Chapter 8

Simple Additive Weighting Method for Evaluation of Service Quality

8.1 Introduction

sets. In 2011, Wei, Cuiping and Xijin Tang [67] proposed an intuitionistic fuzzy group decision making approach based on entropy and similarity measures. In 2011, Wu, Jian-Zhang and Qiang Zhang [68] proposed multi criteria decision making method based on intuitionistic fuzzy weighted entropy. In 2011, Wei, Cui-Ping, Pei Wang, and Yu-Zhong Zhang [66] proposed entropy, similarity measure of interval valued intuitionistic fuzzy sets and their applications. Although researchers have dedicated a lot of attention to service quality using different methods, there are still lacking of application of interval valued intuitionistic fuzzy set in measuring service quality. To the best of authors’ knowledge, there have been not much of progress in the study of interval valued intuitionistic fuzzy sets in different perspectives and its applications in solving problems except data analysis, pattern recognition and multi-criteria decision making. Therefore, this chapter is carried out interval valued intuitionistic fuzzy simple additive weighting method for evaluating service quality. We use the simple operation of interval valued intuitionistic fuzzy arithmetic operation for calculating the aggregation score for each alternative. By comparing the score values, the alternatives are ranked from the lowest to highest. This chapter proceeds as follows. In section 8.2, basic definitions related to the score function are presented. In sections 8.3, summarizes the computational steps of interval valued intuitionistic fuzzy simple additive weighting method for evaluating service quality. In section 8.4, a case study of service quality evaluation of car insurance companies is presented and Summary is made as
the last section 8.5 of the chapter.

8.2 Preliminaries

In this section, basic definitions related to the score function are presented.

Definition 8.2.1. Score function

Let \( A = [\mu^L_A(x), \mu^R_A(x)], [\gamma^L_A(x), \gamma^R_A(x)] \) be an interval valued intuitionistic fuzzy number, a score function \( S \) of an interval valued intuitionistic fuzzy value can be represented as follows:

\[
S(A) = \frac{\mu^L_A(x) + \mu^R_A(x) - \gamma^L_A(x) - \gamma^R_A(x)}{2} \tag{8.1}
\]

Here the score function is applied to compare the grades of the interval valued in intuitionistic fuzzy sets. Hence, the alternative with the largest value of the score is the best alternative for customers.

8.3 Algorithm of Simple Additive Weighting Method

In this section, summarizes the computational steps of interval valued intuitionistic fuzzy simple additive weighting method for evaluating service quality. In 1981 Hwang and Yoon were developed Simple Additive Weighting method (SAW). This method is the best known and widely used Multi Attribute Decision Making (MADM) method. The basic principle of simple additive weighting is to obtain a weighted sum of the performance ratings of each alternative under all attributes. Suppose we have a set of m
alternatives, \( A_i \) \( (i = 1, 2, \ldots, m) \). Let \( C_1, C_2, C_3, \ldots, C_n \) be the criteria to evaluate the service qualities. Furthermore, we assume that the weights of criteria supplied by decision makers are represented by a weighting vector \( W_j = (W_1, W_2, W_3, \ldots, W_n) \) where \( W_1, W_2, W_3, \ldots, W_n \) are represented by intuitionistic fuzzy sets defined as follows:

\[
W_j = [\mu_w(c_j), \gamma_w(c_j), \pi_w(c_j)]
\]

The computational procedure for interval valued intuitionistic fuzzy simple additive weighting is being presented as follows:

**Step 1.** Construct an interval valued intuitionistic fuzzy decision matrix:

\[
D = (d_{ij})_{mn}
\]

is an interval valued intuitionistic fuzzy decision matrix such that:

\[
D = \begin{pmatrix}
   d_{11} & \ldots & d_{1n} \\
   \vdots & \ddots & \vdots \\
   d_{m1} & \ldots & d_{mn}
\end{pmatrix}
\]

Where \( d_{ij} = ([\mu_{ij}^L, \mu_{ij}^R], [\gamma_{ij}^L, \gamma_{ij}^R], [\pi_{ij}^L, \pi_{ij}^R]) \), \( i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n \), where are contained in interval valued intuitionistic fuzzy decision matrix. In \( d_{ij}, [\mu_{ij}^L, \mu_{ij}^R] \) represents the closed interval degree that the alternative \( A_i \) satisfies the attribute \( C_j \) and \( [\gamma_{ij}^L, \gamma_{ij}^R] \) represents the closed interval degree that the alternative \( A_i \) does not satisfy the attribute \( C_j \) and \( [\pi_{ij}^L, \pi_{ij}^R] \) represents the closed interval degree of hesitation (or uncertainty) associated with the alternative \( A_i \) and attribute \( C_j \).

**Step 2.** We assume that the weight of criteria supplied by decision makers is
represented by a weighting vector $W_j = (W_1, W_2, W_3, \ldots, W_n)$ of the criteria.

**Step 3.** We obtain the total interval valued intuitionistic fuzzy scores $V(A_i)$ for individual alternatives by multiplying the intuitionistic fuzzy weight vectors ($W$) by interval-valued intuitionistic fuzzy rating matrix ($D$).

$$V(A_i) = D \otimes W$$

$$= \sum_{j=1}^{m} A_i \otimes W_j$$

$$= \sum_{j=1}^{m} \left( \begin{array}{c}
[\mu_{A_i}^L(c_j), \mu_{A_i}^R(c_j)], [\gamma_{A_i}^L(c_j), \gamma_{A_i}^R(c_j)], \\
[\pi_{A_i}^L(c_j), \pi_{A_i}^R(c_j)] \\
\otimes[\mu_w(c_j), \gamma_w(c_j), \pi_w(c_j)]
\end{array} \right)$$  \quad (8.2)

**Step 4.** Applying equation (8.1) to obtain a crisp score function among $S(A_1), S(A_2), S(A_3), \ldots, S(A_m)$ for the various alternatives.

**Step 5.** Rank the alternatives $A_i$ ($i = 1, 2, \ldots, m$) and then select the most desirable ones. The largest value of $S(A_i)$ among $S(A_1), S(A_2), S(A_3), \ldots, S(A_m)$ represents the best alternative.

### 8.4 A Case Study of Service Quality Evaluation

In this section, a case study of service quality evaluation of car insurance companies is presented. The evaluation of service quality normally involves a set of $m$ alternatives $A_i$ ($i = 1, 2, \ldots, m$). The alternatives in this case are several car insurance companies. The quality of services provided by these alternatives is evaluated by their customers which represented by a set of $n$
criteria $C_j$ ($j = 1, 2, \ldots, n$). A survey questionnaire is used to estimate the quality levels of services perceived by customers. Four selected car insurance companies are VD Insurance ($A_1$), KPN Insurance ($A_2$), LPS Insurance ($A_3$), and BSN Insurance ($A_4$). The questionnaires consist of six evaluation criteria which are Confidence ($C_1$), Responsiveness ($C_2$), Reliability ($C_3$), Assurance ($C_4$), Empathy ($C_5$) and Tangibles ($C_6$). The evaluation is made according to the following steps.

**Step 1.** Construct the interval valued intuitionistic fuzzy decision matrix.

The interval valued intuitionistic fuzzy decision matrix has been constructed and given below.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(0.5, 0.6], [0.2, 0.3], [0.1, 0.3])</td>
<td>(0.4, 0.5], [0.2, 0.3], [0.2, 0.4])</td>
<td>(0.3, 0.4], [0.2, 0.3], [0.3, 0.5])</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.5, 0.6], [0.2, 0.3], [0.1, 0.3])</td>
<td>(0.6, 0.7], [0.0, 0.1], [0.2, 0.4])</td>
<td>(0.3, 0.4], [0.0, 0.1], [0.5, 0.7])</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(0.3, 0.4], [0.1, 0.2], [0.4, 0.6])</td>
<td>(0.3, 0.4], [0.2, 0.3], [0.3, 0.5])</td>
<td>(0.3, 0.4], [0.2, 0.3], [0.3, 0.5])</td>
</tr>
<tr>
<td>$A_4$</td>
<td>(0.5, 0.6], [0.2, 0.3], [0.1, 0.3])</td>
<td>(0.2, 0.3], [0.0, 0.1], [0.6, 0.8])</td>
<td>(0.0, 0.1], [0.3, 0.4], [0.5, 0.7])</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Alternative</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(0.4, 0.5], [0.3, 0.4], [0.1, 0.3])</td>
<td>(0.8, 0.9], [0.0, 0.1], [0.1, 0.2])</td>
<td>(0.1, 0.2], [0.4, 0.5], [0.3, 0.5])</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.4, 0.5], [0.2, 0.3], [0.2, 0.4])</td>
<td>(0.6, 0.7], [0.1, 0.2], [0.1, 0.3])</td>
<td>(0.1, 0.2], [0.3, 0.4], [0.4, 0.6])</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(0.5, 0.6], [0.1, 0.2], [0.2, 0.4])</td>
<td>(0.7, 0.8], [0.0, 0.1], [0.1, 0.3])</td>
<td>(0.1, 0.2], [0.4, 0.5], [0.3, 0.5])</td>
</tr>
<tr>
<td>$A_4$</td>
<td>(0.6, 0.7], [0.1, 0.2], [0.1, 0.3])</td>
<td>(0.4, 0.5], [0.1, 0.2], [0.3, 0.5])</td>
<td>(0.2, 0.3], [0.2, 0.3], [0.4, 0.6])</td>
</tr>
</tbody>
</table>
Step 2. We assume that the weight of criteria supplied by decision makers is represented by a weighting vector $W_j = (W_1, W_2, W_3, W_4, W_5, W_6)$.

<table>
<thead>
<tr>
<th>$W_j$</th>
<th>$W_1$</th>
<th>$W_2$</th>
<th>$W_3$</th>
<th>$W_4$</th>
<th>$W_5$</th>
<th>$W_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_w(c_j), \gamma_w(c_j), \pi_w(c_j)$</td>
<td>(.2,.4,.4)</td>
<td>(.2,.2,.6)</td>
<td>(.1,.5,.4)</td>
<td>(.15,.5,.35)</td>
<td>(.25,.3,.4)</td>
<td>(.1,.3,.6)</td>
</tr>
</tbody>
</table>

Step 3. The total interval valued intuitionistic fuzzy score value $V(A_i)$ for each alternative is calculated is using equation (8.2) Definitions 1.3.3 and 1.3.4 and is as follows:

$$V(A_i) = \sum_{j=1}^{6} A_i \otimes W_j$$

$$= \sum_{j=1}^{6} \left( \left[ \left[ \mu_{A_i}^L(c_j), \mu_{A_i}^R(c_j) \right], \left[ \gamma_{A_i}^L(c_j), \gamma_{A_i}^R(c_j) \right] \right] \right) \otimes \left[ \left[ \mu_w(c_j), \gamma_w(c_j), \pi_w(c_j) \right] \right]$$

$$= \sum_{j=1}^{6} \left( \left[ \left[ \mu_{A_i}^L(c_j) \mu_w(c_j), \mu_{A_i}^R(c_j) \mu_w(c_j) \right], \left[ \gamma_{A_i}^L(c_j) + \gamma_w(c_j) - \gamma_{A_i}^L(c_j) \gamma_w(c_j) \right] \right], \left[ \left[ \mu_{A_i}^L(c_j) \mu_w(c_j), \mu_{A_i}^R(c_j) \mu_w(c_j) \right], \left[ \gamma_{A_i}^R(c_j) + \gamma_w(c_j) - \gamma_{A_i}^R(c_j) \gamma_w(c_j) \right] \right] \right)$$

For $i = 1, j = 1, 2, 3, 4, 5$ and 6. We get $V(A_1)$
\[
V(A_1) = \begin{bmatrix}
([5, 6], [2, 3], [1, 3]) \otimes (2, .4, 4) \\
+([4, 5], [2, 3], [2, 4]) \otimes (2, 2, 6) \\
+([3, 4], [2, 3], [3, 5]) \otimes (1, 5, .4) \\
+([4, 5], [3, 4], [1, 3]) \otimes (15, 5, .35) \\
+([8, 9], [0, 1], [1, 2]) \otimes (25, 3, .4) \\
+([1, 2], [4, 5], [3, 5]) \otimes (1, 3, .6)
\end{bmatrix}
\]
\[
V(A_1) = \begin{pmatrix}
[.5 \times .2, .6 \times .2], \\
[.2 + .4 - .2 \times .4, .3 + .4 - .3 \times .4], \\
1 - (.5 \times .2 + .2 + .4 - .2 \times .4), \\
1 - (.6 \times .2 + .3 + .4 - .3 \times .4)
\end{pmatrix}
\]

\[
+ \begin{pmatrix}
[.4 \times .2, .5 \times .2], \\
[.2 + .2 - .2 \times .2, .3 + .2 - .3 \times .2], \\
1 - (.4 \times .2 + .2 + .2 - .2 \times .2), \\
1 - (.5 \times .2 + .3 + .2 - .3 \times .2)
\end{pmatrix}
\]

\[
+ \begin{pmatrix}
[.3 \times .1, .4 \times .1], \\
[.2 + .5 - .2 \times .5, .3 + .5 - .3 \times .5], \\
1 - (.3 \times .1 + .2 + .5 - .2 \times .5), \\
1 - (.4 \times .1 + .3 + .5 - .3 \times .5)
\end{pmatrix}
\]

\[
+ \begin{pmatrix}
[.4 \times .15, .5 \times .15], \\
[.3 + .5 - .3 \times .5, .4 + .5 - .4 \times .5], \\
1 - (.4 \times .15 + .3 + .5 - .3 \times .5), \\
1 - (.5 \times .15 + .4 + .5 - .4 \times .5)
\end{pmatrix}
\]

\[
+ \begin{pmatrix}
[.8 \times .25, .9 \times .25], \\
[.0 + .3 - .0 \times .3, .1 + .3 - .1 \times .3], \\
1 - (.8 \times .25 + .0 + .3 - .0 \times .34), \\
1 - (.9 \times .25 + .1 + .3 - .1 \times .3)
\end{pmatrix}
\]

\[
+ \begin{pmatrix}
[.1 \times .1, .2 \times .1], \\
[.4 + .3 - .4 \times .3, .5 + .3 - .5 \times .3], \\
1 - (.1 \times .1 + .4 + .3 - .4 \times .3), \\
1 - (.2 \times .1 + .5 + .3 - .5 \times .3)
\end{pmatrix}
\]

119
\[
V(A_1) = \begin{bmatrix}
([0.1, 0.12], [0.52, 0.58], [0.38, 0.3]) \\
+([0.08, 0.1], [0.36, 0.44], [0.56, 0.46]) \\
+([0.03, 0.04], [0.6, 0.65], [0.37, 0.31]) \\
+([0.06, 0.075], [0.65, 0.7], [0.29, 0.225]) \\
+([0.2, 0.225], [0.3, 0.37], [0.5, 0.475]) \\
+([0.01, 0.02], [0.58, 0.65], [0.41, 0.3])
\end{bmatrix}
\]

\[
V(A_1) = [[0.48, 0.58], [0.0127, 0.0279], [0.5073, 0.3921]]
\]

Similarly, we get

\[
V(A_2) = [[0.47, 0.57], [0.0059, 0.0103], [0.5241, 0.4197]]
\]

\[
V(A_3) = [[0.41, 0.51], [0.0095, 0.0215], [0.5805, 0.4685]]
\]

\[
V(A_4) = [[0.35, 0.45], [0.0061, 0.0153], [0.6439, 0.5347]]
\]

**Step 4.** The score function for each alternative is calculated using equation (8.1) and is as follows:

\[
S(A_1) = \frac{0.48 + 0.58 - 0.0127 - 0.0279}{2}
\]

\[
S(A_1) = 0.5097.
\]

Similarly, we get

\[
S(A_2) = 0.5119,
\]

\[
S(A_3) = 0.4445,
\]

\[
S(A_4) = 0.3893.
\]

**Step 5.** Rank the alternatives \(A_i\) \((i = 1, 2, 3 \text{ and } 4)\) and then select the most
desirable ones. By comparing the score values, the alternatives are ranked from the lowest to highest. Thus the optimal ranking order of the alternatives is given by $A_4 < A_3 < A_1 < A_2$. So, the most desirable alternatives are $A_2$.

In other words, the company KPN Insurance achieved the highest ranking of service quality among the four selected car insurance companies.

8.5 Summary

In this chapter, we have presented application of an interval valued intuitionistic fuzzy set approach using the interval valued intuitionistic fuzzy arithmetic operations in a simple additive weighting method. The result of the car insurance companies, case study has shown that the company KPN Insurance has achieved the highest ranking in service quality. This implicates that the company KPN Insurance is the first choice of customers when they decided to purchase car insurance. This chapter has provided evidence that the interval valued intuitionistic fuzzy set is a suitable tool to solve the uncertainty and fuzziness in the multiple criteria decision making of service quality problem.