next Piaget investigates the relation between ordination and cardination. Sticks of increasing length are again used along with the metaphor of a staircase. The child seriates the sticks as before but this time is also asked how many steps have been walked up to get to a given step (stick). The instability of seriated
elements before the 3rd stage make cardination difficult. The child might count from the wrong end, and if the elements are mixed up is unable to state the cardinality of a given element. Only at the 3rd stage do cardinality and ordinance become operational and therefore coordinated. Experiments 5 and 6 raise the issue of classes. Elements with a natural ordinance are not equivalent but are members of a class such as stick.

*Number* is indeed possible to the extent that the elements are viewed no longer as being either equivalent or non-equivalent, but as being *at the same time* equivalent and non-equivalent. (p. 156). A cardinal number corresponding to a seriated element implies an asymmetrical relation because a set A consists of two unequal sets $A'$ (the cardinal number) and $A'' (A - A')$. A cardinal number is a class whose elements are conceived as 'units' that are equivalent, and yet distinct in that they can be seriated, and therefore ordered. Conversely, each ordinal number is a series whose terms, though following one another according to the relations of order that determine their respective positions, are also units that are equivalent and can therefore be grouped in a class. Finite numbers are therefore necessarily at the same time cardinal and ordinal, since it is of the nature of number to be both a system of classes and of asymmetrical relations blended into one operational whole. (p. 157)

**Relations between Class and Number**

Where as the previous experiment implied that the logical structures of class and asymmetry are the basis of number, this experiment shows that
number may equally be regarded the basis of logical structures. It involves the child being asked to make statements and groupings of sets of elements comprising subsets of elements (such as a set of flowers comprising pink and blue flowers.) Piaget concludes that “class and number have a common mechanism, that of their additive and multiplicative operations” (p. 181). To seriate elements the child attends to differences between elements. To group elements the child attends to the equivalence of elements. Number is at the same time a class and an asymmetrical relation, the units of which it is composed being simultaneously added because they are equivalent, and seriated because they are different from one another (p. 184).

**Arithmetical Relations of Part to Whole**

The child is presented with two lots of identical elements. Each lot is grouped as additive compositions such as ‘4 + 4’ and ‘1 + 7’ and the child is (i) questioned about their equivalence. In two further variations of this experiment the child is asked (ii) to equalise unequal sets and (iii) to divide one set into two equal groupings. The purpose is to investigate the difficulties the child has with the “inclusion of classes into a total class” (p. 185) and vice versa. Predictably, the three stages emerge once again.

Piaget concludes that this experiment again confirms his view that number identity arises from grouping and seriating, or colligation and enumeration as he now calls it, with reversibility distinguishing operations from empirical construction, and being the key to achieving the third stage.
2.3.3.2 Studies of Piagetian tasks in relation to development of number concept – Children with Visual Handicap

Number of researchers have tried to replicate the experiments conducted by Piaget (1952) to understand how the child develops the concept of number? Some of the researches are highlighted below:

An adaptation of the original Piagetian number task used by Good (1973) to study the development of Conservation of Number, revealed that, before the age of 7 to 8 years a majority of the children were able to only establish one-to-one correspondence, but were not, able to conserve number concept. But, among 7 to 9 years of age more than 3/4th of the subjects have attained conservation of number, favourable comparing with those of Piaget and others.

Piaget (1952) holds that for conservation to appear the child must be able to perform the following operation: multiple classification, seriation and reversibility. In this regard, Canning (1957) and Tobin (1972) studied the Conservation of Number and Substances respectively, confirming that blind children attain conservation later than the Sighted children. A lag of almost three years is observed. Phemister (1962) indicated that in free play that involved certain contrived situation, might help in fostering the conservation of number.

Friedman and Pasnak (1973) in their study on attainment of seriation and classification concepts by blind and Sighted children confirmed a significant difference between the two groups and the developmental lag being two to three years in the case of blind children.
Similar observations were made by Higgins (1973) while he investigated the two samples on classification ability. His interpretation is even more specific and assuring the condition of total congenital blindness per se is not sufficient to produce a delay in the formation of intellectual structures underlying classification (the deficiency) appeared to be of perceptual (figurative) and symbolic rather than intellectual (operative) origin.

The most noteworthy research in this area in India has been done by Mandarvalli (1990). This study has clearly demonstrated the validation of Piaget's stages of cognitive development among Children with Visual Handicap wherein the development of number concept in particular is highly relevant to this study. Mandarvalli’s (op cit) study focussed on attainment of concrete operational stage among Children with Visual Handicap. The study was conducted on a sample of 190 visually handicapped (age range 6 – 14 years) studying in five residential special schools for the blind. The sample was also representative of the nature of visual handicap (Totally blind at birth; Totally blind after birth; Partially blind at birth; Partially blind after birth) and gender - boys and girls. Two of the major objectives of her study were to

i). analyse the developmental trend among visually handicapped children in acquiring selected concepts and operations to attain concrete operational stage as described by Piaget and his associates on the basis of the stage criteria /classification given by them for each sub-stage with qualitatively different characteristics
ii). Analyse the developmental trend among visually handicapped children in acquiring the selected concepts and mental operations as represented by their performance level on a test of cognitive capabilities with sub-test scores as well as a total test scores as an index of their cognitive development status.

As the current focus of the study was on enquiring the development of number concept among Children with Visual Handicap hence, among the various cognitive tasks selected for the study by Mandarvalli (1990) i.e., Number, Length, Area, Quantity – Clay, Quantity – Sand, details of the results and analysis of the performance of visually handicapped on the 'Number Conservation' task in her study is discussed below.

Table 15

Results of “Number Conservation” (Mandarvalli, 1990)

| ‘M’ Age in years (N) | NA | Percentage of Visually Handicapped Three Main Stages | NC | TC | CC | NC | RA | WA | I | CC | E |
|---------------------|----|--------------------------------------------------|----|----|----|----|----|----|----|----|----|---|
| G₁ 7 (40)           | 33 | 47 10 10       | 5  | 42 | 6  | 4  |
| G₂ 9 (39)           | 17 | 62 5 16       | 27 | 35 | -  | 16 |
| G₃ 11 (39)          | 24 | 50 - 26      | 13 | 37 | 6  | 20 |
| G₄ 13 (35)          | -  | 43 9 48      | 15 | 28 | 20 | 28 |
| G₅ 15 (37)          | -  | 24 - 76      | 14 | 10 | 24 | 52 |

NA – No Answer  
NC – Non-Conservation (RA – Romancing Answer; Wa – Wrong Answer)  
TC – Transitory Conservation  
CC – Complete Conservation (I – Identity Thinking; E – Equality Thinking)

From the percentage analysis of the performance of visually handicapped (Table 15) on the 'Number Conservation' task at each stage of development with respect to their age it was observed by Mandarvalli (1990):
- eventhough the percentage of visually handicapped in NA, was quiet high in the first three age groups G₁, G₂ and G₃ (with fluctuations), visually handicapped of higher age groups G₄ and G₅ did not reveal any sort of inability / reluctance to involve themselves in the task as there was non in these groups with 'NA'.

- There is steady increase in the percentage of visually handicapped in stage III and a steady decrease in stage I (except a variation at G₂) implying a gradual development of conservation of number with respect to their age.

- The rate of increase in the percentage of visually in stage III is quite rapid after 11 years as against a very slow progress indicated from 7 to 9 years group.

- The operational conservation increases with age in both the modes of thinking - identity and equality

- There is a clear dominance of equality reasoning over identity reasoning in all the age groups except in G₁; it is more than double in the highest age group – 15 years, implying that the visually handicapped use equality reasoning more commonly than the identity reasoning in conserving the concept of number.

The results of the number task in the study by Mandarvalli (1990) on the whole agreed with the observations noted by Piaget. The visually handicapped too discover the invariance of number taking note of the fact that grouping together or spreading out the elements of a set no way affects its number. Mandarvalli (1990) concluded from her study that visually handicapped follow the
same sequence of developmental stages as ascertained by Piaget for normal children in attaining the conservation of number. But there exists a developmental lag in visually handicapped with respect to their age. Most of the visually handicapped pass on from stage I to stage III implying that transitory stage is not an essential sub-stage for the visually handicapped to attain operational conservation of number. Further, there appears to be a rapid growth in the development of conservation of number in visually handicapped after 11 years of age.

Some of the other noteworthy findings of the study conducted by Mandarvalli (1990) are:

- A clear developmental lag with respect to their age is conspicuous in attaining the concrete operational level of thinking among Children with Visual Handicap.

- Children with Visual Handicap attain Cognitive Development Status at a later stage as compared with the Sighted children and also the chronological age of Children with Visual Handicap and their CDS is not linear.

- A relationship between the Nature of Handicap and Cognitive Development Status of Children with Visual Handicap was found to be insignificant, implying that the Nature of Handicap does not influence the cognitive development of Visually Handicapped to a considerable extent.

- A definite developmental lag was observed among Children with Visual Handicap in all the five conservation tasks (number, length, area, quantity –

- Interestingly, owing to the action programme that contained structured activities designed to foster cognitive development among Children with Visual Handicap reveals that a properly designed special program of activities with focus on appropriate concepts and operations can foster cognitive development to a considerable extent over a reasonable period of time.

Lister, et al. (1996) investigated extent of Similarity in Concept Development for Visually Impaired and Sighted Children. Through seriation, verbal seriation, and conservation tasks, investigated blind, partially Sighted, and Sighted children's understanding of quantity. Subjects were 81 children equally dispersed through these 3 groups. Age range was 4 to 17 years. Found similarity in concept acquisition among three groups that extended beyond quantity conservation concepts.

Lister, et al. (1993) studied development of quantity concepts in children with special needs between 7 and 11 years of age with moderate learning difficulties were tested for conservation. Nonconservers were divided into experimental and control groups. Experimental group children performed conservation tasks with conserving children in their class. Post tests indicated
that the increase in performance in conservation tasks was greater for the experimental than the control group.

The above studies are noteworthy from the point of view that concrete operational stage, according to Piaget (1952) is notable by the discovery of conservation has implications in the development of Arithmetic Skills among Children with Visual Handicap. The studies (Canning, 1957; Friedman and Pasnak, 1973; Gottesman, 1973; Hatwell, 1966; Higgins, 1973; Mandarvalli 1990; Phemister, 1962; Piaget, 1952; 1988; Simpkins & Stephens, 1973; and Tobin, 1972) reveal that there is a clear lag in the acquisition regarding conservation of concepts among Children with Visual Handicap. Conservation of number forms the foundation for the development of other arithmetical concepts and the lag in the former has implications for the later concepts also. An in-depth study is augmented in this area that would help in knowing the development of skills in Arithmetic among Children with Visual Handicap and to design suitable program to foster the same.

Drawing heavily upon the study conducted by Mandarvalli (1990), investigator has made an earnest attempt at enquiring the attainment of conservation of number as suggested by Piaget and probe its influence if any on the performance of the Children with Visually Handicapped on the arithmetic tasks during the formative years of education – lower primary.
2.4 Studies on Inclusive Education

2.4.1 Inclusive education – studies on general practice

Inclusive education as defined by Helander et al. (2000) focuses on building of support system that is integral component of the overall educational system. Development is ultimately about people having control over their lives. Charity is about people remaining as victims, controlled by others remarks, Coleridge 1993). Emphasising on the integration of the disabled, CBR states that we have to refocus our efforts away from adapting the people to society and towards adapting society to the people instead. To achieve the goals of ‘full participation and equality’, it is not enough to rehabilitate the disabled person. Denying the disabled person opportunities that are available to other members of the society, too, is disabling.

In the past, services were segregated which often led to isolation, neglect and prejudice. Today’s efforts are towards active and outward looking services that recognise persons with disability as natural members of society. This is where the new thinking differs profoundly from the traditional concept of segregated education.

In this new perspective, integration ceases to be the primary concern. Creating a school for all – an inclusive School – becomes a new priority. Such a school will recognise and respond to the diversity of the individuals that make up the school population at large. It will accommodate individual differences. As a
result, it will create a climate of Inclusiveness where nobody is pushed out and no group of children is denied access.

Foreman (1996) remarks on the integrated education that the last 20 years have seen strong movements toward the integration of students with special needs in regular classrooms. There is now widespread acceptance at a theoretical and administrative level that students with special needs have the right to participate in mainstream schooling. The research outcomes and social justice arguments, however, often fail to persuade teachers who feel that they are unprepared to teach, or that their school lacks the physical facilities to include children with special needs.

Discussing barriers to inclusive education, Ring and Travers (2005) conducted a case study of a pupil with severe learning difficulties. The study was conducted in a school in Ireland. The aim of this study was to examine the inclusion of a pupil with a severe general learning difficulty in a four-teacher mainstream primary school, located in rural Ireland. The research employed a qualitative multiple operationism approach to data collection. Data were analysed qualitatively, and quantitative reporting and display procedures were also employed. This paper focuses on curricular and social access, the pupil's perception, peers' perception and the impact on peers. The study identified the existence of a number of dilemmas in seeking to secure successful inclusion. These included concerns over specialist teaching materials, mainstream teachers' perception of meeting the needs of pupils with special educational
needs as constituting an esoteric specialist domain, non-disabled pupils' lack of knowledge and understanding of learning disability, and the extent to which the pupil was included socially. Questions are raised about the model of support for inclusive education in Ireland.

For successful implementation of inclusive practices in schools, Waite, Bromfield and McShane (2005) argues that comparison of perceptions helps to clarify some of the factors that may be influential in successful development of inclusion in schools. A phenomenological approach places value on the affective component of "voice" as a "true" measure of success. In regards to strategies for successful implementation of inclusive practices in school, Smith and Leonard (2005) notes that the challenge for educational practitioners is to find ways to implement high-quality special education programs collaboratively amid the public call for school efficiency and accountability. In a qualitative research conducted by Smith and Leonard (op cit), examined inherent challenges in the implementation of school inclusion programs in ten public schools in North Louisiana over a three-year period. Data collection methods included participatory observation, semi-structured interviews with nine teachers and three principals in four schools, two focus groups, and a document search. The findings revealed the critical and challenging role of the principal for establishing collaborative cultures for successful school inclusion. Additionally, special education teachers and general education teachers experienced intrapersonal and interpersonal value conflicts in the pursuit of educational equity amidst a climate of school accountability.
Wall (2002) investigated whether the amount of teachers' previous exposure with visual impairments influenced their point of view about the inclusion of students with visual impairments in general education classrooms. Surveys were sent to three different groups of general education teachers in Canada. A total of 96 teachers participated in this study, each group of teachers had different experiences with students with visual impairments in school settings. Teacher experiences ranged from having working with students with visual impairments in their classrooms to no experience at all (randomly selected). Wall divided participants' responses about including students with low vision and/or blindness in general education classrooms into 3 different categories: positive, negative, and qualified. A major finding of this study was that teachers who have had direct or indirect experience with students with visual impairments were more positive about inclusion than the randomly selected teachers. Wall recommends that the amount of positive contact between teachers and students with visual impairments should be augmented before the teacher is expected to teach a student with visual impairment.

2.4.2 Teacher preparation and attitude towards inclusive education

The dramatic shift that the implementation of policies of inclusion has brought requires the involvement of all stakeholders. Teachers in particular are in the frontline of implementing whatever inclusive practices are adopted, and it is for this reason that their perspectives on crucial issues are essential if successful inclusion is to be achieved (Fuschs & Fuschs, 1994).
Emphasising on teacher preparation for inclusion practice in elementary schools Varma (2007) states: Quality education for everyone mean special approaches for those with disabilities and addressing to their needs. The emphasis is therefore laid on inclusive education that uses the existing Infrastructure / institutions and professionals to the extent possible. The inclusion demands for preparing teachers with initial induction and sensitizing them through in-service education programmes to help them attend to the special needs of the learners with various special needs of the learners with various challenges and equipping these teachers with skills, competencies and strategies required to cater to the diversity in an inclusive setting control.

Hodkinson (2006) in a small-scale and rather limited study analysed whether Newly Qualified Teachers’ (NQTs’) conceptualisation of inclusive education is mediated by prolonged classroom exposure to this government initiative. The study, whilst arguing that further research is necessary, suggests that NQTs’ conceptualisation of inclusion becomes more negatively based over the course of their first year of employment owing to perceived lack of support and resources to ensure its successful implementation. The study also offers confirmation of the findings of previous researchers who contend that teachers conceptualise inclusive education in terms of children with special educational needs and further that they believe that not all such children can benefit from being educated in mainstream schools.
Berry and Englert (2005) on teachers’ role in inclusive practice studied examined the nature of student talk and the teacher’s role during book discussions. The participants were 17 first- and second-graders with and without disabilities in an inner-city inclusion classroom. Applied conversation analysis techniques were employed to analyze two videotaped book discussions. Results indicated that student-selected topics and contingent talk were necessary for fluent conversational discourse. Additionally, the teacher’s role was crucial in apprenticing students to deal with a novel participant structure and its attendant complex linguistic and cognitive requirements. Results also demonstrated the competence with which students with disabilities assumed influential and decisive roles in the discussions. Implications for students with disabilities are discussed in terms of opportunities for self-expression and involvement in constructing and negotiating the activity.

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Wilczenski (1992) views inclusion, as an extension of mainstreaming where students with disabilities becomes the responsibility of the regular class teacher who is supported by specialists in the classroom. However, Talmor, Rieter and Feigin (2005) attempted to identify the environmental factors that relate to the work of regular school teachers who have students with special needs in their classroom, and to find out the correlation between these factors and teacher burnout. A total 330 primary school teachers filled in a questionnaire that had three parts: (1) personal background data; (2) the Friedman's burnout questionnaire; and (3) environmental features typical of the work of school teachers that include students with special needs in their classroom, in four areas: psychological features, organizational, structural and social. Results show that the background data that related significantly to burnout was teachers' attitudes towards inclusion. The more positive the attitude was, the more the teacher experienced burnout in the category of self-fulfilment. Two other findings that were found to correlate with higher burnout rates were the number of students with special needs in class (more than 20%) and very little assistance provided to the teacher. Three environmental factors were found to have
negative correlation with burnout: the organizational factor, the psychological and the social, with the latter being the most significantly negatively correlated with burnout - i.e. the less social support the teacher experienced, the higher was her level of burnout.

A further consideration in discussing the centrality of resource allocation to the inclusion debate is that the provision of resources for the support of students with special needs should not be achieved to the detriment of the resource base for schools in general. Cormack and Winter (1997) have emphasised that inclusive education advocacy should invoke two accountability principles: not only should resources allocated to students with disabilities lead to the achievement of appropriate learning outcomes, but such resources should also contribute to the effective improvement of the educational system for all students, including those with disabilities. In this second principle lies the kernel of solutions to the pragmatic problems of implementing inclusive education for students with disabilities.

2.4.3 Approaches to inclusive educational practices

Two major approaches to providing inclusive education is practiced all over the world. They are: Individualised Education as practice in US and Inclusive School model as practiced in United Kingdom.

The individualised education approach is characterised by the provision of an Individualised Education Program (IEP) for each child with a disability. Federal legislation in the US, known as the Individuals with Disabilities Education
Act (IDEA), mandates that every child identified with a disability must have an IEP and be assessed by the Basic Special Education process, which has 10 steps (Office of Special Education and Rehabilitation Services [OSERS], 2000):

i). Child is identified as possibly needing special education

ii). Child is evaluated

iii). Eligibility is decided

iv). Child is found eligible: IEP required within 30 days

v). IEP meeting is scheduled, and a minimum of eight specified persons are invited

vi). IEP meeting is held and the IEP is written (with 11 separate pieces of information) not clear

vii). IEP is implemented

viii). Progress is measured and reported to the parents

ix). IEP is reviewed at least annually

x). Child’s disability status is re-evaluated at least every three years

The Council of Exceptional Children (CEC) has conducted a major research investigation of progress in individualised education for students with disabilities in the US nationwide over a two-year intensive period. (CEC, 2000) The outcomes of the Council’s survey are concerning and suggest that adoption of the individualised model will contribute to declining quality of the teaching workforce and impoverishment of educational conditions. In these circumstances efforts to include students with disabilities in regular schools are likely to be
overwhelmed by the cost, in economic, social and environmental terms, to the whole community.

Further, difficulties associated with the individualised approach are realised by considering that, taken to its extreme, the individualised approach is driving education towards individualised instruction for all students. One of the prime motives for the inclusion of students with disabilities in regular schools has been to return them to the community to which they belong. Their individualised education placement in regular schools may lead, ironically, to segregation from the main body of students and fail to provide the socially just and equitable education that was sought. It is proposed that a shift in emphasis is required, from education individuals who compete for scarce resources, to improving the quality of education for all students, including those with disabilities (Jenkins, 2002).

Inclusive Schools model to integrated education seem to address the issue raised by Individualised education approach. The inclusive schools’ movement, currently, promoted in the United Kingdom, proposes an alternative solution to the education of students with disabilities. Ainscow (1999), leader of the Inclusive School’s model has identified three major dimensions of inclusivity:

1. Dimension A: Creating inclusive cultures and values
2. Dimension B: Producing inclusive policies
3. Dimension C: Evolving inclusive practices
Acknowledging the above approach, Jenkins (2002) proposes the Continuum-based model and stresses that rather than considering individualised education and the inclusive schools movement to be opposed to each other, it may be more helpful to consider them as two poles of a continuum of educational policy and practice with respect to students with disabilities. At the individualised extreme, the fundamental unit of educational attention is the individual student. Here, all policies and practices are focused on assessing each student's special needs, designing an IEP to meet the needs, and then drawing on the school's human and financial resources to meet each individual set of needs. At the other extreme of the continuum, a totally inclusive schools approach, defines the fundamental unit of educational attention as a whole school. In this approach, programs are planned to encompass all students' needs and the work of teachers is governed comprehensively by school-wide policies. Different schools will then be located at different points along this continuum of educational policy, depending on their mix of individualised and inclusive approaches.

Signs that inclusive practices are becoming self-sustaining may occur, for example, when a teacher realises that a reading comprehension strategy originally trialled with one student in an IEP is readily applicable to a much larger group of students (Chard, Vaughn, Tyler, & Sloan, 2000).

While many schools would welcome the opportunity to become more inclusive, the demands of teaching and the current pace of change in education sometimes seem to be so great as to preclude new developments that provide
significant advantages for the education of students with disabilities. Reviews of the related literature indicate that general and special educators have mixed reactions to inclusion, related to the efficacy of implementation and the degree of administrative support, resources and training they have received (Daane, Beirne-Smith & Latham, 2000; Salend & Duhaney, 1999).

2.4.4 Inclusive education for Children with Visual Handicap

British and European Missionaries started the concept of educating the blind children in India. The first school for blind in India was set up at Amritsar, in 1887. It is now shifted to Dehradun and is called the Sharp Memorial School. In 1890 the first schools in south India was established at Palam Cottah. The 20th century witnessed a steady growth in the number of schools for the blind spread all over the country. The integrated system of education was introduced in 1956 and by 1979 around 200 Children with Visual Handicap were integrated in the schools for the Sighted. It is significant that National institute for Visually handicapped (NIVH) has been established at Dehra Dun to provide leadership in research, development and training. In early 1990's National Council of Education and Research Institute (NCERT) and several University Departments have been funded for developing training programme for the implementation of the Integrated Education for the Disabled. Beside, a number of District Rehabilitation Centres (DRC) are set up for providing Infrastructure facilities for assessment, support and rehabilitation of the disabled including the visually handicapped.
Goel (1985) highlighting the issues of integrating the Child with Visual Handicap in the regular schools comments – the outward physical differences between the blind and Sighted influence their attitude towards one another. The blind develop physical, social and psychological insecurity due to which they tend to devaluate themselves and develop negative attitude of hatred, suspicion and hostility towards the Sighted. On the other hand, the Sighted treat them as inferior and marginal beings and try to isolate or segregate them as far possible from the community. This makes the task of integration of two groups difficult. The full integration has to be achieved at various levels such as family, school, vocational training, employment and social life. Blindness does not act as any constraint in pursuing day to day activities if proper guidance, aids and encouragement are provided. A blind can study and work with a Sighted person, move around in various situations confidently, he must be helped to integrate with the general society.

Dash (2003) remarks that the adjustment of the visually impaired child in a regular classroom situation has now become a regular feature in the modern schools. When he is in school, whether he is assigned to a regular grade with extra assistance in a resource room or to a special class for the partially Sighted, will depend on many factors, among which are his visual acuity, his interests and abilities, his orientation to class activities, the size of the class, the attitudes and skills of the teacher or teachers and the availability of suitable materials and physical surroundings which will enable him to utilise his limited vision.
Commenting on education of Children with Visual Handicap, Dash (op cit) highlights that the visually limited have the same needs for basic mastery of the skills subject (the three R's, they also need social studies, science, English and so on, the methods for good teaching in these subject matter are as essentially the same as those for children with sight. However, effective programmes for the visually limited must provide additional emphasis.

It is proposed that the conceptualisation of inclusive education as a continuum-based approach to policy and practice provides the opportunity to critically examine alternative outcomes and identify more and less sustainable options, to the ultimate benefit of students with and without disabilities in schools of the future.

2.5 SUMMARISING THE SIGNIFICANCE OF THE STUDY IN RELATION TO THE LITERATURE REVIEWED

**Education of Children with Visual Handicap**

Education of children with special needs in general and Children with Visual Handicap in particular is a broad area and has been addressed at national level through various policies (National Policy for Persons with Disabilities, 2006;), acts (Persons with Disabilities Act, 1995a) and also accommodating in the curriculum framework (National Curriculum Framework, 2005). Further, keeping in mind the international efforts (The Dakar Framework of Action, 2000; The Millennium Development Goals, 2006; UNESCO, 1994;) in relation to education of Children with Disabilities, this study aimed to understand the educational needs of
Children with Visual Handicap particularly in relation to their educational needs for developing arithmetic competence. Studies on development of number concept among Children with Visual Handicap (Ahlberg & Csocan, 1997; Klingenberg, 2000; Yakubu, 1997) highlights that blind children are an extremely heterogeneous group and it is more appropriate to consider visual impairment a risk factor then a cause of children’s difficulties in learning numerical concepts.


*Development of arithmetic skills*
Counting skill is considered as basic to learning of arithmetic. Children make use of their counting skills in the early stages of learning arithmetic (Butterworth, 2005). A variety of models (Dehaene & Cohen, 1995; Siegler & Shragar, 1984) of the mental organisation of arithmetical facts has been proposed. According to these models, mathematics fact retrieval will depend on the learning history of the individual. Thus, facts that are learned earlier or practised more will show greater accessibility. However, problem-size effect has major influence on retrieval times (Ashcraft et al., 1992; Butterworth, Girelli, Zorzi, & Jonckheere, 2001). Principal milestones in the normal development of arithmetic has been summarized by Butterworth (2005). Accordingly, at age 3 years, a child can counts out small numbers of objects (Wynn, 1990); at age 3 and 4 years a child can add and subtract one with objects and number words (Starkey & Gelman, 1982) and also use cardinal principle to establish numerosity of set (Gelman & Gallistel, 1978); at age 4 year a child can use fingers to aid adding (Fuson & Kwon, 1992); at age 5 years a child can add small numbers without being able to count out sum (Starkey & Gelman, 1982), between age 5 and 6 years a child understands commutativity of addition and counts on from larger (Carpenter & Moser, 1982) and also can count correctly to 40 (Fuson, 1988); at age 6 years 'Conserves' number (Piaget, 1952) and age between 6 and 7 years understands complementarity of addition and subtraction (Bryant et al., 1999) and also can count correctly to 80 (Fuson, 1988). Thus, the development of arithmetic can be seen in terms of an increasingly sophisticated understanding of numerosity and its implications, and in increasing skill in manipulating
numerosities. Understanding of such milestones in terms of learning arithmetic skills has major implications on designing the teaching-learning experiences for children at lower primary grades.

From a developmental perspective, research in simple cognitive arithmetic (and in many other domains) established that performance at any particular age is commonly a mixture of processing strategies (e.g., Ashcraft & Fierman, 1982; Koshmider & Ashcraft, 1991; Lemaire et al., 1994; Lemaire & Siegler, 1995; Siegler, 1987, 1988a; Siegler & Shipley, 1995). The development of simple arithmetic has been characterised by the fact that younger children are more reliant on slower implicit counting strategies, and with development, children retrieve a declarative representation of numerical facts from memory (Ashcraft, 1982, 1983, 1987, 1992; Ashcraft & Fierman, 1982; Campbell & Graham, 1985; Cooney, Ladd & Swanson, 1988; Siegler, 1988a; Siegler and Shrager, 1984; Svenson & Sjoberg, 1983). The shift from counting strategies to memory retrieval is determined by the strength of fact representation in memory (e.g., Geary & Burlingham-Dubree, 1989; Lemaire et al., 1991, 1994; Lemaire & Siegler, 1995; Siegler, 1987; Siegler & Jenkins, 1989; Siegler & Shrager, 1984). That is, facts stored with high associative strengths are routinely, rapidly and accurately retrieved from memory. Those with lower associative strengths are more prone to solution via counting. A study (Lemaire et al., 1996) on working memory resources and arithmetic suggests that working memory resources could also be an important contributor to the achievement and development of numerical
cognition (see Brainerd, 1983; Geary, Brown and Samaranayake, 1991; Kaye, 1986, for a similar argument).

From the review of literature on various computational models (Aschcraft, 1992; Campbell, 1995; Dehaene, 1992, 2001; Gelman, 1992, 2000; Viscuso et al., 1989; Zorzi et al., 2005; Gallistel & Gelman, 1992, 2000; Stoianov, Zorzi, Becker, & Umiltà, 2002; Zorzi & Butterworth, 1997, 1999;.) it is apparent that such studies might be crucial for understanding the neuropsychological and neuroimaging patterns of association and disassociation among different arithmetic operations. Such studies among Children with Visual Handicap would be helpful to understand the mental pattern of arithmetic operations thus, enabling educators to adopt suitable modalities for teaching-learning mathematics to Children with Visual Handicap. This view is supported by Siegler (2003). He summarises the implications of research in cognitive science in relation to mathematics as: Neither controlled scientific experimentation nor theoretical analyses automatically translate into prescriptions for classroom instruction. They can provide useful frameworks for thinking about teaching and learning, can indicate sources of difficulty that children encounter in learning particular skills and concepts, and can demonstrate potentially effective instructional procedures. However, a process of translation into the particulars of each classroom context is necessary for even the most insightful frameworks and the most relevant findings to be utilized in ways that improve learning. Both institutional support for such continuous improvement and teacher dedication to meeting this goal are essential if research is to lead to superior instruction.
There are multiple links between teaching, testing and learning, but the exact nature of these connections is still unclear (Cooney, Grouws & Jones, 1988). As research continues in search of a deeper understanding of successful mathematics teaching, the understanding of relationships becomes more important. Understanding what constitutes effective mathematics teaching and how to foster it is an essential goal for mathematics education research. As research proceeds, the classroom context and the nature and means of assessment will be central issue (Grouws et al., 1992). Increasingly, meaningful learning is being recognised as an active, constructive process through which students develop their own interpretations, approaches and ways of viewing phenomena, and through which learners relate new information to their existing knowledge and understandings (Masters & Mislevy, 1991).

In regards to gender differences in arithmetic achievement, there are contradictory research results in children's mathematical performance and gender (Aunio, 2006). Some research shows that girls and boys possess identical primary numerical abilities (e.g., Dehaene, 1997; Nunes & Bryant, 1996). The researchers analysing the British National Curriculum Key Stage 1 measurements (children aged four to seven years) have mostly reported girls outperforming the boys in basic arithmetic (Demie, 2001; Gorard, Rees, & Salisbury, 2001; Strand, 1997, 1999). Carr and Jessup (1997) report contradicting outcomes, as in their first school year, boys and girls may use different strategies for solving mathematical problems, but there is no difference in the level of performance. Gender differences in general and specific numerical
skills at preschool age have attracted little attention (Torbeyns et al., 2002; Van de Rijt et al., 2003), and it would therefore be worthwhile to check such differences in young children’s number sense.

As the study envisaged to enquire if there were any differences in arithmetic achievement among Children with Visual Handicap – boys and girls, the review of related literature is envisaged to support the achievement of boys and girls may not significantly differ (Aunio, 2006; Van Den Heuvel-Panhuizen, 2004; TIMSS, 2003) however, there may be difference in the strategies (Carr & Jesup, 1997; Fennema, Carpenter, Jacobs, Franke, & Levi, 1998; Wakefield, 2006) that they adopt in solving the mathematical problems.

Further, in order to enquire into the errors committed by the Children with Visual Handicap in arithmetic skills, a diagnostic form of testing was envisaged. This necessitated the review of literature in relation to the various forms of assessment being done to diagnose arithmetic errors. Such a review of literature on assessment was conducted for the Sighted as well as for the Children with Visual Handicap. From the review it is seen that though lot work is being done to identify the best form of assessment (Baker, O’Neil, & Linn, 1993; Collison, 1992; De Lange, 1999; Herman & Winters, 1994; Linn, Baker, & Dunbar, 1991; Lajoie, 1995; Mumme’, 1990; Van den Heuvel-Panhuizen, 1996; Van den Heuvel-Panhuizen, Becker, 2003; Watson, 1999; 2000; Webb, 2001; Wiggins, 1989b; Wiliam & Black, 1996; Wolf, 1989) that will help the teacher to enhance the learners’ achievement level is being done, a theory and a model of mathematical
assessment must be developed to unify these efforts and to provide a base on which to build.

It has long been 'known' that children's errors and misconceptions can be the starting point for effective diagnostically-designed mathematics teaching. Diagnostic tests aims at interpreting the error response. Review of available literature including historic trail on error categorization models (Attisha, 1982, 1983; Attisha & Yazdani, 1984; Buswell, 1926; Reisman, 1982) provides a significant base for diagnostically evaluating the arithmetic errors among Children with Visual Handicap selected for the study.

Operating from the point of view that instruction and assessment are closely linked, that good teachers constantly assess students informally, that student self-evaluation is a vital part of learning, that formal assessments are stronger if they relate closely to the content and form of classroom instruction, and that documentation of assessment is important in connecting classroom work to external evaluation, mathematics educators may indeed be able to formulate assessment practices that will elicit improved mathematical achievement (Stenmark, 1991).

Piaget’s Theory of Cognitive Development

Sensation, perception and cognition are essential for any learning to take place. This study focused on the Piaget’s theory of Cognitive Development in relation to the development of arithmetic skills among the Children with Visual

The studies thus highlight the lapse in the current research regarding cognitive development among Children with Visual Handicap particularly in relation to development of number concept. Thus, need for more research in the area of cognitive development among Children with Visual Handicap in relation to acquisition of knowledge and skills of a content strand e.g., arithmetic is warranted. This study is an earnest attempt to consider Piaget's stages of development and probe the development of cognitive capabilities (classification,
seriation, conservation of quantity, conservation of number) among Children with Visual Handicap in relation to development of number concept

Based on the number of studies reviewed in this chapter, following conclusions can be drawn:

- Research on the primary school education particularly with respect to development of arithmetic skills among Children with Visual Handicap has been totally neglected and most of the research is adult oriented. Hence, more research at primary – compulsive years of schooling is defensible.

- In-depth research into the developmental process of acquiring Arithmetic Skills among Children with Visual Handicap is limited. Reported studies focus mainly on the cognitive capabilities and its relation to the development of arithmetic concepts, however, very few studies have dealt into the error analysis of the problem-solving skills among Children with Visual Handicap particularly at lower primary level.

- Diagnostic approach to inquire into the errors committed by Children with Visual Handicap is very scarce. Decorte and Verschaffel (1981) studied the problem-solving skills of first and second grades with an error analysis approach for Sighted. Similar studies for Children with Visual Handicap at lower primary level of education would be helpful in designing the appropriate instructional pedagogy and further building a strong foundation for developing higher arithmetic concepts.
• Consolidation of modalities for improving the teaching – learning process in relation to Arithmetic Skills is hardly been a concern. With the emerging new technologies, it is a challenge to the Child with Visual Handicap to acquire the knowledge using all the modalities and at the same pace as his Sighted peer would do!

• Experimental studies on introducing concretised experience for developing the basic arithmetic concepts finds very few takers. Lot of research is being done in the area of determining the efficacy of Braille reading and writing, however such a focus on the usage of the aids for teaching-learning arithmetic is not observed.

• Studies on biological aspects – specifically on neural network theories for determining the spatial correlation abilities, using of Braille and further cognitive mapping for mobility is being carried out. Similar, studies on neural network basis of problem-solving among Children with Visual Handicap is warranted.

• With respect to the Piagetian tasks of cognitive development, it is evident from the various studies that conservation of various concepts like number, area, quantity emerge in Children with Visual Handicap with a developmental lag ranging from four to eight years.

• Studies on remediation of developmental lag have not drawn much attention of research workers on Children with Visual Handicap
• Research into the relationship between the cognitive development and the acquisition of Arithmetic Skills has been mainly dealt with respect to children with sight. Similar studies on Children with Visual Handicap are lacking.

• Studies on teacher training in context of special school setting and integrated setting has not been dealt with. Status report on the availability of teacher training and suggestions to improve the teaching practice of teachers in teaching Children with Visual Handicap has been emphasised in the reported studies. Teacher morale, willingness and the practical issues faced by the teacher calls for indepth inquiry and formulation of appropriate workshop to address their needs.

• Integrated education via inclusive education is emphasised strongly by various policies and acts for mainstreaming the Children with Visual Handicap. However, integrating Children with Visual Handicap are a major challenge, as it demands allocation of time and resources to fully meet the special needs of the group. This aspect is very evident from the reported studies and calls for looking into options that will help to integrate Children with Visual Handicap on a continuum of segregated to integrated setting with the community and other paraphanelaria supporting their special needs.

The overall significance of this study in relation to the reviewed literature was that it studied the development of arithmetic skills among Lower Primary (5 – 11 ages) Visually Handicapped Children. Through the analysis of curriculum prescribed for the Children with Visual Handicap at Lower Primary Level and
study of certain cognitive capabilities, the study sought to identify the educational needs of Children with Visual Handicap for developing arithmetic capabilities among them. Outcome of the study were envisaged to inform the educators of the difficulties that teachers and students have in learning mathematics - arithmetic in particular. Such information would also be beneficial to mathematics education of Children with Visual Handicap in the integrated setting. As development of arithmetic skills is hierarchical (meaning unless lower skills are mastered, learning of higher skills becomes difficult) the study aimed to understand the teaching-learning of mathematics for Children with Visual Handicap at Lower Primary Level in particular.

2.6 JUSTIFICATION OF THE STUDY

It is evident from the significance of the study in relation to the literature reviewed, that despite the significance of learning Arithmetic Skills (Butterworth, 2005; Fuson, 1988; Lemaire, Abdi & Fayol, 1996; Starkey & Gelman, 1982; Wynn, 1990) and developing the needed cognitive capabilities (Friedman & Pasnak, 1973; Jordan & Jensen, 1979; Higgins, 1973; Mandarvalli, 1990; Warren 1984) during the primary school grades of Children with Visual Handicap in comparison to the Sighted, research in this area has been scant. The need for regular assessment (Romberg & Carpentar, 1986; J. Williams & Ryan, 1997) of individual progress for all students is greater than ever before (Bell, Swan, Onslow, Pratt & Purdy, 1985; Dallaway, 1994; Singh & Prakash, 2000); periodic checks on the child's understanding of new learning are essential; and such checks must usually involve working directly with each child to detect strengths
and weaknesses in performance. Such a diagnostic evaluation of arithmetic skill performance involves individual testing to: determine in general the achievement level at which the child is functioning and detect the failures/errors in basic Arithmetic Skills.

Keeping in mind the context of the study and the extensive review of related literature presented in this chapter, the current study aimed towards inquiring into issues such as: How do Children with Visual Handicap develop basic arithmetic concepts and skills in the absence of visual perception? Do they acquire these concepts and skills at the same pace as the Sighted children do? Further, as most of the research related to visually impaired is aimed at adult education, and the primary school education of the Children with Visual Handicap seem to be neglected, this study focused mainly on acquisition of arithmetic concepts and skills at primary level. Further, mathematics curriculum for the Sighted is prescribed for Children with Visual Handicap with minimal or no adaptation and adjustment in the curriculum, does this have any bearing on the acquisition of arithmetic concepts and skills? How can educators benefit from the knowledge of learning of mathematics through various cognitive stages? What are the common errors committed by Children with Visual Handicap in the acquisition of arithmetic concepts and skills? What adaptation and modification is required in the curricular inputs for instruction of Arithmetic Skills to Children with Visual Handicap? What type of individualised education plan is required for Children with Visual Handicap in an integrated setting? To address these questions a comprehensive qualitative study with a diagnostic and evaluative
approach was undertaken to understand the development of arithmetic skills among Children with Visual Handicap. Further, the information garnered through this study will also be useful to the practitioners of the field for reviewing the existing mathematics curriculum for Children with Visual Handicap at lower primary level - improving its content, sequence and modalities to reduce the mismatch if any between the special needs and the curricular demand if necessary, adding to its potential and raising its overall quality.

Therefore, the study was aptly titled "Development of Arithmetic Skills among Lower Primary Visually Handicapped Children".

The Next Chapter

This thesis records the development of arithmetic abilities among Children with Visual Handicap by studying the mathematics curriculum prescribed for them, comparing their arithmetic achievement with that of Sighted children at lower primary level and further by inquiring into the cognitive capabilities necessary for developing numerical abilities. The research questions upon which this qualitative-analytical study is framed and the various limitations of study are discussed in Chapter 3.