2.1 INTRODUCTION

The deposition of atherosclerotic plaques in the lumen of an artery is called stenosis or, atherosclerosis. Stenosis may be formed at one or, more locations of the cardiovascular system and its subsequent and severe growth may lead to serious circulatory disorders (Caro\textsuperscript{2}, Fry\textsuperscript{3}, Young\textsuperscript{33}, MacDonald\textsuperscript{41}). As a result of this unbecoming formation at the innermost blood vessel wall, there will arise a reduction and an obstruction to the regular blood flow in the constricted regions of an artery and a blockage of the arteries and, the presence of stenosis in one or, more of the major blood vessels supplying blood to the brain, heart, different organs etc., may result in different cardiovascular and arterial diseases (haemorrhage, stroke,
myocardial infarction, cerebral accident, coronary thrombosis, angina pictoris etc.). Although an exact mechanism of such deposits is somewhat unclear, it has been observed that certain parts of the human arterial tree, like the carotid, coronary and femoral arteries as well as the bifurcation of the aorta at the iliac arteries, are predisposed to deposits of this kind. It has been reported that hydrodynamic factors can take an important role in the initiation, development and progression of an arterial stenosis (Caro, Young and Tsai). Further, many researchers (Fry, Dintenfass, Chien, Caro, Schmid-Schonbein) have indicated that the rheologic and fluid dynamic properties of blood and blood flow could play a significant role in the fundamental understanding, diagnosis, prognosis and treatment of many cardiovascular, cerebrovascular and arterial diseases.

The study of blood flow in channels with constriction has drawn considerable attention from the researchers in recent years due to its potential applications to biological systems. Young has proposed a mathematical model to study the hydrodynamical aspects of blood flow through an artery with the presence of mild stenosis. Young and Tsai have examined theoretical and experimental models on various features of blood flow in a partially occluded tube. Lee and Fung have investigated various effects of stenosis on the cardiovascular system by conformal mapping technique. Morgan and Young have obtained theoretical results for flow variables in the
stenosed model of blood. MacDonald\textsuperscript{41} has analysed the steady flow through a mild axisymmetric stenosed artery. Shukla \textit{et al.}\textsuperscript{43} and, Chaturani and Biswas\textsuperscript{46-47} have considered various effects of an arterial stenosis on the cardiovascular system. Misra and Kar\textsuperscript{50} have developed a mathematical model to study the different characteristics of blood flow through stenosed vessels with the slip velocity at wall. Many other researchers, Forrester and Young\textsuperscript{34}, Chow and Soda\textsuperscript{36}, Ehrlich\textsuperscript{39}, Deshpande and Giddens\textsuperscript{40}, Yongchareon and Young\textsuperscript{42}, Doffin and Changneau\textsuperscript{44}, Polard\textsuperscript{45}, Perkkio and Keskinen\textsuperscript{49} and others\textsuperscript{50-53} have proposed mathematical models for blood flow through stenosed arteries. In the aforementioned models\textsuperscript{35-53}, various aspects of blood flow in stenosed tubes have been studied, by considering blood to behave as a Newtonian fluid. Human blood is regarded as a suspension of different tiny cells in a continuous aqueous substance plasma (Fung\textsuperscript{24}, Guyton\textsuperscript{5}). Although, blood exhibits a non-Newtonian character at low shear rates (Merrill\textsuperscript{31}), at high shear rates generally found in the larger arteries (diameter nearly above 1 mm), blood behaves as a Newtonian fluid (Taylor\textsuperscript{32}). It has been shown by Pedley\textsuperscript{26} that flow of blood in large arteries (e.g., aorta), can be modelled as a Newtonian viscous fluid. Since stenosis normally generates and advances in large diameter arteries (500 to 2000 \textmu m) where blood shows a Newtonian nature, it seems to be rationale to assume blood to be homogeneous, isotropic, incompressible
Newtonian continuum having a constant viscosity and density for flow through a stenotic artery.

Further, in the above mentioned works (Young, Morgan and Young, Shukla et al., Deshpande and Giddens, Perkkio and Keskinen, and others), the boundary condition taken at the uniform and narrowed artery walls, is the usual no-slip condition at the boundary. The existence of velocity slip at the flow boundaries has already been mentioned by many authors (Vand, Nubar, Jones, Brunn, Bloch). Experimental observations (Bugliarello and Hayden, Bennet) on blood flow also indicate the likely presence of a slip at the blood vessel wall. However, in the works of Chaturani and Biswas, and Misra and Kar, a velocity slip at the constricted wall is adopted. In view of the above theoretical models and experimental works, a slip condition at the stenotic wall is taken in the present modelling.

The aim of this theoretical analysis is to study the effects of slip at the stenotic wall on the flow variables (velocity profiles, flow rate, pressure drop, resistance to flow, wall shear stress) in case of mild, moderate and severe stenoses and to show, how the analysis developed with the slip, could be useful in the obstructed circulatory system.
2.2 BASIC EQUATIONS OF THE FLOW

The basic equations governing the fluid flow include the conservation equations of mass and momentum and, the constitutive equation of the fluid. The conservation equation of mass and linear momentum\(^n\)\(^n\) are the equation of continuity:

\[
\dot{\rho} + \rho \dot{V}_i = 0, \tag{2.2.1}
\]

and the equation of motion:

\[
T_{ij} + \rho \ddot{V}_i = \rho \dot{V}_i. \tag{2.2.2}
\]

where \(T_{ij}\) is the usual stress tensor of second order, \(\rho\) the density of the fluid, \(\ddot{V}_i\) represents the body force per unit mass, \(\dot{V}_i\) the usual velocity field and the superimposed dot denotes the material time derivative.

The constitutive equation for Newtonian viscous fluid is in the form:\(^n\)

\[
T + \rho I = 2\mu D. \tag{2.2.3}
\]

where \(T\) is the stress tensor, \(\rho\) the pressure, \(I\) the Kronecker delta, \(\mu\) the traditional shear viscosity coefficient and \(D\) the rate of strain tensor which is here the symmetric part of the velocity gradient is expressed as

\[
D_{ij} \equiv V_{(ij)} \tag{2.2.4}
\]

where the parentheses stand for the symmetric part of \(V_{i,j}\).
The conservation equations (2.2.1-2.2.2) above, for incompressible viscous fluid flows, reduce to the following vector form:

\[ \mathbf{V} \cdot \mathbf{V} = 0 \quad (2.2.5) \]

and

\[ \rho \left[ \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right] = \mathbf{f} - \mathbf{V} p + \mu \mathbf{V}^2 \mathbf{V} \quad (2.2.6) \]

where nebla \( \mathbf{V} \) is the vector differential operator and \( \mathbf{V}^2 \) the Laplace operator.

In absence of body force and the inertial terms, the equations for a linear incompressible fluid for steady flow, is obtained from equation (2.2.6) in the form

\[ \mu \mathbf{V}^2 \mathbf{V} = \mathbf{V} p \quad (2.2.7) \]

2.3 FORMULATION OF THE PROBLEM

Let us consider an axially symmetric, laminar, steady, fully developed and one-dimensional flow of blood, obeying the constitutive equation for a Newtonian-fluid; through an artery (uniform circular tube) with stenosis (constriction) as shown in Fig. 2.1. Fluid velocity vector has the form \( \mathbf{V}(0,0,u(r)) \) in cylindrical polar system \((r, \theta, z)\) representing the radial, circumferential and axial coordinates respectively.

The basic equation governing the unidirectional flow, obtained from equation (2.2.7), is
where the shear stress $\tau_{rz}$ for a Newtonian fluid (obtained from equation 2.2.3) becomes

$$\tau_{rz} = \mu \frac{\partial u(r)}{\partial r}$$  \hspace{1cm} (2.3.2)

where $u = u(r)$ is the axial velocity, $p$ the fluid pressure and $\mu$ the viscosity of the fluid.

As a result of equations (2.3.1-2.3.2), there yields the following equation:

$$C + \mu \frac{\partial}{\partial r} \left[ r \frac{\partial u(r)}{\partial r} \right] = 0,$$

$$\hspace{1cm} (2.3.3)$$

where, $C = \left( -\frac{dp}{dz} \right)$ is the pressure gradient which is constant for a uniform tube.

The geometry of an arterial stenosis which is assumed to be symmetrical, is given by Young$^{33}$

$$\frac{R(z)}{R_0} = 1 - \frac{\delta s}{2R_0} \left[ 1 + \cos \frac{2\pi}{L_0} \left( z - d - \frac{L_0}{2} \right) \right], \hspace{1cm} d \leq z \leq d + L_0,$$

$$\hspace{1cm} = 1, \hspace{0.5cm} \text{otherwise}, \hspace{1cm} (2.3.4)$$

where $R(z)$ is the radius of the artery in the stenotic region, $R_0$ is the constant radius of the normal artery region, $L_0$ is the length of the stenosis, $d$ indicates its location and $\delta s$ is the maximum height of the stenosis.

The boundary conditions of the problem are
\[ u = u_s \text{ at } r = R(z), \quad \text{(slip condition)} \quad (2.3.5) \]

And

\[ \frac{\partial u}{\partial r} = 0 \text{ at } r = 0, \quad \text{(symmetry condition)} \quad (2.3.6) \]

where \( u_s \) is the slip velocity at the stenotic wall and \( u_s = u_s(R(z)) \).

### 2.4 SOLUTION

Integrating equation (2.3.3), its solution becomes

\[ u(r) = B + AI_r^r - \frac{C}{4\mu} \quad (2.4.1) \]

where \( A \) and \( B \) are the constants of integration, to be determined with the help of given conditions.

Using the boundary conditions (2.3.5-2.3.6) in equation (2.4.1), the solution of the equation (2.3.3) becomes

\[ u(r) = u_s + \left( \frac{C}{4\mu} \right) \left( R^2(z) - r^2 \right), \quad 0 \leq r \leq R(z). \quad (2.4.2) \]

The volume flow rate

\[ Q = \int_{r=0}^{R(z)} \int_{\theta=0}^{2\pi} r u(r) \, dr \, d\theta, \quad (2.4.3) \]

for this case, may be computed (using equation (2.4.2) in the form

\[ Q = \pi \left[ R^2(z) u_s + \frac{C}{8\mu} R^4(z) \right]. \quad (2.4.4) \]

It can be noted from the equation of continuity that \( Q \) is a constant.

From the equation (2.4.4), the pressure gradient can be expressed as
\[
\frac{dp}{dz} = \left(-\frac{8\mu}{R^4(z)}\right) \left(\frac{Q}{\pi} - u_s R^2(z)\right)
\]  
\text{(2.4.5)}

Integrating equation (2.4.5) between the limits

\[p = p_i \text{ at } z = 0\]

and

\[p = p_0 \text{ at } z = L,\]

where \(L\) is the length of the tube,

we obtain pressure drop in the form

\[p_i - p_0 = \frac{8\mu}{\pi} \int_{z=0}^{L} \left[Q - u_s R^2(z)\right] dz\]
\text{(2.4.7)}

The resistance to flow \((\lambda)\) is defined as follows

\[\lambda = \frac{p_i - p_0}{Q} \]  
\text{(2.4.8)}

Using equations (2.3.4), (2.4.7) and (2.4.8) resistance to flow takes the form

\[\lambda = \frac{8\mu}{\pi} \left[R_0^{-4} (\pi^{-1} - Q^{-1} R_0^2 u_s) (L - L_0) + \right.\]
\[\left. \int_{z=L_0}^{R(z)} \left[Q - u_s R^2(z)\right] dz\right]\]
\text{(2.4.9)}

where \(Q_1 = Q\) at \(R(z) = R_0\).

The average pressure gradient in the axial direction may be defined as

\[\left(\frac{dp}{dz}\right)_{av} = \int_{r=0}^{R(z)} \frac{(dp)}{dz} dr = \int_{r=0}^{R(z)} \frac{dp}{dz} dr,\]
\text{(2.4.10)}
and with the help of equation (2.4.5), it reduces to

$$\left(\frac{dp}{dz}\right)_{av} = \left(-\frac{8\mu}{R^4(z)}\right)(Q, \pi^1 - u_b R^2(z)) = \frac{dp}{dz}. \quad (2.4.11)$$

Expression for wall shear stress obtained from the formula

$$\tau_{R(z)} = \left(-\mu \frac{\partial u(r)}{\partial r}\right)\bigg|_{r=R(z)} \quad (2.4.12)$$

becomes

$$\tau_{R(z)} = \frac{C}{2} R(z). \quad (2.4.13)$$

Apparent viscosity can be computed from the formula

$$\mu_a = \frac{C\pi R^4(z)}{8Q} = \left[\frac{8u_b}{CR^2(z)} + \frac{1}{\mu}\right]^{-1} \quad (2.4.14)$$

where Q has been given in equation (2.4.4)

Another representation of the flow variables:

**Velocity function**

$$\bar{u}(r) = u_b + \bar{R}^2 \left\{ \frac{1}{R} \right\}, \quad 0 \leq \frac{1}{R} \leq 1 \quad (2.4.15)$$

**Flow Rate**

$$\bar{Q} = 2u_b \bar{R}^2 + \bar{R}^4 \quad (2.4.16)$$

**Pressure gradient term**

$$\left(\frac{dp}{dz}\right) = (\bar{R})^4 \left(\bar{Q} - 2u_b \bar{R}^2\right) = \left(\frac{dp}{dz}\right)_{av}. \quad (2.4.17)$$
Resistance to flow

\[ \bar{\lambda} = \left(1 - \frac{2\bar{u}_s}{Q}\right) \left(\frac{L - L_0}{L}\right) + \frac{1}{L} \int_{z=d}^{d+L_0} (\bar{R})^{-4} \left(1 - \frac{2\bar{R}^2\bar{u}_s}{Q}\right) dz, \]  

(2.4.18)

Wall shear stress

\[ \tau_R = \left(\frac{dp}{dz}\right) \bar{R}, \]  

(2.4.19)

Apparent viscosity

\[ \bar{\mu}_a = \left[1 + 2\frac{u_s}{R^2}\right]^{-1} \]  

(2.4.20)

where

\[ R = R(z), \quad \bar{R} = \frac{R}{R_0}, \quad \bar{s} = \frac{\bar{s}}{R_0}, \quad \frac{CR_0^2}{4\mu} = \bar{u}'_0, \quad Q_0 = \pi CR_0^4/(8\mu), \quad \left(\frac{dp}{dz}\right)_0 = -\frac{8\mu Q_0}{\pi R_0^4}, \]

\[ \lambda_0 = \frac{8\mu L}{\pi R_0^4}, \quad (\tau_R)_0 = \left(\frac{dp}{dz}\right)_0 \frac{R_0}{2}, \quad \bar{u} = u/u'_0, \quad \bar{u}_s = u_s/u'_0, \quad \bar{Q} = Q/Q_0, \]

\[ \left(\frac{dp}{dz}\right)_0 = \left(\frac{dp}{dz}\right)/\left(\frac{dp}{dz}\right)_0, \]

\[ \bar{\lambda} = \frac{\lambda}{\lambda_0}, \quad \bar{\tau}_R = \frac{\tau_R}{(\tau_R)_0} \quad \text{and} \quad \bar{\mu}_a = \frac{\mu_a}{\mu} \]  

(2.4.21)

(\mu being the viscosity of the Newtonian fluid)

2.5 RESULTS AND DISCUSSIONS

Throughout the present work for blood flow through an obstructed artery (\(d \leq z \leq d+L_0\), the following viz., stenosis length \(L_0 = \text{eight} = \text{its} \)
location $d$, maximum height of stenosis $\delta s$ (in dimensionless form) equals to $\left(1-\sqrt{3}/2\right)\left(1-1/\sqrt{2}\right)$ and $\frac{1}{2}$ corresponding to an unnatural growth of 25 percent, 50 percent and 75 percent in three respective cases of mild, moderate and severe stenosis, are used in the mathematical analysis. In this chapter, body fluid blood (- a red cell suspension) is assumed to behave like a Newtonian fluid with constant viscosity $\mu = 2$ centipoise (cp) (Fig. 2.1). The geometry of an arterial stenosis in non-dimensional form, is presented in eq. (2.3.4) of Section 2.3. Also, in boundary condition, an axial velocity slip at the stenotic wall, is employed. Analytic expressions are obtained for velocity, flow rate, pressure gradient, pressure drop, resistance to flow, wall shear stress and apparent viscosity, with the slip condition and, a second representation of these flow variables, in their dimensionless form, is presented in the analysis. Velocity distribution obtained in eq. (2.4 2) is found to be a function of radial $r$ and axial $z$ coordinates, tube radii $(R_0, R)$ in uniform and constricted artery regions, three dimensions $L_0$, $d$ and $\delta s$ relating to the geometry of an arterial stenosis, pressure gradient $\gamma$, shear viscosity $\mu$ of blood and slip velocity $u_s$. This is in contrast to Poiseuille flow wherein velocity function depends on $r$, $R_0$, $\gamma$ and $\mu$ only. For $u_s=0$, the present analysis leads to Newtonian model with zero-slip at a stenotic wall. In case, $R(z) = R_0$ and $u_s \geq 0$, it leads to Poiseuille flow of blood through a uniform tube with slip or, no-slip at flow boundary.
Also, an existence of velocity slip at blood vessel wall, has been reported, both theoretically\textsuperscript{60,101,105} and experimentally\textsuperscript{103-104} and, methods to detect (Astarita \textit{et al}\textsuperscript{109}) and determine (Cheng\textsuperscript{110}) slip experimentally, have been indicated in literature, however, the magnitudes of wall slip are not yet determined. In order to study the effects and influence of slip on the flow parameters, in gradual advancement of stenosis and on Newtonian and non-Newtonian behaviour of blood, three cases of slip $\bar{u}_s = 0.00, 0.05, 0.10$ are taken throughout the investigation. In the proceeding text, we have attempted to examine the variations in flow characteristics, due to slip at stenotic wall.

\subsection*{2.5.1 Velocity Profiles}

A comparison of velocity profiles (obtained from eq 2 4 15) in a steady state artery inhabiting three kinds of stenosis development for slip and no-slip cases and different axial locations, is shown in Figs 2 2-2 4. In the figures, variation of axial velocity against tube radius for stenosis heights and axial distances, are considered. For all types of constriction, dimensionless velocity profiles are drawn for stenotic artery locations $z = d = 8, d + L_0/8, d + L_0/4, d + 3L_0/8, d + L_0/2$, $z = d + L_0 = 16$, slip and zero-slip $\bar{u}_s = 0.00, 0.05, 0.10$ and mild, moderate and severe formations at lumen of an artery. Wherefrom, the following observations can be made.
(i) Profiles are parabolic in uniform arteries, whereas parabolic and deviations from this trend, are noticed in stenotic regions. Non-parabolic trend is found prominent in case of severe stenosis.

(ii) In normal and constricted tubes, velocity is maximum at axis and minimum (zero or, slip velocity) at wall. Its magnitude is greater in a uniform tube than that in a non-uniform channel.

(iii) As tube radius $r/R$ increases in full scale form 0 to 1, at upper and lower sides of the axis, velocity reduces from greater values at axis to smaller ones at tube wall.

(iv) Velocity is maximum at maximum cross-sectional area (i.e., at initiation and termination of stenosis region) at $z = d = d + L_0$ and minimum at minimum cross-sectional area (i.e., at the throat of a stenosis) at $z = d + L_0/2$. However, as axial coordinate $z$ increases from $z = d$ to $d + L_0/2$, velocity decreases from its highest value to a lowest one and for remaining part when taken from other end $z = d + L_0$ to $d + L_0/2$, the trend in velocity profile remains alike and, so profiles drawn at equal axial distances apart from the throat and at its either side, overlap each other.

(v) As expected, velocity increases with slip at wall. Its values are higher for flows with slip $\bar{u}_s > 0.00$ than those in
no-slip case $\overline{u}_s = 0.00$. As slip increases from $\overline{u}_s = 0.005$ to $0.10$, it attains a higher and more higher values, in all three stages of stenosis considered.

(vi) With an increase in stenosis height or, for a reduction in tubular area, velocity $\overline{u}$ decreases for slip and no-slip cases. However, for all three cases of stenosis,

$$\overline{u}|_{\overline{u}_s = 0.00} < \overline{u}|_{\overline{u}_s = 0.05} < \overline{u}|_{\overline{u}_s = 0.10}.$$  

(vii) At the minimum cross-sectional area i.e., at the throat of a stenosis, velocity is minimum and this magnitude, although not so significant in mild formation but in moderate and severe stenosis cases, it seems to be appreciably low

2.5.2 Flow Rate

Variation of the rate of flow $\overline{Q}$ (in eq. 2.4.16) against axial distance $z$ in full scale $z = d$ to $d + L_0$, for all kinds of stenosis and slip, is presented in Fig. 2 5. It can be easily noticed from the profile

(i) $\overline{Q}$ is constant for a uniform tube whereas in a constricted artery, it changes with axial locations

(ii) It is minimum at minimum cross-sectional area (i.e., at the throat) and maximum at the two ends (i.e., at beginning and end) of stenosis. In the two equal portions $d \leq z \leq d + L_0/2$ and $d + L_0/2 \leq z \leq d + L_0$, $\overline{Q}$
increases from the least value at the throat to the greatest value at the initiation or, termination of stenosis

(iii) Values of flow rate obtained with wall slip is greater than that computed with zero-slip at vessel wall. As slip increases in magnitude, $\bar{Q}$ acquires higher and more higher values

(iv) When the growth advances from mild to severe stenosis, $\bar{Q}$ reduces from a higher value to a lower one and this trend remains similar for both slip and zero-slip cases

(v) As axial coordinate $z$ increases in constricted region, flow rate decreases from a great value at one end $z=d$ to a lower one of the throat $z = d+L_0/2$ and for other equal non-uniform region, $\bar{Q}$ increases from the throat, reaching a higher value at the other end $z = d+L_0$

(vi) In case of a fixed magnitude of slip $\bar{u}_s \geq 0$, it is noticed from the behaviour of flow rate in all three kinds of stenosis growth,

$$\bar{Q}_{\text{mild case}} > \bar{Q}_{\text{moderate stage}} > \bar{Q}_{\text{severe form}}$$
2.5.3 Pressure Gradient

Variation of pressure gradient (obtained from eq. 2.4.17) with slip $\bar{u}_s$, stenosis height $\delta s$ and axial distance $z$ (in $d \leq z \leq d + L_0$), is shown in Fig. 2.6. It can be observed that:

(i) There is no appreciable change in its value, for changes in variation of slip, axial locations and heights of stenosis.

(ii) The pressure gradient is expected to be less with an introduction of wall slip and also to attain a maximum value at the minimum cross-sectional area of stenosis.

(iii) The profile is linear in case of a uniform tube whereas for non-uniform artery region, it is found to be slightly non-linear.

2.5.4 Wall Shear Stress

The wall shear stress, $-\tau_R$ and its variation with slip and heights of stenosis, have been computed from eq. (2.4.19) and plotted against axial coordinate $z$ in the scale $d \leq z \leq d + L_0$ (Fig. 2.7). The profile clearly indicates the following:

(i) Wall stress reaches a maximum at minimum area of stenosis and rapidly decreases in the diverging section of stenosis.
(ii) As stenosis height increases, it increases from mild form to severe stage of stenosis.

(iii) As axial coordinate \( z \) changes from \( z = d \) to \( d + L_0/2 \), \( -\tau_R \) increases from the minimum value at the mouth to a maximum one at throat of stenosis. On the remaining equal portion, it decreases from the maximum value at throat to a minimum one at the end of stenosis \( (z = d + L_0) \).

(iv) As stenosis height increases, wall stress at the throat, increases from mild stage to severe form. However, there does not occur any change in wall stress at the ends of stenosis.

(v) Wall stress is expected to be lower due to a slip and as slip will increase in magnitude, it will decrease further.

2.5.5 Pressure Drop and Resistance to Flow

The variations of pressure drop (computed numerically from eq 2.4.7) with stenosis height, slip and axial distance, shows that

(i) Pressure drop is less with slip than that with no-slip

(ii) Its maximum value will be attained at the throat of stenosis.

(iii) The corresponding pressure drop in normal artery region will be straight lines.

(iv) However, it will increase with increasing stenosis height but decreases with increasing slip.
Expression for resistance to flow $\lambda$ is obtained in eq. (2.4.18) and its variation with axial distance $z$ in $d \leq z \leq d+L_0$ shows:

(i) The behaviour of $\lambda$ is similar to that of pressure drop.

(ii) It will attain a maximum value at the throat of stenosis and a minimum one at its two ends.

(iii) Its magnitude obtained with wall slip will be lower than that computed with zero slip at boundary.

(iv) As stenosis height increases, $\lambda$ will enhance in magnitude from mild case to a severe stenosis.

(v) However, with an increase in slip magnitude, there will be a further reduction in resistance to flow.

### 2.5.6 Apparent Viscosity

Apparent viscosity $\bar{\mu}_a$ is computed from eq. (2.4.20) and its variation in accordance with different heights of stenosis and slip or non-slip cases, has been plotted against axial distance $z$ (in full scale $d \leq z \leq d+L_0$) graphically in Fig. 2.8. The observations from the figure are the following:

(i) Profile drawn for zero-slip case is linear but $\bar{\mu}_a$ exhibits a non-linear trend for wall slip.

(ii) As expected, it decreases with slip at boundary and as slip increases in magnitude, $\bar{\mu}_a$ reduces further.
(iii) As stenosis height increases from mild form to severe stage, $\tilde{\mu}_a$ decreases from a higher magnitude to a lower one.

(iv) Magnitudes of viscosity obtained with slip are found to be lower than those, obtained with zero-slip. This behaviour in $\tilde{\mu}_a$ is reflected in all three stages of stenosis.

(v) As axial distance $z$ increases from one end $z = d$ to the throat $z = d + L_0/2$ of stenosis, viscosity decreases from a higher value at initiation of a growth to a lower one at throat of a stenosis. In the next equal portion, $\tilde{\mu}_a$ increases from throat to attain a higher value at the termination $z = d + L_0$ of stenosis.

(vi) As $\tilde{\mu}_a$ decreases as tube radius decreases in the constricted region it shows Fahraeus-Lindqvist Effect (FLE) here.

2.6 CONCLUSION

In the present chapter, steady flow of blood (a Newtonian fluid) through a stenosed vessel, under the condition of slip as suggested in the models at boundary of mild, moderate and severe stenosis, has been investigated (Fig. 2.1). Analytic expressions for velocity, flow rate, pressure drop, resistance to flow, wall shear stress...
and apparent viscosity, are obtained. It can be noticed that velocity is a function of pressure gradient $C$, fluid viscosity $\mu$, tube radii in uniform $R_0$ and constricted $R(z)$ regions, coordinates radial $r$ and axial $z$, stenosis length $L_0$, location $d$ and height $\delta s$ and, slip velocity $u_s$. This is in contrast to Poiseuille flow of blood (- behaving as a Newtonian fluid)$^{67}$, in which case, velocity depends on $C$, $R_0$, $\mu$ and $r$ only. In existing models$^{46,47,50,106}$, either slip or, no-slip and only one kind of growth or, unbecoming formation at lumen of an artery, are considered. Here, three gradual developments for an abnormal growth at inner wall and cases of slip or, zero-slip at vessel boundary, have been included.

Important observations of the present analysis are:

(a) The model includes Poiseuille flow of blood (- a Newtonian fluid) with slip or, zero-slip at tube wall$^{67}$ and Newtonian fluid flow with no-slip at stenotic wall, as its special cases.

(b) Axial velocity increases with an employment of slip at stenotic wall (Figs. 2.2-2.4). Velocity profiles are parabolic in uniform tubes whereas deviations from this trend are noticed in constricted arteries. It attains a maximum value at tube axis and a minimum value at wall and, it is greater in a uniform artery than that in a stenosed tube.
Further, velocity is maximum at initiation and termination of a stenotic region and, minimum at throat of a stenosis. Its magnitude at equal axial locations (on either equal portion) from the minimum constricted area of a stenosis, is observed to be the same.

As magnitude of slip increases, velocity attains a more higher value, in all three stenotic regions. Also, due to an increase in stenosis height, followed by a gradual reduction in tubular area, velocity goes on decreasing, for both slip or, no-slip cases. However, velocity in the cases of moderate and severe stenosis is appreciably lower than that, in case of a mild stenosis.

Flow rate is observed to be a constant, in case of a uniform tube but it varies in an obstructed artery (Fig. 2.5). Its magnitude is minimum at the throat and maximum at the two ends of stenosis.

Rate of flow increases with slip but decreases with an increase in stenosis height. As slip increases, flow rate increases accordingly. For a fixed value of slip, it is the greatest in mild stage and the smallest in severe stenosis.

Although, no appreciable change in pressure gradient could be noticed, due to a variation of slip, stenosis heights and axial locations in this non-uniform artery region (Fig. 2.6), pressure gradient is expected to be
reduced as a result of wall slip and, to attain a maximum value at the minimum cross-sectional area of stenosis

(h) Wall shear stress increases as stenosis develops from mild form to moderate case and then, moderate stage to a severe one (Fig 2.7). It is expected to be lower, as a result of slip at vessel wall. Its magnitude will be minimum at the two ends and maximum at the throat of stenosis.

(i) Pressure drop is reduced with slip and it attains the maximum value at the throat of stenosis. Behaviour of resistance to flow is similar to pressure drop. It will attain a maximum value at the throat of a stenosis and increase with stenosis height. Resistance will be lower due to slip and as slip increases, it will reduce further in magnitude.

(j) Apparent viscosity $\mu_a$ decreases with slip and reduces further as slip increases (Fig. 2.8).

(k) As stenosis height increases from mild form to severe stage, $\mu_a$ decreases form a greater value to a smaller one. However, its minimum magnitude is attained at the throat of a stenosis and the maximum value at its two ends i.e., at initiation and termination of stenosis.

(l) As in the stenotic region, $\mu_a$ decreases as tube radius decreases and it increases with vessel diameter, so
apparent viscosity exhibits Fahraeus-Lindqvist Effect (FLE) in this case (Fung).

From the present analysis of flow variables, viz., velocity function, flow rate, wall shear stress, pressure drop, resistance to flow and apparent viscosity it may be concluded that with slip damages to vessel wall, could be reduced. Also, resistance (or, impedance) to flow and wall shear stress will increase with an increase in stenosis size (or, height) and therefore, this theoretical analysis is in agreement with Young's mathematical model for analysing theoretically the effects of stenosis on flow characteristics of blood. In our investigation, on account of slip, velocity and flow rate will increase and, pressure drop and wall shear stress will decease which is in a good agreement with the observations made in the models of Chaturani and Biswas and, Misra and Kar. Thus, this theoretical analysis firmly establishes that a velocity slip at vessel boundary of an arterial segment, possessing a mild, moderate and severe kinds of stenosis, enhances the axial velocity and flow rate of blood in one hand and, bringing down the resistance to flow, wall shear stress and apparent viscosity, on the other hand. Further, it is noticed, pressure loss increases with an increase of channel constriction, which statement is in agreement with the analysis of Sanyal and Maji. It may also be noted that presence of stenosis increases the pressure drop, which is in agreement with the theoretical analysis of Misra and Kar and, experimental observation of Forrester and Young.
Thus, this reduction in pressure drop, shear stress, resistance to flow and apparent viscosity and, enhancement in velocity and rate of flow, due to an employment of slip, may be exploited for better functioning of diseased arterial system. Hence, one may look forward for such kind of drugs or tools which could produce slip and use them for treatment and cure of peripheral and arterial diseases. Also, it has been reported that hydrodynamic factors could play an important role in the development and progression of arterial stenosis (Young¹, Caro², Fry³) and also, in the basic understanding and cure of cardiovascular (cvs), arterial, haematological etc diseases. Many researchers (Fry⁷, Dintenfass⁸, Chien⁹ and Caro¹⁰) have indicated that the rheologic and fluid dynamic properties of blood and its flow, could play a vital role in the fundamental understanding of cardiovascular diseases. In view of the importance of blood rheology in the fundamental understanding of many cvs and arterial diseases and, their cure, it seems very important to use an appropriate boundary condition and the accurate flow parameters and, proper variables. In the analysis, velocity slip has been treated as constant and as a result, there could not be noticed any effect of slip on wall shear stress. Further, flow is considered one-dimensional and as a result, certain flow characteristics (e.g., back flow, separation and reattachment phenomena) as is reported, in the works of Morgan and Young³⁸ and, Chaturani and Biswas⁴⁶-⁴⁷, could not be indicated in the present analysis.
This theoretical analysis could be improved if the effects of the following factors viz., slips in both axial and rotational velocities for blood flow, flow as two-dimensional, visco-elastic nature of vessel wall, unsteady nature of flow, size and entrance effects, layered models with variable viscosity, wavy wall of stenosis, presence of inertia and body acceleration terms in the basic equation etc., are included.

The existing experimental work\textsuperscript{34} on blood flow through stenosed vessels, consider only pressure drop, it could be interesting to determine wall shear stress, slip at wall, velocity and flow rate etc. Such investigations may be helpful in determining the growth, development and progression of an arterial stenosis and that, in turn may be useful for a better understanding of stenotic and arterial diseases like, angina pectoris, myocardial infarction, sickle cell diseases, stroke, thrombosis etc.

In blood flow modelling through narrow tubes, experimental results\textsuperscript{61} have indicated the existence of a cell free Peripheral Plasma Layer (PPL) in the neighbourhood of vessel wall, in order to account for PPL, a two-layered model for blood flow in a stenosed artery may be more appropriate which has been considered in the next chapter.
Fig. 2.1 Flow Geometry of an Arterial Stenosis with Velocity Slip at Wall
Fig. 2.2 Variation of Velocity with Tube Radius for Different Stenosis Heights and Axial Distances (No-slip Case)
Fig. 2.3 Variation of Velocity with Tube Radius for Different Stenosis Heights and Axial Distances (with Slip)

WITH SLIP ( \( \bar{u}_y = 0.05 \))

<table>
<thead>
<tr>
<th>Stenosis</th>
<th>Nature</th>
</tr>
</thead>
<tbody>
<tr>
<td>25%</td>
<td>Mild</td>
</tr>
<tr>
<td>50%</td>
<td>Moderate</td>
</tr>
<tr>
<td>75%</td>
<td>Severe</td>
</tr>
</tbody>
</table>

\[ r/R \]

\[ \bar{u} \]
WITH SLIP ($\bar{u}_c = 0.10$)

Stenosis Nature

- 25% Mild
- 50% Moderate
- 75% Severe

Fig. 2.4 Variation of Velocity with Tube Radius for Different Stenosis Heights and Axial Distances (Slip case)
Stenosis type | With no-slip | With slip $\bar{u}_s = 0.0$ | With slip $\bar{u}_s = 0.05$ | With slip $\bar{u}_s = 0.10$
--- | --- | --- | --- | ---
25% | Mild | | | 
50% | Moderate | | | 
75% | Severe | | | 

Fig. 2.5 Variation of the Rate of Flow with Axial Distance for Different Heights of Stenosis and Slip Velocity
Table: Variation of Pressure Gradient with Axial Distance for Different Stenosis Heights and Slip Velocity

<table>
<thead>
<tr>
<th>Stenosis type</th>
<th>With no-slip $u_s = 0.0$</th>
<th>With slip $u_s = 0.05$</th>
<th>With slip $u_s = 0.10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25% Mild</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50% Moderate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>75% Severe</td>
<td></td>
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</tbody>
</table>

Fig. 2.6 Variation of Pressure Gradient with Axial Distance for Different Stenosis Heights and Slip Velocity
Fig. 2.7  Variation of Wall Shear Stress with Axial Distance for Different Stenosis Heights and Slip Velocity
Stenosis type | With no-slip | With slip | With slip
---|---|---|---
25% | Mild | $\bar{u}_s = 0.0$ | $\bar{u}_s = 0.05$ | $\bar{u}_s = 0.10$
50% | Moderate | | |
75% | Severe | | |

Fig. 2.8 Variation of Apparent Viscosity with Axial Distance for Different Stenosis Heights and Slip Velocity