Chapter 2

Analytical Bases and Empirical Frameworks

2.1 Introduction

This chapter deals with the analytical and empirical framework for convergence to provide a systematic analysis for the proposed objectives of the study. It is based on the analytical framework of neoclassical growth theory to explain the determinants of inter-regional (state) economic growth and convergence in Indian federal context. It identifies the basic determinants of economic growth and establishes relationship between the economic variables over time, and explains the differences in inter-state growth rates and levels towards the steady state. This framework provides for an empirical analysis to predict convergence or divergence in growth rates and explains the differences in standards of living across Indian states. Using this empirical framework of convergence analysis, impacts of net factor incomes and federal fiscal transfers are examined to explain the variations in inter-state growth rates and standard of living in Indian federalism. Therefore, testable hypotheses such as absolute and conditional, and sigma convergence (divergence) are examined in per capita real income, per capita real consumption and per capita real state disposable income across the states. Using econometric techniques, absolute and conditional convergence are estimated to find out the speeds of convergence or divergence. Thus, this analytical bases and empirical frameworks will help understand the differences in the inter-state growth rates and levels as well as the trends in inter-state inequalities in India.

2.2 Estimation of determinants of steady state per capita income

Considering a set of states of Indian federation, \( i = 1, 2, \ldots, N \), over a number of years, \( t = 1, 2, \ldots, T \), the differences in growth rates and levels are explained within the analytical framework of Solow-Swan (1956) growth model. It assumes a Cobb-Douglas production function with exogenous labour augmenting technical progress. With two inputs, capital and labour, output of Indian state \( i \) at time \( t \) is

\[
Y_{it} = K_{it}^{\alpha} (A_{it}L_{it})^{1-\alpha} \quad 0 < \alpha < 1
\]  

(2.1)

where
Y_{it} is output,

A_{it} is the level of technology,

K_{it} is the stock of capital, and

L_{it} is the quantity of labour of Indian state \( i \) at time \( t \).

The coefficients \( \alpha \) and \( (1- \alpha) \) represent the elasticities of output with respect to capital and labour, respectively. L and A are assumed to grow exogenously at the rates \( n \) and \( g \):

\[
L_{it} = L_{i0} e^{nt} \quad (2.2)
\]

\[
A_{it} = A_{i0} e^{gt} \quad (2.3)
\]

The number of effective units of labour, \( A_{it}L_{it} \) grows at the rate \( n_i + g_i \). Assuming that \( s_i \) is the constant fraction of output that is saved and invested, and letting \( \hat{k}_i = K_{it}/A_{it}L_{it}, \hat{y}_i = Y_{it}/A_{it}L_{it}, \hat{c}_i = C_{it}/A_{it}L_{it} \) that represent the capital, output and consumption per efficiency units of labour, respectively, the evolution of \( \hat{k}_i \) over time is given by

\[
\dot{\hat{k}}_i = s_i \hat{y}_i - (n_i + g_i + \delta) \hat{k}_i
\]

It is assumed that production function parameter, \( \alpha \), and depreciation of capital, \( \delta \) are constant across Indian states but, all other parameters allow to differ. Although the variations in parameters across \( i \) play central role in empirical analysis, for the time being and clarity of exposition, the above equation is specified as follows for a single Indian state after dropping the subscript \( i \).

\[
\dot{\hat{k}} = s \hat{k}_i^{\alpha} - (n + g + \delta) \hat{k}_i \quad (2.4)
\]

where dot (.) represents the changes in a variable with respect to time. Given the preceding assumptions and definitions, MRW (1992) show that \( \hat{k}_i, \hat{y}_i \) and \( \hat{c}_i \) converge to the steady state values of \( \hat{k}^*, \hat{y}^* \), and \( \hat{c}^* \) as defined by

\[
\hat{k}^* = \left[ \frac{s}{n + g + \delta} \right]^{1/(1-\alpha)} \quad (2.5)
\]
\[ y^* = \left[ \frac{s}{n + g + \delta} \right]^{\frac{\alpha}{1-\alpha}} \] (2.6)

and

\[ c^* = y^* - (n + g + \delta) k^* \] (2.7)

The steady state capital-labour, and output-labour ratios are directly related to the rate of saving and inversely related to the rate of population growth (or effective depreciation rate). The central predictions of Solow-Swan growth model concern the impact of saving, and population growth on real income.

Substituting equation (2.5) into the production function in equation (2.1), and taking natural logarithm of the resultant equation, the following expression for the steady state value of output per worker \( (\nu = Y/L) \) is obtained.

\[ \ln y_t = \ln (A_0) + gt + \frac{\alpha}{1-\alpha} \ln(s) - \frac{\alpha}{1-\alpha} \ln(n + g + \delta) \] (2.8)

where \( A_0 \) is the initial level of technology. \( A_0 \) also represents the resource endowments, climatic conditions, institutions and so on. Therefore, it allows for differences across states. Thus, MRW assume that

\[ \ln (A_0) = a + \varepsilon, \]

where \( a \) is a constant and \( \varepsilon \) is a country-specific shock. Substituting this into the equation (2.8) and subsuming \( gt \) into the constant term \( a \), the empirical specification of natural log of per capita real income, \( y \) of Indian state \( i \) at a given time \( t \) is given by

\[ \ln y_t = a + \frac{\alpha}{1-\alpha} \ln(s_t) - \frac{\alpha}{1-\alpha} \ln(n_t + g + \delta) + \varepsilon_t \] (2.9)

Assuming \( (g + \delta) \) is constant, the equation (2.9) predicts that natural log level of output per worker is positively associated with the natural log difference between the gross investment rate, \( s \), and the effective depreciation rate \( (n + g + \delta) \). This equation (2.9) can be estimated by using econometric techniques with \( \varepsilon_t \) as random disturbance terms in the model. Assuming that factors are paid, according to their marginal products and the states (regions) are currently in their steady states, MRW show that it predicts not only the signs but also the magnitudes of the coefficients on saving rate and population growth.
MRW assume $e$ to be independent of the explanatory variables, $s$ and $n$ to apply OLS estimation of equation (2.9). They give three reasons for making this assumption. First, it is common and is made not only in the Solow-Swan model, but also in other growth models. Even if saving and population growth are endogenous, under the isoelastic preference conditions, $s$ and $n$ are independent of $e$. Second, this independency assumption of $e$ with $s$ and $n$, help test the various informal hypotheses that have been made regarding the relationship between income, saving and population growth. Third, the regression results allow testing of the joint hypothesis of validity of the Solow-Swan model due to the specifications of signs and magnitudes of the coefficients.

2.3 Estimation of determinants of steady state per capita income with human capital variable

Estimation of equation (2.9) may not provide the complete explanation for differences in levels of per capita real income across states of Indian federation due to the problems of specification bias. For instance, Human capital may be positively correlated with saving rate and negatively related with population growth. Thus, omission of this capital variable causes the omitted variable bias in equation (2.9). MRW find that augmenting the Solow-Swan model with measures of human capital improves its predictive power of explaining inter-state (regional) growth rates. For clarity of explanation, dropping the $i$ subscript, the production function with the inclusion of human capital for single Indian state $i$ is given as follows:

$$Y_i = K_i^a H_i^{A - \lambda} (A_i L_i)^{\lambda}$$

(2.10)

where $H$ is the stock of human capital, and all other variables are the same as defined before. Let $s_k$ and $s_h$ be the fractions of income invested in both physical and human capital respectively. Then the evolution of $\dot{k}$ and $\dot{h}$ over time are determined by

$$\dot{k} = s_k \dot{y}_i - (n + g + \delta) \dot{k}_i$$

(2.11)

$$\dot{h} = s_h \dot{y}_i - (n + g + \delta) \dot{h}_i$$

(2.12)

where $\dot{h} = H/AL$ is the human capital per effective unit of labour. Therefore, in the augmented Solow-Swan model, the economy converges to a steady state as defined by
\[ \hat{k}^* = \left[ \frac{s_k^{1-\lambda} s_h^\lambda}{n + g + \delta} \right]^{-\frac{1}{1-\alpha-\lambda}} \]  
(2.13)

\[ \hat{h}^* = \left[ \frac{s_h^{-\lambda} s_k^{1-\lambda}}{n + g + \delta} \right]^{-\frac{1}{1-\alpha-\lambda}} \]  
(2.14)

Substituting the values of \( \hat{k}^* \) and \( \hat{h}^* \) in the production function equation (2.10) augmented by human capital the steady state value of output (income) per capita \( (v = Y/L) \) is obtained as given in equation (2.15).

\[
\ln y = \ln(A_i) + g t + \frac{\alpha}{1-\alpha-\lambda} \ln(s_i) + \frac{\lambda}{1-\alpha-\lambda} \ln(s_h) - \frac{\alpha + \lambda}{1-\alpha-\lambda} \ln(n + g + \delta) 
\]  
(2.15)

However, while testing augmented Solow-Swan model, MRW raise an important question: whether the available data on human capital correspond more closely to the rate of accumulation of \( (s_h) \) or to the level of human capital \( (h) \). In the present context, the second specification of MRW is followed after incorporating human capital variable, \( \hat{h}_i \). Therefore, introducing the subscript \( i \), equation (2.9) with human capital is modified as

\[
\ln y_i = a + \frac{\alpha}{1-\alpha} \ln(s_i) - \frac{\alpha}{1-\alpha} \ln(n_i + g + \delta) + \frac{\lambda}{1-\alpha-\lambda} \ln(\hat{h}_i^*) + \epsilon_i 
\]  
(2.16)

where \( \lambda \) represents the output elasticity of level of human capital or the stock of human capital. The equations (2.9) and (2.16) are the bases for estimation of fixed effects panel data model for \( i=1, 2, ..., 14 \) major Indian states over the time: \( t= 1977-78,..., 2000-01 \) to provide the explanation for differences in the steady state levels of per capita real income (as given in Chapter 6).

2.4 Estimation of determinants of transitional growth and convergence

Assuming Indian states (economies) are in their steady state and using equation (2.9) or (2.16), differences in the levels of per capita real income across states (economies) can be examined over a period of time. In order to study transitional growth rate and hence, convergence phenomenon from an initial position to a steady state position of different states (economies), understanding of the transitional growth path within the neoclassical growth framework is important. The essence of neoclassical growth theory is that growth of per capita real income is a
The transitional growth rates of capital, output, and consumption per effective labour within the Cobb-Douglas production function with exogenous labour augmenting technical progress are given as

\[ \gamma_k = \alpha s A \left( \frac{k}{\dot{k}} \right)^{(1-\alpha)} - (n + g + \delta) \]  \hspace{1cm} (2.17)

\[ \gamma_y = \alpha s A \left( \frac{k}{\dot{k}} \right)^{(1-\alpha)} - (n + g + \delta) \]  \hspace{1cm} (2.18)

\[ \gamma_c = \alpha s A \left( \frac{k}{\dot{k}} \right)^{(1-\alpha)} - (n + g + \delta) \]  \hspace{1cm} (2.19)

where, \( \gamma \) refers to growth rate of a variable, \( A > 0 \) and \( 0 \leq \alpha \leq 1 \). Equation (2.17), (2.18), and (2.19) show that growth rates of capital, output, and consumption per effective labour are directly proportional to the saving rates and levels of technology, and inversely proportional to the effective depreciation rate (that constitutes population growth rate, rate of technological progress and rate of depreciation of capital). All these per capita variables in efficiency units exhibit same dynamics during the transition within the neoclassical growth framework.

Now the transition of Indian state \( i \) is given by taking a first order Taylor approximation of \( \dot{y}_i \) around the steady state value of \( \bar{y} \) as given in equation (2.6):

\[ \delta(\ln(\dot{y}_i))/\delta t = \beta(\ln(\dot{y}_i) - \ln(\dot{y}_i)) \]  \hspace{1cm} (2.20)

where \( \beta = (1-\alpha)^*(n+g+\delta) \). The parameter \( \beta \) is the speed of convergence\(^\text{13} \). It measures how much the growth rate declines as the per capita real income increases toward its steady state value. Growth rate of per capita real income depends on the distance that

\(^{13}\) An important property of convergence coefficient \( \beta \) is that the saving rate and the level of technology do not affect the speed of convergence, \( \beta \). It is because of two offsetting forces, which exactly nullify to each other. In the first place, given \( \dot{k} \) (capital per effective unit of labour), a higher 's' leads to greater investment and therefore to a faster speed of convergence. Second, a higher 's' raises the steady state capital intensity \( k^* \) and thereby lowers the average product of capital in the vicinity of the steady state. This reduces the speed of convergence. Hence, for a given value of parameters \( g, n, \delta \), the convergence coefficient \( \beta \) is determined by the capital-share parameter, \( \alpha \). Similarly, the differences in the level of technology, \( A \), have same offsetting effects on \( \beta \). However, in Ramsay-Cass-Koopmans model, \( \beta \) depends on the parameters of technology and preferences.
separates between the current and steady state levels of per capita real income during
the transition.

Solving this differential equation (2.20) and subtracting ln \( \dot{y}_0 \), the following
growth equation (2.21) is obtained.

\[
\ln(\dot{y}_i) - \ln(\dot{y}_0) = \alpha - (1 - e^{-\beta t}) \ln(\dot{y}_0)
\]  \hspace{1cm} (2.21)

where 't' and '0' refer to the terminal and initial points. Dividing by \( t \) (time
intervals) in both sides, the following equation (2.22) for average growth rate is
obtained.

\[
\frac{1}{t} \ln(\dot{y}_i) - \ln(\dot{y}_0) = \alpha' - (1 - e^{-\beta t}) \ln(\dot{y}_0)
\]  \hspace{1cm} (2.22)

where \( \dot{y}_0 \) is income per effective worker at some initial date (time) 0, \( \dot{y}^* \) is the steady
state level of income per effective worker and \( \alpha' = g + \frac{1}{t} (1 - e^{-\beta t}) \ln(\dot{y}^*) \) is a
constant across \( i \) and \( g \) is the rate of technical progress. Equations (2.21) and (2.22)
are the analytical basis for empirical estimation of transitional dynamics from an
initial time 0 to any future time \( t \geq 0 \).

Keeping the steady state growth rate, \( g \), the convergence speed, \( \beta \), and
averaging interval, \( t \) as constant; the equation (2.22) implies that the average per
capita growth rate of output/income depends negatively on the ratio of \( \dot{y}^* \) to \( \dot{y}_0 \). It is
interesting to explain the different parameters that effect \( \beta \). The coefficient \( [(1 - e^{-\beta t})/t] \)
that relates the growth rate of \( \dot{y} \) to \( \ln(\dot{y}^*/\dot{y}_0) \) in equation (2.22) declines with \( t \) for
given \( \beta \). Therefore, the average growth rate declines as \( t \) rises. As \( t \to \infty \), the steady
state growth rate \( g \), dominates the average; and hence, \( [(1 - e^{-\beta t})/t] \to 0 \), and the average
growth rate of \( \dot{y} \) in equation (2.22) tends to the steady state growth rate, \( g \). If we are
given \( t \), a higher \( \beta \) implies a higher coefficient \( [(1 - e^{-\beta t})/t] \) (as \( t \to 0 \), the coefficient
approaches \( \beta \)).

Assuming \( i = 1, 2, ..., 14 \) major Indian states are in their steady state, and using
data on per capita real income, per capita real consumption and per capita real state
disposable income, the speed of absolute (unconditional) \( \beta \)-convergence (or
divergence) is estimated in the univariate equations of types (2.21) and (2.22) for 14
major states of India.
In the empirical specification of the econometric equation (2.21) or (2.22), it is implicitly assumed that all 14 major states in India approach to the same steady state or, at least, the steady state is not correlated with the initial levels of per capita real income. If $y_i *= y*$ ($*$ refers to steady state levels of per capita real income and $i$ stands for state), then this term gets absorbed by constant $\alpha$ in the equation (2.21) or (2.22). On the contrary, if it is assumed that states in India converge to the same steady state levels of per capita real income in the long run and they don’t satisfy the assumption made, then the equation (2.21) is misspecified and the error term becomes correlated with determinants of steady state. When the steady state is correlated with initial level of per capita real income, the error term is correlated with the explanatory variables; then the estimated coefficient of $\ln(y_0)$ is biased towards zero irrespective of techniques used.

In order to overcome this estimation problem, state-wise data on determinants of steady state (saving rates, population growth rates, human capital) can be used to estimate their impacts on growth of income. That is, instead of estimating the univariate regression like equation (2.21) or (2.22), a multivariate regression forms of equations (2.23) and (2.24) can be estimated.

States in India may differ with respect to their tastes and preference, saving behaviour, institutions and production technologies. The neoclassical (Solow-Swan, 1956) model predicts that different economies reach different steady state levels of income per person, depending on the level of rate of investment, depreciation rate, population growth, human capital, quality of institutions and government policies and so forth. It predicts that states (regions) will have different rates of growth, depending upon each state’s (region) initial deviation from its own steady state. If these differences are accounted for, the model predicts that growth of a state is positively related to the distance that separates it from its own steady state (MRW, 1992). This is the idea of conditional convergence.

Now substituting equation (2.6) in equation (2.22), the following relationship is obtained.

$$
\frac{1}{t}(\ln y_r - \ln y_0) = \alpha + \left(\frac{1-e^{-\beta t}}{t}\right) \frac{\alpha}{1-\alpha} \ln(s_i)
$$

$$
- \left(\frac{1-e^{-\gamma t}}{t}\right) \frac{\alpha}{1-\alpha} \ln(n_i + g + \delta) - \left(\frac{1-e^{-\beta t}}{t}\right) \ln(y_0)
$$

(2.23)
where, \( \tilde{\alpha} = \left( \frac{1-e^{-\beta t}}{t} \right) \ln(A_0) + gt \) and \( g + \delta \) is assumed constant.

Equation (2.23) predicts that states with low initial output per labour (worker) possess faster transitional growth rates than the states with higher initial output per labour (worker), conditioned upon the values \( s, n, g, \) and \( \delta \).

The transitional equation for the Solow-Swan model augmented with human capital is given by

\[
\frac{(1/n)}{\ln(y) - \ln(y_0)} = \tilde{\alpha} + \left( \frac{1-e^{-\beta t}}{t} \right) \frac{\alpha}{1-\alpha} \ln(s_u) - \left( \frac{1-e^{-\beta t}}{t} \right) \frac{\alpha}{1-\alpha} \ln(n + g + \delta) + \left( \frac{1-e^{-\beta t}}{t} \right) \lambda \frac{\lambda}{1-\alpha} \ln(\hat{h}_*) - \left( \frac{1-e^{-\beta t}}{t} \right) \ln(y_0) 
\]

(2.24)

where \( \beta = (1-\alpha \lambda) (n+g+\delta) \) = speed of convergence.

In empirical estimations, the equations (2.23) and (2.24) are estimated in stochastic form under the assumption that the production structure is common to all states. This assumption is necessary because it is difficult to observe the efficiency function of \( \ln(A) \) and \( \tilde{\alpha} = \left( \frac{1-e^{-\beta t}}{t} \right) \ln(A_0) + gt \) is assumed to be constant across the states.

Given the limitations of data such as saving (investment) at the state levels, it is useful to focus on the estimations of conditional convergence regression (2.23) (or 2.24) in Indian federal context. Inclusion of private investment and public investment or intergovernmental transfers variable in the conditional convergence regression equation (2.23) (or 2.24) may pose a theoretical question within the neoclassical growth framework. It is because the capital shares components of either public capital due to public investment or exogenous investment components of intergovernmental transfers, or private capital in state income may not be compatible since the only exponent of aggregate capital is, \( \alpha \) in the model. The conditional convergence regression equation (2.23) derived from the neoclassical form of production function of Cobb-Douglas type that uses aggregate factors of production such as capital and labour inputs with labour augmenting technical progress (Harrod neutral) is indirectly helpful to estimate the shares of capital and labour in the national (state) income. This estimate of capital share (\( \alpha \)) is obtained from the aggregate capital (investment) used
in the production process for an economy. Therefore, estimation of a separate parameter due to public investment rate in the presence of other (private) investment variable in this convergence regression (2.23) may be inconsistent with the neoclassical growth model (Solow-Swan, 1956) and may result in ambiguity in the determination of magnitudes of parameters.

Nevertheless, introducing both private and public investment rates with other determinant of steady-state and growth in the conditional convergence regression (2.23) calls for the alternative formulation of production function that could be helpful to estimate both the shares of private and public sector capital stock in the national (state) income within the neoclassical growth framework. While extending the neoclassical growth model in a Cobb-Douglas technology to capture both private and public sector capital stock can be used to understand the contribution to the growth performance because of private and public sector capital formation in an economy (see, Khan and Kumar, 1997), the following analysis adheres to the original neoclassical growth framework of Solow-Swan (1956) to examine the conditional convergence with a view to account for intergovernmental transfers (treated as exogenous investment) in the model. In this context, total investment state as a whole includes private investment and intergovernmental transfers to obtain aggregate investment made in the state.

Besides the theoretical and empirical problems associated with this analysis, problems may also arise due to different estimation techniques used to estimate the speed of convergence with the Indian data set such as ours. Given the one set of variables in the convergence regression equation (2.23), the speed of conditional convergence (divergence) coefficient, \( \beta \) to be retrieved from the OLS estimate of negative (positive) coefficient of natural log of initial levels of per capita real income \([\ln(y_0)]\), is sometimes very difficult.

\[
\text{If } b = -\left(1 - e^{-\beta t}\right) \Rightarrow \beta = -\frac{\ln(1 + bt)}{t} 
\]

where \( b \) is the OLS coefficient of natural log of initial level of per capita real income, and \( t \) is the time interval. When \( |bt| > 1 \) for \( t > 0, b < 0 \); then \( \beta \) cannot be computed.

This problem can be sorted out by applying NLS method of estimation since the equation (2.23) and (2.24) are in non-linear in parameters. Estimation of the
equation (2.23) by NLS method will give parameters of the model like the speed of conditional convergence, $\beta$, capital share in the state income, $\alpha$ and the share of population (labour) in state income, $1-\alpha$. However, we may face the problem of determining exact capital share in the state income, due to indeterminate volume of total investment made in the state as a whole, and explaining the impact of intergovernmental transfers for understanding the differential growth of income across the states. Since we are interested to know the impact of intergovernmental transfers with other control variables, employing OLS method of estimation may give us to assess the impact of intergovernmental transfers through the elasticity measures of respective coefficients of the variables used for the purpose. Despite the fact the NLS estimate can offer an unbiased and consistent estimate of $\beta$, and $\alpha$; OLS method is chosen to estimate equations (2.23) and (2.24) for understanding and assessing the impacts of different variables better through elasticity measures.

2.5 Estimation of determinants of transitional growth using a dynamic panel technique

Estimation of equations (2.23) and (2.24) in a cross-section of states (economies) gives rise to empirical framework for studying convergence. Since the presumed correlation with lagged (initial) per capita real income is positively related with state-specific effect (due to differences in technology), the coefficient for the initial levels of per capita real income would be biased upward in a cross-section study. This means that the speed of convergence $\beta$ may be underestimated. Therefore, single equation analysis of growth and convergence in the form of equations (2.23) or (2.24) suffers from omitted variable bias due to unobserved state-specific effects and endogeneity of explanatory variables. These problems can be overcome by using dynamic panel growth framework.

Islam (1995) develops a dynamic panel growth framework as derived from basic neoclassical growth model by (a) converting the output (income) per effective worker, $\hat{y}$ to per worker output (income), $y$ and (b) representing with $(1-e^{-\beta t}) \ln(A_0)$ as the time-invariant individual state-specific effect term.

Let $\tau = (t) - (t-\tau)$ be the time interval between the time $t$ and $t-\tau$ (say five years), where $\hat{y}_{t-\tau}$ is income per effective labour at some initial point of time, $t-\tau$. In the new notations of time interval $\tau$, the equations (2.23) and (2.24) can be
transformed into following dynamic panel growth equation as given by equation (2.25).

\[ y_{i,t} = \psi y_{i,t-1} + \sum_{j=1}^{J} \theta_j x^j_{i,t} + \eta_i + \mu_i + v_{it} \quad (2.25) \]

where \( y \) is the per capita income (output)

- \( y_{it} = \ln y_{it} \)
- \( y_{i,t-\tau} = \ln y_{i,t-\tau} \)
- \( \psi = e^{-\beta \tau} \)
- \( \theta_1 = \left( 1 - e^{-\beta \tau} \right) \frac{\alpha}{1 - \alpha} \)
- \( \theta_2 = \left( 1 - e^{-\beta \tau} \right) \frac{\alpha}{1 - \alpha} \)
- \( \theta_3 = \left( 1 - e^{-\beta \tau} \right) \frac{\lambda}{1 - \alpha} \)
- \( x^1_{i,t} = \ln (s) \)
- \( x^2_{i,t} = \ln (n + g + \delta) \)
- \( x^3_{i,t} = \ln (h^*) \)
- \( \mu_i = \left( 1 - e^{-\beta \tau} \right) \ln A_0 \)
- \( \eta_i = g \left( t - e^{-\beta \tau} (t - \tau) \right) \)
- \( v_{it} = \text{idiosyncratic error} \)
- \( \tau = \text{time interval of five-year period.} \)

Estimation equation (2.25) with fixed effects panel data model may overcome the problems of omitted variable bias and to some extent endogeneity of regressors. The value of \( \beta \) tends to be higher in panel data analysis in comparison to the cross-section study since the bias in the later arises from the omission of appropriate conditioning variables (see CEL, 1996). The implied (parameters) \( \beta, \alpha, \) and \( \lambda \) retrieved from the OLS estimates of equations (2.25) are given as follows:

\[ \beta = - \frac{\ln (\psi)}{\tau} \quad (2.25a) \]

\[ \alpha = \frac{\theta_1}{\left( 1 - e^{-\beta \tau} + \theta_1 \right)} \quad (2.25b) \]

\[ \lambda = \frac{(\theta_3 - \alpha \theta_2)}{\left( 1 - e^{-\beta \tau} \right)} \quad (2.25c) \]
2.6 Estimation of convergence by types

2.6.1 Absolute convergence

Estimating equations (2.21) and (2.22) gives rise to absolute or $\beta$-convergence. It suggests that poor states (economies) tend to grow faster than rich states (economies) independent of initial conditions of their incomes if the economies have similar structural parameters (saving rate, consumption behaviour, technology, government policies, and factors market). Empirically, absolute convergence is applied in a group of states (economies) if there is an inverse relationship between the initial levels of per capita real income and its average growth rates independently of their initial conditions. It measures how rapidly (slowly) the per capita real income of the economies is converging to the common steady state value.

2.6.2 Conditional convergence

Similarly, equations (2.23) and (2.24) provide with a framework for estimation of conditional $\beta$-convergence depending upon the model under consideration (i.e., whether Basic Solow-Swan or Augmented Solow-Swan model). This hypothesis is defined as the per capita real income of states (economies) that are identical in their structural characteristics (preferences, technology, the rates of population growth, and government policies etc.) converges to one another in the long run independently of their initial conditions. Once we control for differences in determinants of the steady state, the initial per capita real income is inversely related to the growth rate of per capita real income. The inclusion of variables (government policies, institutional choices involving property rights and free market, accumulation of human capital, fertility decisions and diffusion of technologies) that serve as proxy for differences in steady state position makes major differences in the results across the broad cross-section of countries. When these additional variables are held constant, the relation between the per capita growth rates and the natural log of initial per capita real income becomes significantly negative, as predicted by the neoclassical growth model.

14 Another compatible hypothesis is club convergence hypothesis. It can be defined as the per capita real incomes of states (economies) that are identical in the structural characteristics (saving rate, consumption behaviour, state of technology, government policies, and factor markets) converge to one another in the long run provided that their initial conditions are similar as well (Galor, 1996).
2.6.3 Sigma convergence

Sigma convergence relates to reduction in cross-sectional dispersion over time. It is said to occur if the dispersion measured by standard deviation (or variance) of the natural log of per capita real income or product across a group of states (economies)—declines over time (Barro and Sala-i-Martin, 2003). The relationship between absolute and sigma convergence is very interesting to study the cross-sectional dispersion (inequality) across states or regions. It happens that even if absolute convergence holds, the sigma convergence may not occur over time.

Let us assume that the absolute convergence applies for a group of states (economies) \( i = 1, 2, \ldots, n \), where \( n \) is a large number. For discrete time period like yearly data, the per capita real income for economy \( i \) can then be approximated within the growth predictions of neoclassical growth model by the process

\[
\ln \left( \frac{y_{it}}{y_{i,t-1}} \right) = a_t - (1 - e^{-\beta}). \ln (y_{i,t-1}) + u_{it} \quad (2.26)
\]

where subscripts \( t \) and \( i \) are the year and the country (region). \( u_{it} \) is the random disturbance term. The neoclassical growth theory implies that

\[
a_t = [g_t + (1 - e^{-\beta}) \ln (\hat{y}_t^*) + g_t (t - 1)],
\]

where \( \hat{y}_t^* \) is the steady state level of \( \hat{y}_t \) and \( g_t \) is the rate of technological progress. Assuming \( u_{it} \) has 0 mean, variance \( \sigma_{u_{it}}^2 \), and is distributed independently of \( \ln (y_{i,t-1}) \), \( u_{jt} \) for \( j \neq i \), and lagged disturbances. \( a_t \) is assumed to be same for all economies and is equal to \( a \). If \( \beta > 0 \), equation (2.26) implies that poor economies tend to growth faster than rich ones. It is because the annual growth rate, \( \ln \left( \frac{y_{it}}{y_{i,t-1}} \right) \) is inversely related to \( \ln (y_{i,t-1}) \). However, if there are two economies, the one that begins behind is predicted to remain behind at any future time.

In order to measure cross-sectional dispersion in per capita real income or consumption, the framework of \( \sigma \)-convergence is used. This is measured by taking the sample variance of natural log of per capita real income/consumption as given in equation (2.27). This equation (2.27) is also used to find sigma convergence (or divergence) in per capita disposable income across states in India.

\[
\sigma_i^2 = \frac{1}{n} \sum_{i=1}^{n} \left( \ln (y_{it}) - \bar{\mu} \right)^2 \quad (2.27)
\]
where $\bar{\mu}_t$ is the sample mean of $\ln (y_{it})$ at time $t$. The equation (2.27) assumes that each person in a state has the same income, and that all 14 states have the same population. While dealing with the standard of livings of people, a better measure of the evolution of personal inequality can be used as population-weighted variance of $\ln$ of per capita real income (Barro and Sala-i-Martin, 2003). The same equation (2.27) is weighted with population share in 14 states’ total and re-estimated. This population-weighted-variance analysis assumes that each person within a state has the same level of income. But, the population figures differ from one state to other since some states may have more people than others. The analysis of absolute ($\beta$) and sigma ($\sigma$) convergence is often called as dynamic measures of inequality in the convergence literature.

When sample size, $n$ is large, $\sigma^2_t$ is close to population variance and its evolution can be derived over the time in accordance with the first-order difference equation,

$$\sigma^2_t = e^{-2\beta} \sigma^2_{t-1} + \sigma_u^2 \tag{2.28}$$

If the variance of the disturbance term, $\sigma_u^2$ is constant over time for all $t$, then the solution of the first-order difference Eq. (2.28) will be given by

$$\sigma^2_t = \frac{\sigma_u^2}{1-e^{-2\beta}} + \left(\sigma^2_0 - \frac{\sigma_u^2}{1-e^{-2\beta}}\right)e^{-2\beta t} \tag{2.29}$$

where $\sigma^2_0$ is the variance of $\ln (y_{i0})$. We can verify the solution Eq. (2.29) that satisfies Eq. (2.28) and this $\sigma^2_t$ monotonically approaches its steady state value:

$$\sigma^2 = \frac{\sigma_u^2}{1-e^{-2\beta}} \tag{2.30}$$

Therefore, the steady state value $\sigma^2$ rises with $\sigma_u^2$ but, declines with $\beta$. If $\beta > 0$, then over time $\sigma_t^2$ falls (rises) if the initial value $\sigma^2_0$ is greater (less) than the steady state value of $\sigma^2$. Hence, a positive coefficient $\beta$ dose not necessarily result in a falling $\sigma_t^2$ ($\sigma$-convergence). In other words, $\beta$-convergence is a necessary but not a sufficient
condition for $\sigma$-convergence. However, it is to be noted that the steady state dispersion is positive even if $\beta$ is positive as long as $\sigma_u^2 > 0$.

2.7 Summary

The analytical bases and empirical frameworks for estimations of various convergence hypotheses in Indian federal context are presented in this chapter. Proper linkages of analytical base of Solow-Swan growth model and empirical analysis for convergence are established to provide with empirical explanation for differences in determinants of inter-state growth and levels of income, and consumption towards the steady state during the transition. Attempt is also made to include federal fiscal transfers to account for differences in the steady state levels and growth of income within a dynamic panel growth framework. In addition, the chapter provides with a framework for the estimation of convergence or divergence (i.e., absolute, conditional and sigma) in per capita real income, per capita real consumption and per capita state disposable income to explain the differences in the standards of living and hence, to analyse the evolution of regional inequalities across the states of Indian federation.

15 It is noted that $\sigma$-convergence and $\beta$-convergence analyses may not give similar results. It is because with a change in the ranks of the regions, there could be a significant decrease in $\sigma$-convergence (divergence). A poorer region could grow so fast that it could overshoot on the richer side, leading to greater overall dispersion. $\beta$-convergence analysis may capture this divergence as convergence or it may show an absence of either (Shankar and Shah, 2003). Therefore, it is essential to estimate both the concepts convergence.