APPENDICES
Dear Students,

This is a simple test in Arithmetic.
This is not an examination to score marks.
This test is to find out if you have any doubt/confusion in solving arithmetic problems.
So, relax try to solve each problem in the test.
There is no hurry, you can take as much as time you need.
<table>
<thead>
<tr>
<th>Concept / Operation</th>
<th>No. of Tasks</th>
<th>Total weightage</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>I - Number</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Counting</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2. Odd - Even</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>3. Completion of series</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4. Ascending-Descending order</td>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td><strong>II - Place Value</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Number Naming</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6. Number Expansion</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>7. Number formation</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>8. Place Value Identification</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td><strong>III - 9. Addition</strong></td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td><strong>IV - 10. Subtraction</strong></td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td><strong>V - 11. Multiplication</strong></td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td><strong>VI - 12. Division</strong></td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td><strong>50</strong></td>
<td><strong>50</strong></td>
</tr>
</tbody>
</table>
Count and write the total number of dots in each row in the box.

1) Row 1 = [dots] 2) Row 2 = [dots] 3) Row 3 = [dots] 4) Row 4 = [dots]

Identify 'odd' and 'even' numbers and group them separately.

5) 931  6) 642  7) 7906  8) 8640  9) 1735  10) 4683

Odd Number
Even Number
Complete the series with filling up the missing numbers.

11) 4848, ____, ____, 4851, __

12) ____, 9120, ____, ____, 9123

Write the numbers in 'Ascending' and 'Descending' order.

1057 56,841 11,113 85,305 76,044 10,000

13) Ascending Order

14) Descending Order
Match the **number names** given in column 'A' by drawing a circle around the number in column 'B' (as shown in example).

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example: Two thousand nine hundred eighty eight.</td>
<td>2898 (2988) 2998</td>
</tr>
</tbody>
</table>

15. Six Thousand Seven Hundred and Twenty Five  
6752 6275 6725

16. Thirty Three Thousand Six Hundred and Ninty  
33,690 33,069 33,609

17. Ninety Thousand Three  
9003 90,030 90,003

18. Ten Thousand  
101,000 10,000 11,000

Expand the given numbers as shown in the example.

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundred</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example: 7213 =</td>
<td>7000</td>
<td>200</td>
<td>10</td>
</tr>
</tbody>
</table>

19) 8437 =  

20) 5062 =  

21) 1910 =  

22) 4903 =  

619
Read the numerals carefully and answer the given questions.

23) 6 8 4 2
24) 3 2 6 8
25) 9 7 0 2
26) 9 9 5 0

a) Which numeral has 8 at Units (ones) place?

b) Which numeral has 0 at Tens place?

c) Which numeral has 9 at Hundreds place?

d) Which numeral has 6 at Thousand place?

Form any four, 4 digit number from the given digits (only one digit can be used twice in the number as shown in the example) and write the number names.

Example: Digits 1 8 3

<table>
<thead>
<tr>
<th>Numbers</th>
<th>Digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 1183</td>
<td>1  8  3</td>
</tr>
<tr>
<td>2) 1838</td>
<td></td>
</tr>
</tbody>
</table>

27) 
28) 
29) 
30) 

620
### Add the following:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>31)</td>
<td>2467</td>
<td>32)</td>
<td>8315</td>
<td>33)</td>
</tr>
<tr>
<td>+</td>
<td>3412</td>
<td>+</td>
<td>7463</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34)</td>
<td>4021</td>
<td>35)</td>
<td>8036</td>
<td>36)</td>
</tr>
<tr>
<td>+</td>
<td>2107</td>
<td>+</td>
<td>9906</td>
<td>+</td>
</tr>
<tr>
<td>7850</td>
<td></td>
<td>6470</td>
<td></td>
<td>887</td>
</tr>
</tbody>
</table>

### Subtract the following:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>37)</td>
<td>7468</td>
<td>38)</td>
<td>4637</td>
<td>39)</td>
</tr>
<tr>
<td>-</td>
<td>5134</td>
<td>-</td>
<td>4215</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40)</td>
<td>8657</td>
<td>41)</td>
<td>7516</td>
<td>42)</td>
</tr>
<tr>
<td>-</td>
<td>4968</td>
<td>-</td>
<td>6789</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Multiply the following:

43) \( \underline{2132} \times 3 \)

44) \( \underline{6423} \times 20 \)

45) \( \underline{3965} \times 49 \)

46) \( \underline{9070} \times 65 \)

Divide the following:

47) \( 3 \) \( \underline{3968} \) (47)

48) \( 6 \) \( \underline{5468} \) (48)

49) \( 9 \) \( \underline{7849} \) (49)

50) \( 8 \) \( \underline{6006} \) (50)
ARITHMETIC DIAGNOSTIC TEST
Primary School Level
Score Sheet

Name: ___________________________ Class: ___________________________
School: ___________________________ SES ___________________________ CVH

<table>
<thead>
<tr>
<th>Arithmetic Tasks</th>
<th>No of</th>
<th>Items in each task</th>
<th>Correct</th>
<th>Wrong</th>
<th>Not attempted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>items</td>
<td>1 2 3 4 5 6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I - NUMBER</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1) Counting</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) Odd - Even</td>
<td>6</td>
<td>5 6 7 8 9 10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) Completion of series</td>
<td>2</td>
<td>11 12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) Ascending - Descending order</td>
<td>2</td>
<td>13 14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II - PLACE VALUE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) Number Naming</td>
<td>4</td>
<td>15 16 17 18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6) Number Expansion</td>
<td>4</td>
<td>19 20 21 22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7) Number formation</td>
<td>4</td>
<td>23 24 25 26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8) Place Value Identification</td>
<td>4</td>
<td>27 28 29 30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III - Addition</td>
<td>6</td>
<td>31 32 33 34 35 36</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV - Subtraction</td>
<td>6</td>
<td>37 38 39 40 41 42</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V - Multiplication</td>
<td>4</td>
<td>43 44 45 46</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VI - Division</td>
<td>4</td>
<td>47 48 49 50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Tasks</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Date:
Signature of the tester

* Mark 'V' or 'X' or 'O' for each task in the cell for Correct / Wrong / Not attempted, respectively.
## I Metric Relations

<table>
<thead>
<tr>
<th>Number of situations</th>
<th>Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Length Seriation</td>
<td>4</td>
</tr>
<tr>
<td>2. Area Seriation</td>
<td>2</td>
</tr>
<tr>
<td>3. Volume Seriation</td>
<td>2</td>
</tr>
<tr>
<td>4. Equidistance points Location</td>
<td>2</td>
</tr>
<tr>
<td>5. Distance Estimation</td>
<td>3</td>
</tr>
</tbody>
</table>

## II Spatial Relations

<table>
<thead>
<tr>
<th>Number of situations</th>
<th>Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Haptic Recognition</td>
<td>2</td>
</tr>
<tr>
<td>2. Figure Drawing</td>
<td>2</td>
</tr>
<tr>
<td>3. Shape Recognition</td>
<td>2</td>
</tr>
<tr>
<td>4. Shape Completion</td>
<td>2</td>
</tr>
<tr>
<td>5. Matrix Construction</td>
<td>3</td>
</tr>
</tbody>
</table>

## III Conservation

<table>
<thead>
<tr>
<th>Number of situations</th>
<th>Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Judgment of Invariance of Number</td>
<td>2</td>
</tr>
<tr>
<td>2. Judgment of Invariance of Area</td>
<td>2</td>
</tr>
<tr>
<td>3. Judgment of Invariance of length</td>
<td>2</td>
</tr>
<tr>
<td>4. Judgment of Invariance of Mass</td>
<td>2</td>
</tr>
<tr>
<td>5. Judgment of Invariance of liquid</td>
<td>2</td>
</tr>
</tbody>
</table>

## IV Temporal

<table>
<thead>
<tr>
<th>Number of situations</th>
<th>Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Judgment of Simultaneity, speed and Distance covered in a Doll-Racing</td>
<td>3</td>
</tr>
</tbody>
</table>

## V Belongingness

<table>
<thead>
<tr>
<th>Number of situations</th>
<th>Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Classification of Pictures</td>
<td>1</td>
</tr>
<tr>
<td>2. Classification of Shapes</td>
<td>1</td>
</tr>
<tr>
<td>3. Identification of odd thing in a group</td>
<td>3</td>
</tr>
</tbody>
</table>

## VI Signs Symbols

<table>
<thead>
<tr>
<th>Number of situations</th>
<th>Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Decoding symbols</td>
<td>3</td>
</tr>
<tr>
<td>2. Decoding Signs into Actions</td>
<td>2</td>
</tr>
</tbody>
</table>

### Total: 6 Sub-tests, 21 sets of text-tasks, specific situations, points of scores.
PROFORMA FOR STRUCTURED INTERVIEW – TEACHERS IN SPECIAL INSTITUTIONS

1. School: ____________________________

2. Sex: Male ☐ Female ☐

3. Vision: Sighted ☐ Partially Sighted ☐

   Onset of blindness ______________________

4. Education: Degree ☐ Post-Graduate ☐

   Other ______________________

5. Experience (service period) Less than 10 years ☐

   10 years to 20 years ☐

   30 years and above ☐

6. Special training in the teaching of visually handicapped

   Workshop ☐ Refresher Course ☐

   Orientation Course ☐ Teaching preparation programme ☐

7. Comments on

   a. Arithmetic syllabus ______________________

   b. Teaching methods ______________________
c. Modalities and strategies adapted for CVH. List advantages and limitations of using.

<table>
<thead>
<tr>
<th>Methods/Modalities</th>
<th>Advantages</th>
<th>Limitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Oral</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Mental Mathematics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. Abacus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. Taylor Frame</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e. Nemeth Code</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f. Braille Reading and Writing</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. Teacher perceptions on effective mathematics curriculum

a. Highlight the areas of mathematics that are difficult to teach CVH. Why?

b. Highlight the sections of arithmetic that are generally omitted at lower primary level for CVH.

c. Comment on the time availability per class for teaching arithmetic.

d. Have you attended any inservice training with respect to teaching CVH. What is your opinion on such training?
   a. Content/Syllabus and Textbooks
   b. Teaching-Learning Aids
   c. Cooperation of Head, Colleagues and Students
   d. Time Table

10. Suggest on improving the curricular content area, teaching methods and modalities.

Note: Use additional sheets for taking notes wherever necessary.
APPENDIX-IV

Addition Bugs

In comparison to the research on subtraction, there have been few studies which attempt to pin down multiplication and addition bugs. This lack of data is an obvious problem, although as pointed out in chapter 5, it is not such a serious problem for this particular project. Nevertheless, it is important to know, at least roughly, what kinds of bugs can occur and how frequent they are likely to be.

These appendices list the multiplication and addition bugs found in a search of the literature. All of the studies looked at have problems, and the lists presented here do not remove the need for a large, modern study of multiplication and addition.

A.1 Sources

The first source of bugs comes from Attisha (1983, also Attisha & Yazdani 1984). Attisha built a tutoring system for all four operations, and surveyed the literature for bugs. Although a large number of bugs were listed, Attisha failed to indicate their frequency or give examples with working marks. Without working marks it is difficult to interpret the definitions of certain bugs, namely those which involve carrying. Those definitions that could be interpreted are included here, and working marks have been added where it aids understanding of the bug.

One of the sources used by Attisha was the bug catalogue produced by Cox (1974). She studied the literature on arithmetic bugs from 1900 to 1973, and also conducted a study of bugs found in 564 subjects in grades two to six. The study demanded that the subjects be close to 100 per cent accurate on their number facts, and bugs were only accepted if they occurred at least three out of five times on a given type of problem.

Cox tested children on the four multicoloum tasks by having them complete tests which were based around a number of levels. For addition there were eight levels, starting from addition of two digits to one digit without renaming. Each level became increasingly difficult, up to addition of three two-digit numbers with renaming. For multiplication there were ten levels. Cox listed the bugs found at each level, and then over the levels produced a categorization of the bugs. As can be seen from table A.1, Cox classified the bugs according to the kind of faulty knowledge that caused the error. The meaning behind Cox's labels is obvious, except for "concept", which is the case when the child seem to be lacking a basic understanding of the concept of multiplication or addition.
Cox’s analysis and method of testing makes it impossible to know exactly how general or specific a particular bug is. For example, for level 2 problems (adding a one digit number to a two digit number, renaming needed) if the subject does not carry, is he or she exhibiting the bug does-not-carry-ones or does-not-carry? That is, if the subject was given a problem involving more digits, would they fail to carry just in the first (ones) column, or in all columns? In these appendices the most specific bugs are listed. Perhaps this complication is why Cox lists the global bug frequencies based on bug categories, rather than on specific bugs.

In her literature survey Cox missed the large arithmetic study undertaken by Buswell (1926). Using verbal protocols, Buswell described behaviours for all four operations. The study lists the number of occurrences of various “habits”—consistent behaviours, but not necessarily behaviours that could be classed as “buggy”. For example, one particular habit, “added carried number last”, describes a subject who always added the carry digit after adding up a column, rather than before starting on the column. Buswell notes that occasionally the subject would forget to add the carry, and this could be avoided if the subject added the carry first (ibid., p. 160). This kind of behaviour does not fit with the modern notion of a bug. However, many of the habits described do appear to be bugs, and are included here. Whereas Cox’s descriptions may be over specific, Buswell suffers the opposite problem. For example, Buswell describes the general bug wrong-operator, when other authors give more specific bugs like multiplies-instead-of adding or subtracts-instead-of-adding.

The final source of bugs comes from a small, unpublished undergraduate project (Ainsworth 1991). It is included here because it builds upon the work of Young & O’Shea (1981) and presents a production system model of multiplication, as well as bug frequency information. As such it is the only study to date which can be easily compared to the computational studies of subtraction.

### A.2 Notes on the catalogue entries

These two appendices list 102 bugs. There are 63 multiplication bugs, of which 9 do not have frequency information. For addition, the total is 39, 11 without frequency data.

Each entry in the catalogue is laid out as follows. The bug name, given in bold, is followed by a short description of the bug. Most bugs have one or more examples to clarify the description. The source of the bug is indicated by showing the name of one of

<table>
<thead>
<tr>
<th>Addition</th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Renaming</td>
<td>Concept</td>
</tr>
<tr>
<td>Concept</td>
<td>Partial product</td>
</tr>
<tr>
<td>Wrong operation</td>
<td>× after renaming</td>
</tr>
<tr>
<td>Place value</td>
<td>+ after renaming</td>
</tr>
<tr>
<td></td>
<td>Renaming</td>
</tr>
<tr>
<td></td>
<td>× by zero</td>
</tr>
<tr>
<td></td>
<td>Wrong operation</td>
</tr>
<tr>
<td></td>
<td>Reversal of digits</td>
</tr>
</tbody>
</table>

*Table A.1. Categories of bugs reported by Cox (1974), and the number of different bugs that fell under each category heading.*
the authors mentioned in the previous section. If that author gave frequency information, it is shown as a percentage of all the bugs taken from that author. Note that this will not be a percentage of all the bugs listed by that author, bugs were not included either because they were not clearly described, or because they were not relevant (e.g., number fact errors). For addition, 116 bug occurrences were used from Cox, and 484 from Buswell. A total of 113 occurrences of multiplication bugs were used from Cox, 512 from Buswell, and 76 from Ainsworth.

The catalogue is listed in alphabetical order, but table A.2 (on page 144) lists the most frequent bugs in order of frequency—or an approximation to that given that many bugs have frequency values from two or three authors.

Some of the bugs listed produce results that look identical to other bugs. For example, the bugs does-not-rename-sum and does-not-rename-product both result in the subject writing carry digits in the answer row. However, these bugs qualify as separate bugs because they have been observed independently of each other. That is, a subject can fail to rename a partial product, yet correctly rename when adding the partial product.

The Buswell frequencies are based on the total frequencies made by 263 subjects, spread over grades 3 to 6 (1926, tables XXXV to XXXVII, pp. 136–139). From the Ainsworth study, the frequencies are summed over two sets of 10–11 year olds and one group of 8–9 year olds (Ainsworth 1991, table 2, p. 32).

The model described in chapter 5 does not attempt to model certain kinds of errors, and for this reason some space-saving liberties have been taken in these appendices. In particular, pattern errors, like $N+0=N$, are only given one way round (i.e., $0+N=N$ is not shown). In the Cox, Ainsworth and Buswell studies, they are given both ways as they can occur independently.

**Addition bugs**

**Added-imaginary-column.** The subject went on to write an answer for a column that did not exist.

<table>
<thead>
<tr>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

**Buswell** 0.21%

**Adds-disregarding-columns.** All the digits of the problem are added, without regard for the columns, i.e., $4+7+6+1+7=25$.

<table>
<thead>
<tr>
<th>4</th>
<th>7</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

**Cox** 12.93%

**Buswell** 11.36%

**Attisha**

**Adds-left-to-right.** Addition is done horizontally, left to right. E.g., $2+4=6, 5+3=8$.

<table>
<thead>
<tr>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>

**Buswell** 0.62%

**Attisha**

**Adds-like-multiplication.** Addition is performed using the pattern for multiplication
Column-skipped. One column is ignored and the column's answer is left blank.

```
  3  4  5
+ 2  1  2
-----
    5  7
```

Copies-first-addend. Where there is a single digit addend, that digit is copied as the answer in the ones column. The answer in the tens column is selected from one of the digits in the top row.

```
  4  6
+ 1  3
-----
  4  9
```

Copies-ones-and-increments. The ones addend in the second row is incremented and given as the answer to the ones column.

```
  4  7  6
+ 1  1  7
-----
  4  8  8
```

Copy-addend. One of the addend rows is copied to the answer row, possibly incremented or decremented.

```
  3  7
+ 1  1  1
-----
  4  8  8
```

Copy-lower-addend. The addend in the second row is copied to the answer. If there are digits over empty cells, they are also copied to the answer row.

```
  4  7  6
+ 1  1  1
-----
  4  8  8
```

Does-not-carry. The subject does not carry.

```
  3  4  5
+ 1  7  6
-----
  4  1  1
```

Does-not-carry-ones. If the result of the ones column is a two digit number, the tens are not carried. The rest of the addition is correct.

```
  3  4  5
+ 1  7  6
-----
  4  1  1
```

Does-not-carry-over-blank. The subject does not carry to a number which is over an empty cell.

```
  4  6  8
+ 1  6  1
-----
  5  2  0
```

Busseil   7.44%

Cox      0.86%

Cox      0.86%

Cox      0.86%

Busseil  26.03%

Cox      1.72%

Attisha

Cox      35.34%

Attisha

631
Carries-one-to-100s. One is carried into the hundreds column regardless of whether a carry is or is not needed.

\[
\begin{array}{c}
5 & 0 & 5 \\
\hline
1 & 7 & 4 \\
\hline
6 & 7 & 9 \\
\end{array}
\]

Carries-one-to-10s. One is carried into the tens column when it is not necessary.

\[
\begin{array}{c}
4 & 6 \\
\hline
4 & 3 \\
\hline
5 & 9 \\
\end{array}
\]

Carries-ten. Ten is carried rather than one. E.g., 7+5=12, 2+1+10=13.

\[
\begin{array}{c}
2 & 5 \\
\hline
4 & 7 \\
\hline
6 & 2 \\
\end{array}
\]

Carries-two. The subject carries two in every column.

\[
\begin{array}{c}
7 & 4 \\
\hline
4 & 1 & 2 \\
\hline
8 & 0 & 3 \\
\end{array}
\]

Carries-wrong-digit. When a column result needs to be carried, the wrong digit is carried.

\[
\begin{array}{c}
5 & 4 & 9 \\
\hline
1 & 8 & 6 \\
\hline
6 & 1 \\
\end{array}
\]

Carry-added-to-column. The carry digit is added into the answer for the current column. In this example, 1+3=4, 7+8=15, 1+5=6, 2+5=7.

\[
\begin{array}{c}
2 & 7 & 1 \\
\hline
4 & 5 & 8 & 3 \\
\hline
6 & 6 & 4 \\
\end{array}
\]

Carry-once-always-carry. Once the subject starts to carry a digit, it is always carried.

\[
\begin{array}{c}
1 & 2 & 7 \\
\hline
4 & 5 & 6 \\
\hline
6 & 8 & 3 \\
\end{array}
\]

Carry-zero-units. When renaming, the carry digit is correctly noted, but the subject writes zero in the answer cell.

\[
\begin{array}{c}
7 & 5 \\
\hline
4 & 1 & 8 \\
\hline
0 & 1 \\
\end{array}
\]
Does-not-raise-carry. The final carry at the end of an answer row is not raised onto the answer row.

```
7 8
4 7 1
1 4 9
```

Buswell 7.02%

Does-not-record-100s. The hundreds column answer is not recorded on the answer row.

```
5 0 5
1 7 4
0 3 9
```

Cox 1.72%

Does-not-rename-copy-100s. The sum of the first column is not renamed, the tens column is not processed, and the digit in the hundreds column is copied to the answer row.

```
2 0 5
1 8 6
2 1 1
```

Cox 0.86%

Does-not-rename-quits-100s. The carry digit from the first addition is written in the answer row but the hundreds column is not processed.

```
2 0 5
4 8 6
8 1 1
```

Cox 1.72%

Does-not-rename-sum. During addition, digits to be carried are written on the answer row.

```
4 8
4 3
4 1 1
```

```
2 8
x 1 7
1 9 6
2 8 0
3 1 7 6
```

Cox 18.97%

Buswell 3.1%

Attisha

Ignores-10s-column. The tens column is ignored.

```
4 8
4 3
1 1
```

Cox 0.86%

Ignores-first-column. The first column of the problem is ignored.

```
3 2 5
4 2 7 1
5 9
```

Attisha

Left-alignment. The subject writes the problem aligned against the left column.

```
5 4
4 3
8 1
```

Attisha

632
Multiplies-instead-of-adding. The subject multiplies, rather than adding,

\[
\begin{array}{c}
3 \times 4 \\
\hline
1 \times 2 \\
\hline
6 \times 8
\end{array}
\]

N+N=N. The subject answers that the sum of two identical digits is just one of the digits.

\[
\begin{array}{c}
3 \times 2 \\
\hline
1 \times 2 \\
\hline
4
\end{array}
\]

One-one-too-many. The answer in the ones column is one more than it should be.

\[
\begin{array}{c}
5 \times 2 \\
\hline
8 \times 6 \\
\hline
1 \times 4 \\
\hline
5 \times 3
\end{array}
\]

Quit-after-last-lower. When the last of the numbers in the lower row has been processed, the subject quits.

\[
\begin{array}{c}
2 \times 7 \\
\hline
4 \times 2 \\
\hline
9 \times 7
\end{array}
\]

Quits-when-carry. When a carry is needed, the subject quits.

\[
\begin{array}{c}
2 \times 7 \\
\hline
4 \times 1 \\
\hline
5 \times 3
\end{array}
\]

Renames-to-wrong-column. When renaming, the subject renames the carry to the wrong column. In the example, the carry from the units column was renamed to the hundreds column.

\[
\begin{array}{c}
4 \times 7 \\
\hline
1 \times 1 \\
\hline
5 \times 8 \\
\hline
3
\end{array}
\]

Spurious-carry. At some stage in the sum a carry was added when it was not appropriate.

Buswell 5.99%

Stutter-add. When there is an empty cell in the problem, the last digit in the bottom row is used as the addend.

\[
\begin{array}{c}
4 \times 2 \\
\hline
3 \times 4
\end{array}
\]

Subtract-carry. The carry was subtracted, rather than added to the addend.

\[
\begin{array}{c}
7 \\
\hline
8 \times 9
\end{array}
\]

Buswell 0.21%
Subtracts-instead-of-adding. The subject uses the subtraction algorithm instead of addition algorithm.

\[
\begin{array}{c}
1 \quad 5 \\
1 \quad 2 \\
\hline
1 \quad 3 \\
\end{array}
\]

Cox 9.48%  
Attisha

Wrong-operator. At some stage in the problem the wrong operator was used (e.g., multiplication for addition). Buswell was no more specific than this.

Buswell 16.32%
<table>
<thead>
<tr>
<th>Rank</th>
<th>Bug</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>does-not-carry-ones</td>
<td>15.54</td>
<td>0.34</td>
</tr>
<tr>
<td>2</td>
<td>does-not-carry</td>
<td>62.03</td>
<td>1.72</td>
</tr>
<tr>
<td>3</td>
<td>adds-disregarding-columns</td>
<td>12.36</td>
<td>12.93</td>
</tr>
<tr>
<td>4</td>
<td>does-not-rename-sum</td>
<td>3.1</td>
<td>8.97</td>
</tr>
<tr>
<td>5</td>
<td>carries-wrong-digit</td>
<td>17.98</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>wrong-operator</td>
<td>16.32</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>subtracts-instead-of-adding</td>
<td>7.44</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>column-skipped</td>
<td>7.02</td>
<td>0.00</td>
</tr>
<tr>
<td>9</td>
<td>does-not-name-carry</td>
<td>5.99</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td>spurious-carry</td>
<td>3.08</td>
<td>0.00</td>
</tr>
<tr>
<td>11</td>
<td>renames-to-wrong-column</td>
<td>3.72</td>
<td>0.00</td>
</tr>
<tr>
<td>12</td>
<td>stutter-add</td>
<td>3.43</td>
<td>0.00</td>
</tr>
<tr>
<td>13</td>
<td>adds-like-multiplication</td>
<td>1.75</td>
<td>0.00</td>
</tr>
<tr>
<td>14</td>
<td>carries-one-to-10s</td>
<td>1.75</td>
<td>0.00</td>
</tr>
<tr>
<td>15</td>
<td>carry-added-to-column</td>
<td>1.75</td>
<td>0.00</td>
</tr>
<tr>
<td>16</td>
<td>does-not-record-10s</td>
<td>1.72</td>
<td>0.00</td>
</tr>
<tr>
<td>17</td>
<td>does-not-name-quits-10s</td>
<td>1.72</td>
<td>0.00</td>
</tr>
<tr>
<td>18</td>
<td>one-one-too-many</td>
<td>1.79</td>
<td>0.00</td>
</tr>
<tr>
<td>19</td>
<td>carries-one-to-10s</td>
<td>0.86</td>
<td>0.00</td>
</tr>
<tr>
<td>20</td>
<td>copies-first-operand</td>
<td>0.86</td>
<td>0.00</td>
</tr>
<tr>
<td>21</td>
<td>copies-first-operand</td>
<td>0.86</td>
<td>0.00</td>
</tr>
<tr>
<td>22</td>
<td>copies-ones-and-increments</td>
<td>0.86</td>
<td>0.00</td>
</tr>
<tr>
<td>23</td>
<td>copy-add</td>
<td>0.86</td>
<td>0.00</td>
</tr>
<tr>
<td>24</td>
<td>copy-lower-add</td>
<td>0.86</td>
<td>0.00</td>
</tr>
<tr>
<td>25</td>
<td>does-not-name-copy-100s</td>
<td>0.86</td>
<td>0.00</td>
</tr>
<tr>
<td>26</td>
<td>ignores-1s-column</td>
<td>0.86</td>
<td>0.00</td>
</tr>
<tr>
<td>27</td>
<td>adds-lhs-to-right</td>
<td>0.62</td>
<td>0.00</td>
</tr>
<tr>
<td>28</td>
<td>adds-oa-rior-column</td>
<td>0.36</td>
<td>0.00</td>
</tr>
<tr>
<td>29</td>
<td>subtract-carry</td>
<td>0.21</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table A.1: The 25 most frequent addition and multiplication bugs. *Key: Values are percentages from three authors: A=Ainsworth (1991), B=Buswell (1926), N=512 for multiplication, N=484 for addition; C=Cox (1974); N=113 for multiplication, N=116 for addition.
**Multiplication Bugs**

0×N=0 carry-N. When multiplying by zero, zero is written as the column’s answer, but the multiplicand is carried.

\[
\begin{array}{c}
2 & 0 \\
\times & 3 \\
\hline
6 & 0
\end{array}
\]

Add-carry-and-multicand. The carried digit is added to the multiplicand, and this sum is given as the column answer. E.g., 6×8=48, 3×4=7. The final “5” was copied.

\[
\begin{array}{c}
5 & 3 & 6 \\
\times & 8 \\
\hline
4 & 8 & 8
\end{array}
\]

Add-carry-and-multiplier. The carried digit is added to the multiplier, and this sum is given as the column answer. E.g., 4×5=20, 4×2=6, 4×8=32.

\[
\begin{array}{c}
8 & 0 & 5 \\
\times & 4 \\
\hline
3 & 2 & 6 & 0
\end{array}
\]

Add-carry-and-multiplier-when-zero. When the multiplicand is zero, the subject adds the carry digit and the multiplier to obtain an answer. In the example, 2×7=14, 1×2=3, 2×5=10.

\[
\begin{array}{c}
5 & 0 & 7 \\
\times & 2 \\
\hline
1 & 0 & 3 & 4
\end{array}
\]

Add-carry-to-multicands. A column’s answer is the sum of the carry digit and the multiplicand. E.g., 6×8=48, 3×4=7, 5+4=9.

\[
\begin{array}{c}
5 & 3 & 6 \\
\times & 8 \\
\hline
9 & 7 & 8
\end{array}
\]
---

**Adds-carry-to-product.** When the result of a multiplication is a two digit number, those numbers are added, e.g., $3 \times 5 = 15$.

\[
\begin{array}{c}
  5 & 2 \\
  \times & 3 \\
  \hline
  6 & 6 \\
  5 & 2 & 0 \\
  \hline
  5 & 8 & 6 \\
\end{array}
\]

Ainsworth 1.32%

**Adds-instead-of-multiplying.** The addition algorithm is used instead of multiplication.

\[
\begin{array}{c}
  7 & 2 & 5 \\
  \times & 3 \\
  \hline
  2 & 1 & 5 \\
\end{array}
\]

Cox 2.65%

**Always-carry.** The subject always adds in the carry digit.

\[
\begin{array}{c}
  2 & 4 & 2 & 9 \\
  \times & 2 \\
  \hline
  4 & 9 & 9 & 8 \\
\end{array}
\]

Buswell 5.47%

**Always-carry-one.** When a carry occurs, the subject adds one to a column answer, not the real carry.

\[
\begin{array}{c}
  5 & 1 & 4 \\
  \times & 7 \\
  \hline
  3 & 5 & 8 & 8 \\
\end{array}
\]

Attisha

**Answer-on-one-row.** All the partial products are written on one answer row.

\[
\begin{array}{c}
  2 & 3 \\
  \times & 4 & 8 \\
  \hline
  8 & 6 & 3 \\
\end{array}
\]

Ainsworth 27.63%

**Answers-left-to-right.** The subject writes the answer left to right. In the example, $2 \times 9 = 18$, subject writes 8 carries 1, and so on.

\[
\begin{array}{c}
  7 & 1 & 2 \\
  \times & 9 \\
  \hline
  8 & 6 & 1 \\
\end{array}
\]

Attisha

---
Carries-wrong-digit. When the result of a multiplication or addition is a number that needs to be carried, the wrong digit is carried.

<table>
<thead>
<tr>
<th>Multiplicand</th>
<th>Ainsworth</th>
<th>Buswell</th>
<th>Cox</th>
<th>Attisha</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 2 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>× 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 8 1 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Carries-wrong-number. A composite bug, where some number was carried, but it was the wrong one (e.g., the units number as in carries-wrong-digit, or always a one, as in always-carries-one).

<table>
<thead>
<tr>
<th>Multiplicand</th>
<th>Ainsworth</th>
<th>Buswell</th>
<th>Cox</th>
<th>Attisha</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 2 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>× 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 6 3 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Carry-added-to-multiplicand. The carry digit is added to the multiplicand before multiplying. I.e., $6 \times 7 = 42$, $(2+4) \times 6 = 36$, $(3+3) \times 6 = 36$.

<table>
<thead>
<tr>
<th>Multiplicand</th>
<th>Ainsworth</th>
<th>Buswell</th>
<th>Cox</th>
<th>Attisha</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 8 3 4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2 2 6 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5 1 4 4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Carry-added-to-tens. When adding a carry digit to a product, the carry is added to the tens part, e.g., $4 \times 6 = 24$, $4 \times 2 = 8$, $2 + 8 = 28$.

<table>
<thead>
<tr>
<th>Multiplicand</th>
<th>Ainsworth</th>
<th>Buswell</th>
<th>Cox</th>
<th>Attisha</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>× 1 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 8 3 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 2 6 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 1 4 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Carry-not-raised. The carry digit is not raised at the end of a answer row in the partial product.

<table>
<thead>
<tr>
<th>Multiplicand</th>
<th>Ainsworth</th>
<th>Buswell</th>
<th>Cox</th>
<th>Attisha</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>× 4 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 6 8 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 2 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Carry-once-always-carry. Once the subject starts to carry a digit, it is always carried.

<table>
<thead>
<tr>
<th>Multiplicand</th>
<th>Ainsworth</th>
<th>Buswell</th>
<th>Cox</th>
<th>Attisha</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>× 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 1 8 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Copies-after-first-column. The first column of a problem is solved correctly, but the remaining multiplicands are copied to the answer row.

<table>
<thead>
<tr>
<th>Multiplicand</th>
<th>Ainsworth</th>
<th>Buswell</th>
<th>Cox</th>
<th>Attisha</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 1 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>× 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 1 9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Copies-multiplicand. No multiplication is performed, but the multiplicand is copied to the answer row.

\[
\begin{array}{c}
2 \ 0 \ 0 \\
\times \ 4 \\
\hline
2 \ 0 \ 0 \\
\end{array}
\]

Copies-multiplicand-at-100s. When processing the hundreds multiplier, the subject inserts two zeros and copies the multiplicand.

\[
\begin{array}{c}
5 \ 1 \ 9 \\
\times \ \ 4 \ 0 \ 0 \\
\hline
1 \ 0 \ 3 \ 8 \\
5 \ 1 \ 9 \ 0 \ 0 \\
\end{array}
\]

Copies-multiplicand-including-zero. The multiplicand is copied as the answer, but a zero is first inserted into the answer.

\[
\begin{array}{c}
2 \ 4 \ 7 \\
\times \ 2 \ 0 \\
\hline
2 \ 4 \ 7 \ 0 \\
\end{array}
\]

Copies-multiplicand-less-2. The answer is two less than the multiplicand.

\[
\begin{array}{c}
1 \ 6 \\
\times \ 4 \\
\hline
1 \ 4 \\
\end{array}
\]

Cross-multiplies. The digits of the problem are cross multiplied, e.g., \(1 \times 4 = 4, 3 \times 2 = 6\).

\[
\begin{array}{c}
4 \ 2 \\
\times \ 3 \ 1 \\
\hline
4 \ 4 \\
\end{array}
\]

Digit-omitted. A digit in the product is not written down. In the example, the subject decided not to write down the 5 from 54 (\(8 \times 8 = 64, 8 \times 6 = 48 + 6 = 54\)).

\[
\begin{array}{c}
6 \ 8 \\
\times \ 9 \ 8 \ 7 \ 8 \\
\hline
4 \ 4 \\
\end{array}
\]

Does-not-add-carry. The carry digit is not added to the column product. Cox notes this error when the subject misses just one carry in a problem (not necessarily every carry).

\[
\begin{array}{c}
1 \ 4 \ 9 \\
\times \ 4 \\
\hline
4 \ 1 \ 0 \ 5 \ 6 \\
\end{array}
\]

Does-not-add-partial-product. The subject does not add the partial product, leaving the sum as shown in the example.

\[
\begin{array}{c}
5 \ 3 \\
\times \ 3 \ 2 \ 1 \\
\hline
5 \ 3 \\
1 \ 0 \ 6 \ 0 \\
1 \ 5 \ 9 \ 0 \ 0 \\
\end{array}
\]

640
Does-not-carry-in-partial-product. The subject does not carry when adding the partial product.

\[
\begin{array}{c}
9 & 2 & 7 \\
\times & 7 & 3 \\
\hline
2 & 7 & 8 & 1 \\
6 & 4 & 8 & 9 & 0 \\
6 & 6 & 5 & 7 & 1 \\
\end{array}
\]

Attisha

Does-not-carry-to-10s. The carried digit is not added to the product in the ten column.

\[
\begin{array}{c}
2 & 1 & 6 \\
\times & 6 \\
\hline
1 & 2 & 6 & 6 \\
\end{array}
\]

Ainsworth 3.95%

Cox 0.88%

Attisha

Does-not-rename-copies-10s. The product from the first multiplication is written in the answer row without renaming, and the tens multiplicand is copied into the answer.

\[
\begin{array}{c}
1 & 6 \\
\times & 4 \\
\hline
1 & 2 & 4 \\
\end{array}
\]

Cox 0.88%

Does-not-rename-first-then-copies. The first multiplication is performed, and the answer is written in the answer without renaming, and remaining multiplicands are copied.

\[
\begin{array}{c}
2 & 3 & 7 \\
\times & 4 \\
\hline
2 & 3 & 2 & 8 \\
\end{array}
\]

Attisha

Does-not-rename-product. Digits carried over from a multiplication are written on the answer row.

\[
\begin{array}{c}
1 & 7 \\
\times & 5 \\
\hline
5 & 3 & 5 \\
\end{array}
\]

Ainsworth 6.58%

Cox 0.88%

Forgets-annex. The zero is forgotten. In the example, a zero should have been inserted into the second answer row.

\[
\begin{array}{c}
4 & 5 \\
\times & 2 & 9 \\
\hline
4 & 0 & 5 \\
1 & 9 & 0 \\
\hline
4 & 9 & 5 \\
\end{array}
\]

Buswell 7.62%

Ainsworth 3.95%

Cox 3.54%

Ignores-zero-multiplier. The first multiplier is ignored when it's a zero, and no zero is inserted in the answer row.

\[
\begin{array}{c}
5 & 3 \\
\times & 2 & 0 \\
\hline
1 & 0 & 6 \\
\end{array}
\]

Cox 3.54%

Attisha
**Incorrect-number-of-annex-zeros.** An incorrect number of zeros are inserted into one of the answer rows.

\[
\begin{array}{ccc}
4 & 5 & 6 \\
\times & 2 & 5 \\
\hline
9 & 1 & 2 \\
6 & 5 & 6 \\
\end{array}
\]

**Cox 6.19%**

**Last-digits-multiplied.** The last multiplicand is multiplied by the last multiplier, rather than multiply each multiplier by each multiplicand. In the example, \(2 \times 7 = 14, 2 \times 0 = 0\), then \(5 \times 3 = 15\).

\[
\begin{array}{c}
5 \\
\times 3 \\
\hline
15 \\
14 \\
\end{array}
\]

**Cox 1.77%**

**Last-multiplication-skipped.** The second multiplicand is not multiplied by the second multiplier.

\[
\begin{array}{c}
3 \\
\times 1 \\
\hline
\frac{3}{2} \\
\end{array}
\]

**Ainsworth 1.32%**

**Multiplied-product-by-carry.** The carry digit is multiplied by the product, rather than being added to it. In this example, \(3 \times 9 = 27, 3 \times 1 = 3, 3 \times 2 = 6\).

\[
\begin{array}{c}
1 \\
\times 3 \\
\hline
6 \\
7 \\
\end{array}
\]

**Cox 1.77%**

**Multiplier-all-by-first-multiplier.** The first multiplier is used to multiply all the other digits. In this example, \(1 \times 2 = 2, 1 \times 4 = 4, 1 \times 3 = 3\).

\[
\begin{array}{c}
4 \\
\times 3 \\
\hline
3 \\
1 \\
\end{array}
\]

**Ainsworth 3.95%**

**Multiplies-by-carry-over-blank.** When the multiplicand is over an empty cell, the subject multiplies by the carry digit.

\[
\begin{array}{c}
7 \\
\times 4 \\
\hline
1 \\
4 \\
2 \\
\end{array}
\]

**Attisha**

**Multiplies-carry.** When there is a carry digit in the current column, it is used for multiplication instead of the multiplicand. I.e., \(8 \times 4 = 32, 1 \times 4 = 12\), and so on.

\[
\begin{array}{c}
3 \\
\times 4 \\
\hline
1 \\
2 \\
4 \\
\end{array}
\]

**Cox 5.31%**

**Attisha**
Multiples-last-multiplicand-and-writes-IT. The only multiplication performed is to multiply the multiplier by the last multiplier (3 x 6 in the example). The product is written in the answer row, and ten is written after it.

\[
\begin{array}{c}
3 & 0 \\
\times & 6 \\
\hline
1 & 8 & 8
\end{array}
\]

\[\text{Cox } 0.88\%\]

Multiples-multiplicands. The first multiplication is correct, but the subject then multiplies the multiplicands. In this example, 1 x 4 = 4, 2 x 4 = 8.

\[
\begin{array}{c}
2 & 4 \\
\times & 3 & 1 \\
\hline
8 & 4
\end{array}
\]

\[\text{Ainsworth } 1.32\%\]

Multiples-partial-product. The partial product is multiplied, not added, with the bug multiplies-using-addition-pattern.

\[
\begin{array}{c}
3 & 2 \\
\times & 2 & 1 \\
\hline
3 & 2 \\
6 & 4 & 0 \\
\hline
2 & 0
\end{array}
\]

\[\text{Ainsworth } 2.63\%\]

Multiples-using-addition-pattern. Uses the addition pattern, but multiplies.

\[
\begin{array}{c}
5 & 2 & 4 \\
\times & 7 & 3 & 1 \\
\hline
3 & 5 & 6 & 4
\end{array}
\]

\[\text{Cox } 4.42\%\]

\[\text{Ainsworth } 11.84\%\]

\[\text{Attisha}\]

Multiply-by-carry-when-zero. When the multiplicand is zero, the subject prefers to multiply by the carry digit.

\[
\begin{array}{c}
4 & 0 & 6 \\
\times & 7 & 3 \\
\hline
1 & 2 & 3 & 8 \\
3 & 0 & 8 & 2 & 0
\end{array}
\]

\[\text{Cox } 1.77\%\]

\[\text{Attisha}\]

\[\text{Buswell } 23.44\%\]

\[\text{Ainsworth } 21.05\%\]

\[\text{Cox } 12.39\%\]

\[\text{Attisha}\]

No-annexing-in-third. No zeros were inserted for the third answer row.

\[\text{Cox } 2.65\%\]

Partial-product-confusion. A general combination error in which the subject had difficulty when the problem had two or more multipliers. In the first example, 4 x 5 = 20,
2 \times 4 = 8 \text{ and } 2 \times 1 = 2 \text{ at } 3. \text{ In the second example, the second and third products are written on the same answer row.}

\[
\begin{array}{ccc}
1 & 4 & 1 \\
\times & 2 & 5 \\
\hline
5 & 4 & 2 \\
\end{array}
\quad
\begin{array}{ccc}
2 & 5 \\
\times & 2 & 5 \\
\hline
5 & 1 & 2 \\
\end{array}
\quad
\text{Buswell} \quad 6.25\%

\text{Partial-product-incorrectly-summed. The addition of the partial product is incorrect. Cox apparently used this category to cover a number of addition bugs.}

\[
\begin{array}{ccc}
5 & 3 \\
\times & 7 & 1 \\
\hline
3 & 7 & 1 \\
\end{array}
\quad
\begin{array}{ccc}
2 & 1 & 2 \\
\hline
4 & 4 & 2 \\
\end{array}
\quad
\text{Cox} \quad 4.42\%

\text{Partial-product-reversed. The order of the digits is reversed in the partial product. In the example, the "219" should be "912".}

\[
\begin{array}{ccc}
4 & 5 & 6 \\
\times & 2 & 5 \\
\hline
2 & 2 & 8 \\
4 & 5 & 6 \\
\end{array}
\quad
\begin{array}{ccc}
1 & 1 & 0 \\
\times & 2 & 8 \\
\hline
1 & 7 & 9 \\
2 & 2 & 4 \\
\end{array}
\quad
\text{Buswell} \quad 1.17\%

\text{Quits-after-first-multiplication. Only the first multiplication is completed.}

\[
\begin{array}{ccc}
2 & 4 & 7 \\
\times & 4 \\
\hline
2 & 8 \\
\end{array}
\quad
\text{Attisha}

\text{Quits-after-first-multiplier. Only the first multiplier is used.}

\[
\begin{array}{ccc}
3 & 1 & 6 \\
\times & 2 & 8 \\
\hline
6 & 2 & 1 \\
7 & 3 & 4 \\
\end{array}
\quad
\begin{array}{ccc}
1 & 1 & 8 \\
\times & 2 & 8 \\
\hline
2 & 2 & 4 \\
2 & 2 & 4 \\
\end{array}
\quad
\text{Buswell} \quad 10.16\%

\text{Quits-at-100s. The subject quits multiplying after processing the tens column.}

\[
\begin{array}{ccc}
2 & 2 & 1 \\
\times & 1 & 1 \\
\hline
1 & 7 & 9 \\
2 & 2 & 4 \\
\end{array}
\quad
\text{Cox} \quad 0.88\%

\text{Repeated-multiplication. A multiplication was repeated.}

\[
\begin{array}{ccc}
1 \\
\times & 2 \\
\hline
8 & 8 \\
\end{array}
\quad
\text{Buswell} \quad 0.59\%

644
Skips-zero-multiplicand. When the multiplicand contains a zero, the multiplication is skipped and the remaining digits of the multiplicand are multiplied by the multiplier directly under the zero. In the example, $2 \times 9 = 18$, $8 \times 5 = 40$.

\[
\begin{array}{c}
8 & 0 & 9 \\
\times & 5 & 2 \\
\hline
4 & 0 & 1 & 8 \\
\end{array}
\]

Spurious-zero-in-100s. A zero is inserted in the hundreds column for no apparent reason.

\[
\begin{array}{c}
9 & 0 & 5 \\
\times & 4 & 6 \\
\hline
5 & 4 & 0 & 3 & 0 \\
3 & 6 & 0 & 2 & 0 \\
\end{array}
\]

Subtracts-partial-product. The subject subtracts the partial product rather than adding. In this example the subject also subtracts the smaller number from the larger.

\[
\begin{array}{c}
5 & 3 \\
\times & 7 & 1 \\
\hline
2 & 1 & 2 \\
3 & 7 & 1 & 0 \\
3 & 5 & 0 & 2 \\
\end{array}
\]

Too-many-annex-zeros. Too many zeros are inserted into the answer row when multiplying by a multiple of ten.

\[
\begin{array}{c}
5 & 5 & 3 \\
\times & 2 & 0 \\
\hline
1 & 1 & 0 & 6 & 0 & 0 & 0 \\
\end{array}
\]

Weird-order. The digits are multiplied in a strange order. In this example, the order is: $4 \times 1 = 4$, $2 \times 1 = 2$, $2 \times 3 = 6$, $2 \times 4 = 8$.

\[
\begin{array}{c}
1 & 3 \\
\times & 2 & 4 \\
\hline
8 & 6 & 2 & 4 \\
\end{array}
\]

Works-left-to-right. The subject starts at the left, adding carries to the right. In the example, $5 \times 3 = 15$, $2 \times 3 = 6$, $1 = 7$.

\[
\begin{array}{c}
5 & 2 \\
\times & 3 \\
\hline
\frac{5}{7} \\
\end{array}
\]

Zero-in-first-row. A zero is inserted at the start of the first row. Subsequent rows have the correct number of zeros.

\[
\begin{array}{c}
4 & 3 & 6 \\
\times & 5 & 1 \\
\hline
1 & 3 & 6 & 0 \\
\end{array}
\]

\[
\begin{array}{c}
4 & 2 & 1 & 8 & 0 & 0 \\
\times & 2 & 6 & 1 & 6 & 0 \\
\hline
2 & 6 & 1 & 6 & 0 \\
\end{array}
\]
How to Write
Arithmetic and Algebra
By Means of the Joint Type Method

By
H. M. TAYLOR, M.A., F.R.S.

With examples by
JOHN R. EMBLE, A.O.P.

and approved by
The College of Teachers of the Blind

Price: SIXPENCE

Printed and Published by the
National Institute for the Blind
221-3-8 Great Portland St., London, W.1
1974
THE ARITHMETIC AND ALGEBRA BOARD

Arithmetical and Algebraical Calculations may be made by the Braille System, but where the Calculating Board is used, that invented by the Rev. W. Taylor is recommended, of which the following brief descriptive explanation is given:

The holes in the Board are star-shaped, with eight angles, thus allowing the square Types—of which there are two kinds, Type I and Type II—to be placed each in sixteen different positions, eight in Position a, and eight in the reverse Position b.

The following is a detailed list of the various Positions of the two Types, together with the corresponding uses assigned to them:
<table>
<thead>
<tr>
<th>TYPE I</th>
<th>TYPE II</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Position a</strong></td>
<td><strong>Position b</strong></td>
</tr>
<tr>
<td>§1</td>
<td>§9</td>
</tr>
<tr>
<td>§2</td>
<td>§0</td>
</tr>
<tr>
<td>§3</td>
<td>§-</td>
</tr>
<tr>
<td>§4</td>
<td>§-</td>
</tr>
<tr>
<td>§5</td>
<td>§÷</td>
</tr>
<tr>
<td>§6</td>
<td>§÷ and Ratio</td>
</tr>
<tr>
<td>§7</td>
<td>§Decimal Point</td>
</tr>
<tr>
<td>§8</td>
<td>§= and Recurring Period</td>
</tr>
</tbody>
</table>

**Additional Uses.**—For Trigonometry, the first six positions of Type II (b) may be used respectively for Sine, Cosine, Tan., Cosec., Sec., Cotan.
### ADDITION

<table>
<thead>
<tr>
<th>Add</th>
<th>1948</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>246</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td></td>
</tr>
<tr>
<td>402</td>
<td></td>
</tr>
<tr>
<td>94</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2718</td>
</tr>
</tbody>
</table>

### MULTIPLICATION

<table>
<thead>
<tr>
<th>Multiply</th>
<th>£27 7s. 10½d. by 15</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>27 7 10½</td>
</tr>
<tr>
<td></td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>136 15 3¼</td>
</tr>
<tr>
<td></td>
<td>5958 14 10½</td>
</tr>
</tbody>
</table>

### SUBTRACTION

<table>
<thead>
<tr>
<th>Km.</th>
<th>Hm.</th>
<th>Dm.</th>
<th>M.</th>
</tr>
</thead>
<tbody>
<tr>
<td>From</td>
<td>4</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Take</td>
<td>1</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
Braille Mathematics Code for India

MANUAL

Prepared under:
The Project:
Adoption and Introduction of an Appropriate Braille Mathematics Code for India

Sponsored by: UNICEF

Published by:
National Institute for the Visually Handicapped, Dehradun,
National Association for the Blind, India
Chapter 1
Numerals

List of Signs

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Sign</th>
<th>Dots</th>
<th>Braille Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Numeric Indicator</td>
<td>3-4-5-6</td>
<td>0 0</td>
</tr>
<tr>
<td>2</td>
<td>Mathematical Comma</td>
<td>6</td>
<td>0 0</td>
</tr>
<tr>
<td>3</td>
<td>Decimal Point</td>
<td>4-6</td>
<td>0 0</td>
</tr>
<tr>
<td>4</td>
<td>Punctuation Indicator</td>
<td>4-5-6</td>
<td>0 0</td>
</tr>
</tbody>
</table>

Explanation
(a) The numerals are represented in the lower portion of the braille cell. However, numerals on the title page and at the corners of pages must be transcribed as in English Braille.
(b) The Numeric Indicator must be used at the beginning of a braille line or after a space.
(c) The Numeric Indicator must not be used in tables and in a work arranged in columns.
(9) 10,000,000

(10) 12,34,57,89,12,000

(If a number is too long to be accommodated in a single braille line, it is divided at the end of the first line by putting a hyphen.)

(11) 1st, 5th

(‘st’, ‘th’ must not be contracted.)

(12) 120.

(d) A Braille Indicator is a sign in braille which does not correspond to any sign in ink-print. The Numeric Indicator is such a Braille Indicator.
**Chapter 2**

**Basic Signs**

List of signs

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Sign</th>
<th>Dots</th>
<th>Braille Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Plus</td>
<td>3 4 6</td>
<td>o o</td>
</tr>
<tr>
<td>2</td>
<td>Minus</td>
<td>3 6</td>
<td>o o</td>
</tr>
<tr>
<td>3</td>
<td>Multiplication (Cross)</td>
<td>4, 1-6</td>
<td>o o o o</td>
</tr>
<tr>
<td>4</td>
<td>Division (Divided by)</td>
<td>4-6, 3 4</td>
<td>o o o o</td>
</tr>
<tr>
<td>5</td>
<td>Is equal to</td>
<td>4-6, 1 3</td>
<td>o o o o</td>
</tr>
<tr>
<td>6</td>
<td>Is greater than</td>
<td>4-6, 2</td>
<td>o o o o</td>
</tr>
<tr>
<td>7</td>
<td>Is less than</td>
<td>5, 1 3</td>
<td>o o o o</td>
</tr>
</tbody>
</table>
Examples:

(1)  1  
 (ii) 0  
 (ii) 0  

(3)  125  
 (ii) 0  
 (ii) 0  

(4)  3.5  
 (ii) 0  
 (ii) 0  

(5)  0.25  
 (ii) 0  
 (ii) 0  

(6)  "3"  
 (ii) 0  
 (ii) 0  

(7)  These were 7 bars.  

(8)  3  
 (italicized three)
(7) $x > y$

(8) $3x - 2 > x + 9$

(9) $0 > -1$

(10) $x = 0$

(11) A sign for Plus is $+$

(12) The Operation Signs are $+$, $-$, $\times$ and $\div$.

(13) $-3$

(14) $\times -5$
# Chapter 3

## Brackets

### List of Signs

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Sign</th>
<th>Dots</th>
<th>Braille Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Round Bracket</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Opening</td>
<td>1-2-3-5-6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Closing</td>
<td>2-3-4-5-6</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Square Bracket</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Opening</td>
<td>4, 1-2-3-5-6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Closing</td>
<td>4, 2-3-4-5-6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Curly Bracket</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Opening</td>
<td>4-6, 1-2-3-5-6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Closing</td>
<td>4-6, 2-3-4-5-6</td>
<td></td>
</tr>
</tbody>
</table>
Explaination

(a) The signs for Brackets must be used both for literary and mathematical purposes.
(b) The signs for Brackets in English Braille, however, are used to enclose literary matter on title pages.

Examples

(1) (3)

(2) [x + 7]

(3) {4 + \[7 + (5 - 1)\]}

(4) {x + 5}

(5) (-3, 0)
Chapter 2—Basic Signs

**Explanation**

(a) The signs for Plus, Minus, Multiplication (Cross), Division (Div) and Division are open signs. No space must be left either before or after them.

(b) The signs for 'is equal to', 'is greater than' and 'is less than' are comparison signs. A space be left both before and after them.

**Examples**

(1) $3 + 4$

(2) $8 - 2$

(3) $9 \times 7$

(4) $45 \div 5$

(5) $105 = 3 \cdot 57$

(6) $3 < 4$
Example is a must.
Clear instructions.
If at first attempt, the answer is wrong, repeat instructions ask the child to attempt.
<table>
<thead>
<tr>
<th>Date</th>
<th>Name</th>
<th>Class</th>
<th>Time</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>8/4</td>
<td>Rajamani</td>
<td>Std V</td>
<td>1 pm</td>
<td>Appointment</td>
</tr>
<tr>
<td></td>
<td>Jayalakshmi</td>
<td></td>
<td>2 pm</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Basav</td>
<td></td>
<td>3 pm</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Saiuddin</td>
<td></td>
<td>4 pm</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pawan</td>
<td></td>
<td>5 pm</td>
<td></td>
</tr>
</tbody>
</table>

- While keeping the direction may change the answer might be correct. Probe for correct answer.
- Check the signs for + & = Equal. Reading may be correct.
- They may put the reg completely opp. Eg: for 9 x 0
- Read the problem before solving.
- Unable to solve problems if numbers of diff. combination of digits are eg: 379842, 876594, 865325.

- Concept of borrowing is not clear.
- Child may not be attentive while talking through the problem.

**Appointment**

- 8 am
- 9 am
- 10 am
- 11 am
- 12 noon
- 1 pm
- 2 pm
- 3 pm
- 4 pm
- 5 pm
13 JAN 1998

Smt. Tausha
3, New Thirumalam Rd
Atpeidi 632007

Asian Xenon Machines
NIVH Enlargement
Cost: 35,000

25 JAN 1998

Arithmetic

<table>
<thead>
<tr>
<th>Class: T 1 std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>T 1: Basavanagudi (25th Dec to Jan 2)</td>
</tr>
<tr>
<td>No. of St: 16</td>
</tr>
<tr>
<td>M: O: I - English</td>
</tr>
<tr>
<td>Syllabus: Stati</td>
</tr>
<tr>
<td>Evaluation: T 1, Oral, Braille</td>
</tr>
</tbody>
</table>

Common Errors:
- May miss the place value, e.g., in 704839, 714839
- T 1: Tells the no: in statement problems - how many cells to take e.g. for 0 in 3232
- T 1: Tells no of cells to be used for different mentally.
- When zero is there, may have problems in reading; they are confused with 0 - next to which place value do assign for zero, yes e.g. in 468035
- Curriculum leads should be lessened.
- Playway/Activity method not used for teaching

5. 74003 0.347
5. 48900 8.3600 *
5. 489
3. 74003
38. 17603

Maths Code - Pencils Unior

Appointment

9 am
10 am
11 am
12 noon
2 pm
3 pm
4 pm
5 pm

Can worksheets be provided?

(25th December - 2nd Jan) Remittance: Rs. 300 + 7.74003
Separate Resource Room: 90% of the usual models till 11th Std.

- Workbooks can be prepared wherein only eg. and problems are given. Children will have to answer it on "TF" and then transfer it on "P-Small Sheet".

- Solving Commercial maths or problems like: \( \frac{560}{42} = \frac{8}{7} \)

- Materials used for Tg Arithmetic Skills
  1. String of beads
  2. Cylindrical shapes for place value
  3. Place value box
  4. Ben with pegs

- Apparatus - test tubes, conical flasks, round bottom flasks, water bath vessel, upset stand, disposable pl, metals, non-metals, etc.

Approach: Tg Arithmetic Basavagange

- Oral, TF, Abacus used for I, II Std
- TF, Nenmak Code used for III Std onwards

- Nenmak code is for prescribed purpose and Taylor's frame is for practical learning purpose. Hence, the child has to be proficient in both the modalities.

To start with larger numerals, e.g., 6 digit. Teacher should specify to the students - how many cells are required to be used.

- Students may misread 0 at a place value in larger numerals, hence the concept of 0 should be strengthened.

- For Statement problems, real life situations should be used.

- Sufficient drill work should be given regarding each problem.