7.1 INTRODUCTION

An abnormal growth along the innermost artery wall, reducing the lumen of an artery is called stenosis or, atherosclerosis (Boyd⁴, Guyton⁵). A stenosis may be found at one or, more locations of circulatory system. This unnatural formation at the vessel wall can cause serious circulatory disorders by reducing or, occluding the blood supply to different organs⁶, ⁴¹ and tissues, for instance, stenosis in arteries supplying blood to brain can bring about cerebral strokes, likewise its presence in coronary arteries can cause myocardial infarction leading to a heart failure⁵ etc. It is therefore, important due to its potential physiological significance, to study blood flow through an artery in the presence of an arterial stenosis.
In recent years, many researchers have investigated the flow characteristics of blood through an artery in presence of a stenosis. A serious study of this problem was made by Young\textsuperscript{33, 37} who put forward mathematical models to analyse theoretically the effects of stenosis on flow characteristics of blood. Young\textsuperscript{33} concluded that the resistance to flow i.e., impedance and wall shear stress increase with an increase in stenosis size. Forrester and Young\textsuperscript{34} extended the theory of Young\textsuperscript{33} to include the effects of flow separation and a mild constriction. There have been suggested many other works concerning steady and unsteady flow in stenosis (Fry\textsuperscript{3}, Lee and Fung\textsuperscript{35}, Young and Tsai\textsuperscript{6}, Morgan and Young\textsuperscript{38}, Deshpande and Giddens\textsuperscript{40}, Shukla \textit{et al.}\textsuperscript{43})

It may be noted that little attention has been given to non-Newtonian behaviour of blood in such studies. Although blood behaves as a Newtonian fluid in larger arteries under high rate of shear,\textsuperscript{32} but at low shear rates,\textsuperscript{31} it deviates from this Newtonian behaviour.\textsuperscript{87} The experimental studies on blood flow by several authors like Bugliarello and Sevilla\textsuperscript{61}, Goldsmith and Skalak\textsuperscript{30} and, Cokelet\textsuperscript{29} indicate that under certain flow conditions, blood flow may exhibit strong deviations from Newtonian flow behaviour\textsuperscript{87} e.g., non-parabolic velocity profiles for flow through tubes of small diameter, slip at wall, existence of peripheral plasma layer next to wall etc. Thus in rheological studies, it is customary to consider blood as a concentrated suspension of neutrally buoyant particles in a viscous
In view of above consideration, a non-Newtonian nature of blood has been considered in the present investigation.

One of the simplest constitutive equation describing the flow behaviour of a non-Newtonian fluid is due to power-law fluid which is usually characterised by two flow parameters viz., fluid consistency (k) and behaviour index (n) (Kapur et al.71). At low shear rate, an appreciable increase in apparent viscosity of blood is associated with the formation of rouleaux, a chain like aggregation of red cells and at a vanishing shear rate, blood behaves as an elastic solid (Cokelet et al.56) resulting to a further elevation of blood viscosity. Whereas at high shear rates, aggregates break up yielding to a decrease in blood viscosity. Such type of behaviour of blood corresponds to the shear-thickening or, dilatant Power-law behaviour for n>1 and to the shear-thinning or, Pseudoplastic Power-law fluid behaviour for n<1, i.e., apparent viscosity increases or, decreases with the rise in shear rate according as n>1 or, n<1. Also, blood flow (under high shear) along the large diameter tubes shows Newtonian fluid behaviour87 which case naturally corresponds to n=1. Cho and Kensey75 examined constitutive models including Power-law fluid model for modelling non-Newtonian viscosity of blood. Hussain et al.84 have proposed a Power-law fluid model for blood flow and examined the blood viscosity autoregulation factor in some diseases. Further, for blood flow through narrow tubes, experimental results (Bugliarello and Sevilla61) indicate the existence of a cell poor peripheral layer in the
neighbourhood of a tube wall Shukla et al. extended their Newtonian two-layered model for blood flow by considering a two-layered model in which fluids in both the regions are Power-law in character. Biswas and Mazumdar have considered a two-layered Power-law fluid model for blood flow in a uniform tube with the slip condition at fluids' interface. In this chapter, an attempt has been made to analyse the viscous blood flow characteristics in the presence of mild, moderate and severe arterial stenosis subject to a slip condition at the interface of fluids and in this two-layered model, core is taken as a Power-law fluid and peripheral layer is represented by a Newtonian fluid.

7.2 FORMULATION OF THE PROBLEM

A two-layered model for blood flow in a stenosed tube has been developed. The model consists of a core of red blood cell suspension in the middle layer and the peripheral plasma in the outer layer. It has been assumed that the core is represented by a Power-law fluid and the peripheral plasma layer (PPL) by a Newtonian fluid (Fig. 7.1). The flow is assumed to be steady and laminar and, fluids are incompressible. The geometry of the stenosis which is assumed to be symmetric, has been presented in Chapter 2 and Chapter 3.

The constitutive equation of Power-law fluid and governing equation of the fluid flow, have been included in Chapter 6. For ready reference, these equations are reproduced in the following
equation of motion (6.2.5) for one-dimensional flow through a
tube with axisymmetric stenosis, of Power-law fluid, for the core
region \(0 \leq r \leq R_1(z)\) is
\[
\frac{\partial}{\partial r} \left( r^k \left| \frac{\partial u_2}{\partial r} \right|^{n-1} \frac{\partial u_2}{\partial r} \right) = -C_r, \quad (n > 1)
\]  
\[7.2.1\]
and for the peripheral region \(R_1(z) \leq r \leq R(z)\), it is
\[
\frac{\partial}{\partial r} \left( r \mu_1 \frac{\partial u_1}{\partial r} \right) = -C_r,
\]
\[7.2.2\]
where \(u_2\) and \(u_1\) are axial velocities in the core and peripheral regions
respectively.

7.3 SOLUTION

For solving the above equations (7.2.1-7.2.2) we have used the
following boundary conditions:

I. Zero-slip condition at the stenotic wall:
\[
u_1 = 0 \quad \text{at} \quad r = R(z),
\]
\[7.3.1\]
II. Slip condition introduced at the interface:
\[
u_2 - \nu_1 = u_s \quad \text{at} \quad r = R_1(z),
\]
\[7.3.2\]
III. Velocity \(\nu_1\) must be finite at the centre line:
\[\nu_1 \text{ is finite at } r = 0,
\]
\[7.3.3\]
IV. Core velocity is finite at the tube axis:
\[
\frac{\partial \nu_2}{\partial r} = 0 \quad \text{at} \quad r = 0,
\]
\[7.3.4\]
where \(u_s\) is the apparent slip and \(R(z), R_1(z)\) are defined in Fig 7.1.
Integrating equation (7.2.2) twice and after employing the conditions (7.3.1, 7.3.3), expression for velocity in the outer layer becomes

\[ u_i(r) = \frac{C}{4\mu_1} \left( R^2(z) - r^2 \right) \quad \text{for} \quad R_1(z) \leq r \leq R(z) \]  \hspace{1cm} (7.3.5)

Integrating equation (7.2.1), we get

\[ \varepsilon k r \left| \frac{\partial u_2}{\partial r} \right|^n = A - \frac{Cr^2}{2} , \]  \hspace{1cm} (7.3.6)

where \( A \) is a constant which vanishes \((A=0)\), with the help of condition (7.3.4) and hence, the above equation results to

\[ \left| \frac{\partial u_2}{\partial r} \right|^n = -\frac{Cr}{2 \varepsilon k} , \]  \hspace{1cm} (7.3.7)

where \( \varepsilon \left( \frac{\partial u_2}{\partial r} \right) \left| \frac{\partial u_2}{\partial r} \right| \) is a sign coefficient which takes the value -1 or, +1, in accordance with the sign of the derivative \( \frac{\partial u_2}{\partial r} \).

In the above equation, absolute value sign may now be dropped because \( \frac{\partial u_2}{\partial r} \) is always negative in case of tube flow \((\varepsilon = -1)\) and with this consideration, equation (7.3.7) leads to

\[ - \frac{\partial u_2}{\partial r} = \left( \frac{Cr}{2k} \right)^{1/n} . \]  \hspace{1cm} (7.3.8)

A second integration yields from the above equation

\[ u_2 = B - \left( \frac{n}{n+1} \right) \left( \frac{Cr^{n+1}}{2k} \right)^{1/n} , \]  \hspace{1cm} (7.3.9)
where B is a constant of integration.

Applying the condition (7.3.2) in equation (7.3.9), the velocity function in the core region becomes

\[ u_2(r) = u_s + \frac{CR^2(z)}{4\mu_1} \left( \frac{R(z)}{R_1(z)} \right)^2 \left( \frac{n}{n+1} \right) \left( \frac{CR_1^{n+1}(z)}{2k} \right)^{1/n} \left\{ 1 - \left( \frac{r}{R_1(z)} \right)^{(n+1)/n} \right\} \]

\[ 0 \leq r \leq R_1(z) \quad (7.3.10) \]

The rate of volume flow is obtained from the formula

\[ Q = \int_{r=0}^{R(z)} 2\pi r u_r \, dr = 2\pi \left( \int_{r=0}^{R(z)} r u_2(r) \, dr + \int_{r=R_1(z)}^{R(z)} r u_1(r) \, dr \right) \quad (7.3.11) \]

which, after using the equations (7.3.5) and (7.3.10), results to

\[ Q = \pi R_1^2(z) u_s + \frac{\pi C}{8\mu_1} \left[ R^4(z) - R_1^4(z) \right] + \left( \frac{\pi n}{3n+1} \right) \left( \frac{CR_1^{n+1}(z)}{2k} \right)^{1/n} \quad (7.3.12) \]

The pressure gradient can be put in the form (from equation (7.3.12)) for the peripheral layer and core region,

\[ \frac{dp}{dz} = \left( -\frac{8\mu_1}{\pi} \right) \frac{Q_1}{[R^4(z) - R_1^4(z)]} \quad (7.3.13) \]

and

\[ \frac{dp}{dz} = \left( -\frac{2k}{R_1^{3n+1}(z)} \right) \left[ 3n+1 \right] \left( Q_1 \pi^{n+1} - R_1^2(z) u_s \right)^n \quad (7.3.14) \]

where

\[ Q_1 = \left( \frac{\pi C}{8\mu_1} \right) \left[ R^4(z) - R_1^4(z) \right] \quad (7.3.15) \]

and

\[ Q_2 = \pi R_1^2(z) u_s + \pi n/(3n+1) \left[ CR_1^{n+1}(z)/(2k) \right]^{1/n} \quad (7.3.16) \]
Apparent viscosity $\mu_a$ can be obtained from

$$\mu_a = \frac{\pi CR^4(z)}{8Q} = \left[ \frac{8\mu_s}{CR^2(z)} \left( \frac{R(z)}{R(z)} \right)^2 + \frac{1}{\mu_1} \left( 1 - \left( \frac{R(z)}{R(z)} \right)^4 \right) \right]^{1/4} \left( \frac{R(z)}{R(z)} \right)^4 \left( \frac{8n}{3n+1} \right) \sqrt{\left( 2k \right) \left( CR(z) \right)^{n-1}}. \quad (7.3.17)$$

Integrating equations (7.3.13-7.3.14) between the limits

$$p = p_i \text{ at } z = 0$$

and

$$p = p_0 \text{ at } z = L, \quad (7.3.18)$$

($L$ is the length of the tube)

We can obtain pressure drop $(p_i - p_0)$ for peripheral $(P_1)$ and core regions $(P_2)$ in the forms

$$P_1 = \left( + \frac{8\mu_1}{\pi} \right) \int_{z=0}^{L} Q \left[ R^4(z) - R_i^4(z) \right]^{-1} dz, \quad (7.3.19)$$

and

$$P_2 = (2k) \int_{z=0}^{L} R_i^{(3n+1)}(z) \left[ \left( \frac{3n+1}{n} \right) Q_s \pi^{-1} - R_i^2(z) \mu_s \right]^n dz. \quad (7.3.20)$$

The resistance to flow $(\lambda)$ which is defined by

$$\lambda = \frac{(p_i - p_0)}{Q}, \quad (7.3.21)$$

as a result of equations (7.3.20-7.3.21) and stenotic geometry (Chapter 3), reduces to

$$\lambda = \left[ \frac{8\mu_1}{\pi} Q_4 \left( R_0^4(1 - \alpha^4) \right)^{-1} + 2k(\alpha R_0)^{(3n+1)} \left[ \left( \frac{3n+1}{n} \right) Q_s \pi^{-1} - \alpha^2 R_0^2 \mu_s \right]^n \right]^{1/2}.$$
\[
\left( \frac{L - L_0}{Q_3} \right) + \int_{z=d}^{d+L_0} Q^{-1} \left[ \frac{8 \mu_1}{\pi} Q_1 \left( R_4(z) - R_1^4(z) \right) \right] \, dz + 2k R_1^{(3n+1)}(z) \left( \frac{3n+1}{n} \right) \left( Q_2 \pi^{-1} - R_1^2(z) u_s \right)^n \, dz, \tag{7.3.22}
\]

where,

\[
Q_3 = Q_{R_1(z)=R(z)=R_0, \quad \text{and} \quad Q_5 = Q_{R_1(z)=R_0}.
\]

A second representation of flow variables:

Velocity function

\[
\bar{u}_t = \bar{R}^2 \left[ 1 - \left( \frac{r}{R} \right)^2 \right], \quad \frac{R_1}{R} \leq \frac{r}{R} \leq 1, \tag{7.3.23}
\]

and

\[
\bar{u}_2 = \bar{u}_s + \bar{R}^2 \left[ 1 - \left( \frac{R_0}{R} \right)^2 \right] + \left( \frac{n}{n+1} \right) \left( \frac{\mu_1}{k R_1^{n+1}} \right)^{1/m} \left[ 1 - \left( \frac{r}{R} \right)^{n+1} \right], \tag{7.3.24}
\]

Flow rate

\[
\bar{Q} = \bar{R}_1^2 \bar{u}_s + \frac{1}{4} \left( \bar{R}^4 - \bar{R}_1^4 \right) + \left( \frac{n}{3n+1} \right) \left( \frac{\mu_1}{k} \bar{R}_1^{3n+1} \right)^{1/n}. \tag{7.3.25}
\]

Pressure gradient term

\[
\left( \frac{dp}{dz} \right) = \bar{Q}_1 \left[ R^4 - R_1^4 \right]^{-1}. \tag{7.3.26}
\]
and

$$\left( \frac{dP}{dz} \right) = \left( \frac{k}{\mu_1} \right) \left( R^{3n+1} \right)^{-1} \left[ \left( \frac{3n+1}{n} \right) \left( Q_2 - R^2 \bar{u}_b \right) \right]^n. \quad (7.3.27)$$

**Apparent viscosity**

$$\bar{\mu}_a = 2 R^2 \left[ 8 \bar{u}_b \left( \frac{R}{R} \right)^2 + 2 R^2 \left( \frac{R}{R} \right)^4 \right] +$$

$$\left( \frac{R}{R} \right)^4 \left( \frac{8n}{3n+1} \right) \left[ \left( \frac{\mu_1}{k} \right) \left( R^n \right)^{n+1} \right]^{-1}. \quad (7.3.28)$$

### 7.4 RESULTS AND DISCUSSIONS

In this two-layered model, blood flow, within an obstructed region, arising due to unbecoming growths at inner artery wall, is considered (Fig. 7.1). It comprises of a core region and a peripheral layer which are duly accounted by Power-law fluid and Newtonian fluid, respectively. Also, a slip condition at interface and usual no-slip at stenotic wall, are employed. Further, in Power-law fluid (-characterised by an index n), three cases may arise viz., (i) pseudoplastic or, shear-thinning behaviour, for n<1, (ii) dilatant or, shear-thickening nature, for n>1 and (iii) Newtonian or, linear behaviour for n = 1 (Fig. 7.2). Analytic expressions for velocity, flow rate, shear stresses at wall and interface, pressure drop and pressure
gradient, resistance to flow and apparent viscosity are obtained and their second form is also put forward in a dimensionless manner. Velocity (in eqs. 7.3.5 and 7.3.10) depends on shear viscosities $\mu_1, k$, coordinates $r$ (radial) and $z$ (axial), pressure gradient $C$, fluid parameter $n$, uniform $R_0$ and constricted $R(z)$, $R_1(z)$ tube radii, slip velocity $u_s$ and dimensions $L_0$, $d$, $\delta_s$ associated with stenosis. The geometry of an arterial stenosis and other quantitative measures, are borrowed from previous chapters 3 and 6. If $R(z) = R_0 = R_1(z)$, $n = 1$ and $\bar{u}_s \geq 0$, the present analysis reduces to poiseuille flow of blood (- a Newtonian fluid) with slip or, zero slip at tube wall. For $R(z) = R_1(z)$ and $n = 1$ and, $R(z) \neq R_0 \neq R_1(z)$ and $n = 1$, it leads to one and two-layered Newtonian models of an arterial stenosis with slip at wall and interface respectively. In case $R(z) = R_1(z)$ and $\bar{u}_s > 0$, $R(z) \neq R_1(z)$ and $\bar{u}_s = 0$, it leads to one-layered model with wall slip and a two-fluid model with no-slip at interface, of Power-law fluid flow, in a stenosed vessel.

### 7.4.1 Velocity Profiles

Velocity is computed from eqs. (7.3.23-7.3.24) and its variations with axial locations $z$ for slip at interface and zero-slip at boundary, different stages of stenosis formation and fluid behaviour index $n$ ($<, =, >1$), are presented against radial coordinate $r/R(z)$, in Figs. (7.3-7.5). It can be seen from the figures that
(i) Velocity in all cases of stenosis sizes and for all values of n, increases with slip used at interface,

(ii) As stenosis height increases from mild case to severe stage, velocity decreases to lower values.

(iii) For variations of fluid property n, it shows distinct behaviour. For n = 1, velocity profiles are parabolic at either end of stenosis and deviations from this trend, are indicated at or, near the throat of stenosis. For n<1, profiles deviate from parabolic trend i.e., they are elongated. When n>1, velocity shows both distinct bluntness and partial plugged flow. However, blunted or, flat profiles are prominent in severe stage of stenosis.

(iv) Velocity is maximum at the axis, lower at interface and minimum at tube wall.

(v) Its magnitude is greater in uniform tube than that in obstructed vessels.

(vi) Velocity obtained with slip is greater than that with zero-slip at interface and, it increases with increasing slip

(vii) As axial coordinate z increases from either end of stenosis (i.e., z = d) to its throat (i.e., z = d + L₀/2), velocity decreases to a minimum.

(viii) Among all stenosis sizes, index n and slip values, velocity attains its greatest magnitude, in any form of stenosis (with
highest slip and for \( n = 0.75 < 1 \) and its lowest value, in severe stage of stenosis (with zero-slip and for \( n = 1.25 > 1 \)).

(ix) Between the layered models, it is noticed that axial velocity is greater than or, smaller than, the corresponding magnitude obtained, in one-fluid Power-law stenosed model (in Chapter 6) for the respective cases of fluid behaviour index \( n \geq 1 \) and \( n < 1 \).

7.4.2 Flow Rate

Variation of flow rate (in eq. 7.3.25) with slip or, no-slip cases, stenosis heights and fluid index \( n \) (\(<\), \(=\), \(>1\)) has been presented against axial coordinate \( z \), in Fig 7.6. The observations from the profile, may include the following

(i) Flow rate, for any stenosis growth and any \( n \), is greater with slip than that, with zero-slip at interface

(ii) It decreases from a higher value to lower one, as stenosis size increases from mild form to severe stage of stenosis, but, it increases with increasing slip.

(iii) As fluid index \( n \) increases from \( n = 0.75 \) (\(<1\)) to \( n = 1.25 \) (\(>1\)), values of flow rate are found to decrease for both slip and no-slip cases

(iv) It is the greatest at either end of stenosis and the smallest at its throat.
(v) Its magnitude is constant in uniform tubes but flow rate changes with axial locations, in stenosed vessels.

(vi) Flow rate is higher in tubes, possessing a mild growth at wall than in vessels, with severe formation at wall.

(vii) Its maximum and minimum magnitudes are indicated at the cases of mild form (for \( n = 0.75 < 1 \) and highest slip \( \bar{u}_s = 0.10 \)) and of severe stage in stenosis at artery wall (for \( n = 1.25 > 1 \) and zero-slip \( \bar{u}_s = 0.00 \)) respectively.

(viii) Between the two models, it is seen that flow rate is greater than or, less than, the corresponding values in one-fluid Power-law model (in Chapter 6), in respective cases of fluid property \( n \geq 1 \) or, \( n < 1 \).

7.4.3 Wall Shear Stress

Behaviour of wall shear stress is similar to one-layered model (in Chapter 6) and it can be noticed that

(i) shear stress at wall, decreases with increasing stenosis height. (ii) As \( n \) increases from \( n < 1 \) to \( n > 1 \), it decreases for all cases of stenosis sizes (iii) As \( z \) increases from \( z = d \) to \( d + L_0/2 \), it decreases from a greater value at either end of stenosis to a smaller one, at its throat (iv) Its magnitude is expected to be reduced with slip at interface.
7.4.4 Pressure Gradient

It is obtained from eq (7.8 26) and its variation with slip or, no-slip, stenosis cases and fluid property \( n \), is shown in Fig 7.7 the following are indicated in the profiles

(i) Pressure gradient increases from a lower magnitude at either end to a maximum at the throat of stenosis

(ii) As stenosis size increases from mild form to severe stage, it increases

(iii) Also it increases with a slip at interface.

(iv) However, it decreases as \( n \) rises from a lower magnitude \((n<1)\) to a higher one \((n>1)\)

7.4.5 Pressure Drop and Resistance to Flow

Variation of pressure drop (from eqs. 7 3.19-7.3 20) with fluid index \( n \) \((<, =, >1)\), stenosis sizes and slip or, no-slip cases, indicates that:

(i) Pressure drop decreases with slip at interface but increases with both fluid property \( n \) and stenosis size, from mild form to severe growth

(ii) It is constant in uniform arteries but changes with axial locations in constricted regions of an artery.

(iii) It is seen to be smaller with slip than that with zero-slip.

(iv) As index \( n \) rises from \( n = 0.75 \ (<1) \) to \( n = 1.25 \ (>1) \), it increases to greater magnitudes
(v) It reduces to a minimum at or, near the minimum area of the constricted region.

Variation of resistance to flow (in eq 7.3.22) with stenosis sizes, fluid index $n$ and slip or, no-slip cases, shows that

(i) resistance reaches a maximum at the throat of stenosis and acquires a minimum at its either end

(ii) It decreases with slip at interface, but, increases with increases in both fluid property $n$ and stenosis height

(iii) The magnitudes of resistance, obtained with slip, is lower than that, with zero-slip at stenotic wall.

7.4.6 Apparent Viscosity

Apparent viscosity $\tilde{\mu}$ is obtained from eq (7.3 23) and its variation with slip or, no-slip cases, stenosis sizes and fluid index $n (<, =, >1)$ is drawn against axial distance $z$, in Fig 7.8. The following can be observed from the figure:

(i) As expected, $\tilde{\mu}$ obtained with a slip condition at interface, is found to be smaller than that, with zero-slip, for all cases of $n$ and in all stenosis sizes.

(ii) For any stenosis height and slip or, no-slip cases, it is clearly noticed that as fluid property $n$ increases from $n = 0.75 (<1)$ to $n = 1.25 (>1)$, $\tilde{\mu}$ increases from a lower magnitude to a higher value.
(iii) Apparent viscosity increases with increasing slip at interface for any stenosis size and variation of index n.

(iv) For any magnitude of n and for either slip or, no-slip case, \( \mu_a \) reduces from a greater value, obtained at the initiation or, termination (i.e., at either mouth) of stenosis to a smaller one, at the minimum cross-sectional tube area (i.e., at the throat).

(v) As axial coordinate z increases from either end (\( z = d \)) to the middle of stenosis (\( z = d + L_0/2 \)), \( \mu_a \) reduces from greater values to smaller ones.

(vi) At the throat of a stenosis, the highest and the lowest magnitudes for \( \mu_a \) are observed at mild type of stenosis formation (with zero-slip and for \( n=1.25>1 \)) and at severe stage (with maximum slip \( \mu_a = 0.10 \) and for \( n=0.75<1 \)) of an arterial stenosis.

(vii) As \( \mu_a \) decreases (considering slip or, no-slip case and variation of index n), as bore of the vessel (i.e., tube radius) decreases from mild formation to severe growth, at the lumen of an artery wall, therefore, apparent viscosity exhibits Fahraeus-Lindqvist Effect (FLE) in this case. It is worthwhile to report that, the behaviour of \( \mu_a \) in two-layered Newtonian model (in Chapter 3) and one-fluid Power-law model (in Chapter 6) is identical with the present case i.e.,
shows FLE in all three, one and two-layered stenosed models.

(viii) Between the Power-law models, it is clearly seen that magnitudes of \( \bar{\mu}_a \) in two-layered analysis are lower than or, higher than the corresponding values in one-fluid model (in Chapter 6), in the respective cases of fluid index \( n \geq 1 \) or, \( n<1 \).

7.5 CONCLUSION

An analysis of two-layered blood flow, in a stenosed artery, under the effects of velocity slip at interface and zero-slip at boundary, is brought forward in this chapter (Fig 7.1). The model basically consists of a red cell suspension (- accounted by Power-law fluid) and a cell free layer (- dealt by Newtonian fluid), existing very near to the wall of mild, moderate and severe stenosis. In core region, Power-law behaviour is characterised by fluid parameter \( n \) (Fig. 7.2). Expressions for velocity, flow rate, stresses, pressure drop, resistance to flow and apparent viscosity, are obtained analytically. It may be noticed that velocity is a function of pressure gradient \( C \), shear viscosities \( \mu_1 \) and \( K \), coordinates radially \( r \) and axially \( z \), tube radii in normal \( R_0 \) and constricted \( R(z) \) and \( R_1(z) \) arteries, slip \( u_s \) and, dimensions \( L_0 \), \( d \) and \( \delta_s \) of stenosis. The following observations may be indicated from this analysis.
(a) In core, there happens three flow situations, due to Power-law behaviour index $n (<, =, >1)$ viz., (i) in case $n<1$, power-law fluid shows pseudo-plastic or, shear-thinning trend, (ii) for $n = 1$, it projects Newtonian nature or, linear relationship and when $n>1$, it exhibits dilatant or, shear-thickening profile.

(b) It may be worthwhile in mentioning that the present analysis includes Poiseuille flow models with velocity slip or, zero-slip at wall\(^{87}\), one and two-layered Newtonian fluid flow through a stenosed vessel, with slip at wall and at interface (Chapters 2-3), one-layered Power-law model of an arterial stenosis with wall slip (in Chapter 6) and, two-fluid stenosed model of Power-law fluid, with zero slip at interface, as its special cases.

(c) In all cases of stenosis sizes and for all values of fluid property $n$, velocity enhances with an axial slip, employed at interface. Its magnitude is higher with slip than with zero-slip (Figs. 7.3-7.5)

(d) For variations of index $n$, it exhibits distinct behaviour. As for $n<1$, velocity profiles show deviations from parabolic behaviour i.e., an elongated trend. At $n = 1$, profiles are parabolic at either end of stenosis and a non-parabolic trend at or, near the throat of the constricted channel. When $n>1$,
velocity indicates both distinct bluntness and partial plugged flow.

(e) With a rise in stenosis height, from mild form to severe growth, velocity decreases to smaller magnitudes.

(f) In the stenotic region, velocity is maximum at either end of stenosis and minimum at its throat.

(g) Also, velocity acquires the greatest value at the tube axis, a smaller magnitude at interface and the lowest value at the vessel wall.

(h) Magnitudes of velocity are seen to be lower than or, greater than the corresponding values in one-fluid power-law model (in Chapter 6), in respective cases of $n < 1$ or, $n \geq 1$.

(i) Flow rate increases with slip but decreases with both increasing stenosis size (from mild case to severe stage) and fluid property $n$ (from $n = 0.75 < 1$ to $n = 1.25 > 1$) (Fig 7.6).

(j) It is the highest at either end of any stenosis and lowest at its throat and its maximum and minimum values are noticed in tubes with mild case of growth (for greatest slip and smaller $n < 1$) and with severe stage of growth (for zero-slip and greater $n > 1$) respectively.

(k) Flow rate in this two-layered analysis, is found to be higher than or, lower than the corresponding magnitude in one-
layered. Power-law model (in Chapter 6) in respective cases of fluid index $n \geq 1$ or, $n < 1$.

(I) Wall shear stress decreases with an increase in both stenosis height and behaviour index $n$ but its magnitude with slip, will be smaller than that with no-slip.

(m) Pressure gradient increases with slip, stenosis size and fluid property. It reduces to a maximum, at the throat of stenosis (Fig. 7.7).

(n) Pressure drop decreases with slip at interface but increases with stenosis size. Resistance to flow reduces to a maximum at the middle of stenosis and acquires a minimum value, at the initiation or, termination of a constricted channel.

(o) It increases with fluid property $n$ and stenosis height (from mild form to severe stage) but decreases with increasing slip.

(p) Apparent viscosity, obtained with slip at interface, is lower than that with zero-slip. It increases with fluid property $n$ (from $n = 0.75 < 1$ to $n = 1.25 > 1$).

(q) As axial coordinate increases from either end of stenosis to its throat, it reduces to lower values.

(r) At the middle of stenosis, the greatest and the smallest viscosity, are exhibited in a stenosed artery, with mild
formation (for zero-slip and at \( n = 1.25 > 1 \)) and with severe
growth (for slip \( \bar{u}_b = 0.10 \)) and at \( n = 0.75 < 1 \) respectively.

(s) As apparent viscosity decreases, as bore of the stenotic
vessel (or, tube radius) decreases from mild growth to
severe form at an artery wall, therefore it shows Fahraeus-
Lindqvist Effect (FLE), in this two-layered analysis.

(t) Between the power-law layered models, apparent viscosity
in this case, is noticed to be lower than or, greater than the
_corresponding magnitude at one-fluid Power-law model (in
Chapter 6), in the respective cases of fluid behaviour index
\( n \geq 1 \) or, \( n < 1 \).

In this theoretical modelling, an attempt is made to analyse the
effects of velocity slip and Power-fluid behaviour index on the two-
layered blood flow through a thin artery, in the presence of mild,
moderate and severe stenosis. In the analysis, velocity shows
parabolic and deviations from parabolic profiles, elongated type and,
blunted as well as partially plugged profiles, in accordance with the
fluid property \( n, =, <, >1 \) Further, as a result of employing a slip at
interface and considering a cell poor peripheral plasma layer (ppl), it
has been observed that velocity and flow rate in this case, are greater
than or, less than, the corresponding variables in one-fluid Power-law
model of an arterial stenosis, in the respective cases of fluid property
\( n \geq 1 \) or, \( n < 1 \). Moreover, apparent viscosity exhibits Fahraeus-
Lindquist Effect (FLE) in the present case. Thus, some anomalous
characters of blood flow viz., blunted and plugged profiles, non-parabolic and elongated trend in velocity and FLE could be explained with the help of the present investigation. It is worthwhile to report that this two-layered analysis can take account of the following viz., core region, ppl, slip, flat or, blunted profiles, shear thinning and shear-thickening non-Newtonian fluid behaviour, partially plugged flow and FLE.

In this mathematical model, it is firmly established that the velocity slip at interface, in two-layered fluid flow of a stenosed vessel, enhances the axial velocity and flow rate but decreases the wall shear stress, pressure drop, resistance to flow and apparent viscosity of blood. Further, as a result of PPL and fluid property n (<, =, >1) this analysis has an advantage over the one-fluid approach. This enhancement and reduction in flow variables, could be utilised (i) for the better functioning of the stenosed arteries, (ii) in reducing the stenosis size or, height and (iii) in bringing back, the regular or, usual blood supply in the body.

In the present development of the mathematical model for blood flow through a stenosed segment of an artery, it can be easily visualised that, with an employment of velocity slip at interface of two-fluid modelling, damages to vessel wall could be reduced. Further, it is worthwhile to mention here that there is an appreciable influence of the slip velocity on the actual pressure drop, velocity, flow rate, resistance to flow and apparent viscosity. It has been noticed in the
analysis that wall shear stress and resistance (or, impedance) to
blood flow will enhance with a rise in stenosis growth from mild case
to severe form at artery wall, which is in agreement to the analysis of
Young\textsuperscript{33} and, Sinha and Singh\textsuperscript{111}. Also, velocity and flow rate will
increase and, pressure gradient and wall shear stress will decrease
as a consequence of velocity slip condition, employed in this
modelling which agrees with the results of Chaturani and Biswas\textsuperscript{46-47}
and, Misra and Kar\textsuperscript{50}. Further, pressure loss in this layered
modelling increases with an increase in channel constriction which is
similar to the observation, as pointed out by the authors Sanyal and
Maji\textsuperscript{54}. Again, the observation that presence of stenosis and its
gradual development, increase the pressure drop, is similar to the
theoretical analysis of Misra and Kar\textsuperscript{50} and experimental results of
Forrester and Young.\textsuperscript{34} However the said pressure drop, could be
reduced appreciably with the slip at interface. Like the two-layered
non-Newtonian fluid flow in a stenosed tube, carried out by Prahlad
and Schultz,\textsuperscript{106} it has been clearly observed that (i) axial velocity
attains a maximum value at the axis, decreases across the core
region and reduces to a minimum at the tube wall, (ii) resistance to
flow in a constricted tube will increase with increasing stenosis height
and (iii) the model can be used to explain the anomalous behaviour
of blood flow viz., blunted velocity profiles. In addition to this, another
anomalous tendency in blood flow viz., Fahraeus-Lindqvist Effect
(FLE), is indicated in this analysis. Moreover, like the models brought
forward by Shukla et al.\cite{43,81} and, Chaturani and Samy\cite{115} it has been clearly noticed that the rheological parameters (Power-law index $n$ and shear dependant fluid viscosity $k$) and ppl as well as its viscosity, have a great influence on the flow variables and, that wall shear stress reduces, with a rise in fluid property $n$ ($<, =, >1$). Furthermore, in contrary to the two-layered model for Casson fluid, examined by Chaturani et al.\cite{117} that apparent viscosity decreases as tube radius increases, in this two-layered Power-law model on blood flow, the behaviour of apparent viscosity is Fahraeus-Lindqvist Effect (FLE), instead of its Inverse i.e., IFLE, as predicted in Casson's model.\cite{117}

In this chapter, an effort is undertaken to study the effects of the non-Newtonian nature of blood, in the presence of a velocity slip and a ppl, on the flow through an artery with mild, moderate and severe stenosis. The influence of Power-law fluid behaviour index $n$, slip and stenosis sizes, on the flow variables has been clearly noticed. Dintenfass\cite{120} has shown that blood rheology could play a vital role in the diagnosis, treatment and fundamental understanding of many cardiovascular (cvs) diseases and arterial disorders. Also, there is a strong evidence that hydrodynamic factors, such as wall shear stress and pressure distribution, may play an important role in the development and progression of arterial stenosis (Texon\cite{121}). Further, Young\cite{1}, Caro\cite{2}, Dintenfass, Chien\cite{9}, Biswas\cite{29}, Shukla et al.\cite{43,81} Sinha and Singh\cite{111}, Prahlad and Schultz\cite{106} and others\cite{115-116}, have reported that fluid dynamics of an arterial stenosis plays a very important role
in the prevention and understanding of many cardiovascular, cerebrovascular and arterial diseases (viz., atherosclerosis, myocardial infarction, stroke, coronary thrombosis, anaemia, thalassaemia, polycythemia, Hbss etc.) Therefore, theoretical as well as experimental investigations in stenosis problems, are of physiological and clinical importance. In view of the above, a two-layered stenosed model on blood flow, has been proposed in the present case. In want of appropriate measures of slip, ppl, variation of fluid index n, shear viscosities etc., we have used their crude magnitudes. However with the inclusion of correct measures of flow parameters, visco-elastic wall of arterial segment, size, shape and flexibility of suspended particles, thickness of ppl, unsteady flow and two-dimensional motion etc. in the present modelling, a better and a realistic blood flow model can be suggested. In proposing such an improved model on blood flow, it would be appropriate to conduct experiments in these directions to determine correct measures of flow parameters and flow variables, which in turn could lead to get a deeper insight into the problems, arising due to an arterial stenosis.
BLOOD – A POWER-LAW FLUID

Viscosity $k$

A NEWTONIAN FLUID

Fig. 7.1 Flow Geometry of an Arterial Stenosis in a Two-Fluid Model

Fig. 7.2 Shear Stress Vs. Rate of Strain Relationship for Power-Law Fluids ($n<, =, >1$)
Figure 74 Variation of Velocity with Tube Ratios (for Slip and No-Slip)
\text{\textit{Variation of Velocity with Tube Radius (for Slip and No-Slip)}}
FIG. 7 (a-c): VARIATION OF FLOW RATE WITH AXIAL DISTANCE