CHAPTER III (A)

Flow due to Non-co-axial rotation of porous disk and a fluid at infinity
CHAPTER III (A)

FLOW DUE TO NON-CO-AXIAL ROTATION OF POROUS DISK
AND A FLUID AT INFINITY.

3(a) 1. INTRODUCTION:

The flow due to non-co-axial rotation of the disk and fluid at infinity has been investigated by many workers.

Batchelor (1951), Stewartson (1953) and Pearson (1965) has studied the flow of an incompressible viscous fluid between two c-axially rotating parallel infinite disks. Berker (1963) has considered the flow between the two disks which are rotating with same angular velocity and implied the possibility of an exact solution if the Navier Stokes equation Coirer (1972) has investigated the flow due to disk and fluid at infinity which are rotating non-co-axially at slightly different angular velocity. Abbot and Walters (1970) has given the exact solution of the primary motion between two eccentrically rotating disks. This problem extended by Knight (1980) and he has included the inertia effect Stephenson (1969) has investigated both theoretically and experimentally the effect of the applied magnetic field on the flow due to co-axially rotating disks. The problem of this type is of particular interest to research worker because of its importance in Geophysics and Astrophysics.

Murthy and Ram (1978) have studied the effect of applied magnetic field on the problem considered by Erdogan, Mohanty (1972) has extended some of the results of Abbot and Walters problem to include the magnetic field. Ramchandra Rao and Raghupupathi Rao (1983) have investigated the steady flow of an incompressible viscous, electrically conducting fluid between two parallel infinite, insulated disks rotating with different angular velocities about two non-coincident axes under the application of a uniform magnetic field in the axial direction.

Here we are considering the flow due to non co-axial rotation of the porous disks and fluid at infinity. We assume that they are rotating with the same angular velocity.
3(a)2. **FUNDAMENTAL EQUATIONS**:  

Consider the steady motion of an incompressible fluid due to rotations of a disk and fluid at infinity. They are rotating non-co-axially. The angles of rotates are same. Let it be $\Omega$.  

We introduce a Cartesian co-ordinates system. Let $z$-axis be the axis of motion of the disk and plane of the disk is $z = 0$. The axis of rotation of the disk and that of the fluid at infinity are in the plane $x = 0$.  

We assume that $l$ is the distance between these axes.  

For a viscous incompressible fluid in steady flow, the Navier stokes equation in Cartesian co-ordinate.

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \\
\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \\
\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)
\end{align*}
\]

Where $p$ denotes the Pressure, $\rho$ the density, $\nu$ the Kinematics Viscosity, $u$, $v$ and $w$ are the Velocity components along $x$, $y$ and $z$ directions respectively.  

**Equation of Continuity**:  

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

**To boundary condition of this problem are**  

\[
\begin{align*}
u = -\Omega y, & \quad v = \Omega x, \quad w = -w_0 \text{ at } z = 0 \\
u = -\Omega y + \Omega l, & \quad v = \Omega x, \quad w = -w_0 \text{ at infinity}
\end{align*}
\]
Where \( u, v, w \) are cartesian components of the velocity vector. Equation (3) suggests the form of the velocity components as

\[
\begin{align*}
  u &= -\Omega y + f(z) \\
  v &= \Omega x + g(z) \\
  w &= -w_0
\end{align*}
\] .......(4)

The boundary condition (4) reduces to

\[
\begin{align*}
  f(0) &= 0, & g(0) &= 0 \\
  f(\infty) &= \Omega l, & g(\infty) &= 0
\end{align*}
\] .......(5)

Substituting (4) in eq" (1) we obtain

\[
\begin{align*}
  \frac{1}{\varrho} \partial^2_p - \Omega^2 x + \Omega g &= - \partial_x \\
  \frac{1}{\varrho} \partial^2_p - \Omega^2 y - \Omega f &= - \partial_y \\
  0 &= - \partial_z
\end{align*}
\] .......(6)

3. SOLUTION :

Equation 6 (c) shows that \( p \) is independent of \( z \). Differentiation of equation 6(a) w.r. to \( z \). We get

\[
\frac{1}{\varrho} \partial^3_p + \varrho g' + \Omega g' + w_o f'' = 0
\] .......(7)

Hence, integrating the equation (7) we have

\[
\frac{1}{\varrho} \partial^2_p + \varrho g' + \Omega g' = c_1
\] .......(8)

Similarly, we have from equation 6 (b)

\[
\frac{1}{\varrho} \partial^2_p + \varrho g' - \Omega f = c_2
\] .......(8)

Where \( c_1 \) and \( c_2 \) are constant.

Introducing

\[
\frac{1}{\varrho} \partial^2_p = \left( \frac{\Omega}{2} \right)^2 x^2
\]
The equation (8) becomes

\[ f'' + 2^{3/2} s f' + 2 g = c, \quad \ldots \quad 9 \text{ (a)} \]
\[ g'' + 2^{3/2} s g' - 2 f = c, \quad \ldots \quad 9 \text{ (b)} \]

Where desh means that differentiation w.r. to \( \omega \) we take
\[ S = \frac{\omega}{2 (\Omega)^{1/2}} \]

is the parameter

The boundary condition w.r. to \( f \) becomes
\[ f(0) = 0, \quad g(0) = 0 \]
\[ f(\infty) = \Omega, \quad g(\infty) = 0 \quad \ldots \quad (10) \]

From the equation (9) we have

\[ F'' + 2^{3/2} s F' - 2 i F = c_1 + i c_2 \quad \ldots \quad (11) \]

Where \( F = f + ig \)

The boundary condition (10) reduces to

\[ F(0) = 0, \quad F(\infty) = \Omega \quad \ldots \quad (12) \]

General solution of the eqn. (11) subject of the boundary condition (12) is

\[ F = -\Omega^2 e^{-\sqrt{2} \sigma s} e^{-\sqrt{2} \sqrt{s^2 + 1} \sigma} + \Omega \]
\[ \quad \ldots \quad (13) \]

Writing the real and imaginary parts separately we get

\[ f = \frac{-\sigma F}{\Omega} \quad = \quad 1 - e^{-\sigma} \cos \beta f \quad \ldots \quad (14) \]
\[ g = \frac{\sigma F}{\Omega} \quad = \quad e^{-\sigma} \sin \beta f \]

\[ \Omega \]
The case \( s = 0 \) corresponds to the problem where there is no suction. In this case, \( \alpha = \beta = 1 \) and (14) becomes

\[
\frac{f}{\Omega} = 1 - e \cos \xi,
\]

\[
\frac{g}{\Omega} = e \sin \xi.
\]

and for \( s \neq 0 \) and \( z = 0 \)

\[
\frac{f}{\Omega} = \alpha \xi,
\]

\[
\frac{g}{\Omega} = \beta \xi.
\]

The variation of \( \frac{f}{\Omega} \) and \( \frac{g}{\Omega} \) are plotted in figure (1) for \( \xi = 0, 1 \) against \( \xi \). Near the plate for \( \xi > 0 \) the values \( \frac{g}{\Omega} \) are greater than those for \( \xi = 0 \), where as those of \( \frac{f}{\Omega} \) are less than its values for \( \xi = 0 \) near the plate. It is observed that the values of \( \frac{f}{\Omega} \) for \( \xi > 0 \) become greater than its value for \( \xi = 0 \) as its moves away from the plate.
Fig. 1: Variation of $f/\Omega_1$ and $g/\Omega_1$ for various values of the blowing parameter $\zeta$. 

$x = \frac{f}{\Omega_1}$

$\frac{g}{\Omega_1} \cdot \gamma$
CHAPTER III (B)

Two dimensional unsteady convective MHD flow through a rotating porous medium with variable suction
CHAPTER III (B)

TWO DIMENSIONAL UNSTEADY CONVECTIVE MHD FLOW THROUGH A ROTATING POROUS MEDIUM WITH VARIABLE SUCTION

3 (b) 1. INTRODUCTION

In recent years, unsteady free and forced convective MHD flow through porous medium has been studied widely because of their importance in aeronautics, missile aerodynamics, meteorology, etc.

Rapits, Perdikis & Trivandis (1981) have made a systematic analysis of a two-dimensional convective flow of a viscous incompressible fluid through a porous medium, bounded by a porous and isothermal plate. An analysis of steady convective flow and the mass transfer through a porous medium bounded by an infinite vertical plate has been presented by Raptis, Trivandis and Kafousias (1981). Raptis, K. and Massalas (1982) have extended their studies to unsteady free convective flow in two different papers. Raptis and Perdikis (1983) have also discussed the oscillatory flow in the presence of free convective flow through a porous medium. Bhaghel, Kumar and Sharma (1992) have made an analysis of unsteady two-dimensional free convective flow of a viscous incompressible fluid through a rotating porous medium bounded by a vertical plane surface at constant temperature when the surface absorbs the fluid with a constant velocity and free stream velocity oscillates about a constant non-zero mean.

In this chapter, we propose to investigate the two-dimensional unsteady convective MHD flow through a rotating porous medium with variable suction. Here the suction velocity at the plate is taken in the form

\[ v' = -v_0 \left[ 1 + A \, e^{i \alpha t} \right] \]
In section 3(b) 2 below, the basic equations of the problem with appropriate boundary conditions have been presented. In section 3(b) 3, an approximate solution to the problem at hand is presented and in section 3(b) 4, the results have been discussed.

3(b) 2. BASIC EQUATIONS

We have considered a unsteady two dimensional flow of an electrically conducting incompressible viscous fluid through a porous medium which is bound by vertical infinite plate. The suction velocity at the plate oscillates about a steady constant mean. The system consisting of the plate and the fluid is rotating about a line perpendicular to the plane of the motion. In the investigation the following assumptions are made.

(i) all the fluid properties are taken to be constant except density in the buoyancy force term where the Boussinesq approximation is used
(ii) the magnetic dissipation and viscous terms in the energy equation are negligible.
(iii) the magnetic Reynolds number is small so that the induced magnetic field can be neglected.
(iv) the electric field $\vec{E}$ is zero.
(v) all the quantities except possibly the pressure $p$ is independent of $x$-coordinates.

The $x$-axis is taken along the upward vertical plane surface, the $y$-axis is taken along to the surface in to the fluid and $z$-axis along the axis of rotation. The velocity $\vec{q}$ and the applied magnetic field $\vec{B}$ may be taken as

$$\vec{q} = \hat{i}u' + \hat{j}v' ; \quad \vec{B} = B_0 \hat{k}'$$
where $i, j, k$ are the unit vectors along $x$-axis and $z$-axis respectively.

The equation of continuity is

$$\frac{\partial v'}{\partial y'} = 0$$

$\therefore v'$ is independent of $y'$ and therefore $v'$ is a function of $t'$ only. We take $v'$ as

$$v' = -v_0 [1 + \epsilon A e^{\omega t'}]$$

where $v_0$ is the mean suction velocity, $\epsilon$ a small reference parameter, $A$ and $\omega$ are two constants such that $\epsilon A < 1$

The momentum equations are

$$\begin{align*}
\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} - 2 \Omega' v' &= -\frac{1}{\rho} \frac{\partial p'}{\partial x'} - g + v' \frac{\partial^2 u'}{\partial y'^2} - \frac{\nu u'}{k'} - \frac{\sigma B^2 u'}{\rho} \tag{3(b)2.1}
\frac{\partial v'}{\partial t'} + v' \frac{\partial v'}{\partial y'} + 2 \Omega' u' &= v' \frac{\partial^2 v'}{\partial y'^2} \frac{\nu v'}{k'} - \frac{\sigma B^2 v'}{\rho} \tag{3(b)2.2}
\end{align*}$$

where $\rho$ is the density, $\nu$ the kinematic viscosity, $p$ the pressure, $k'$ the permeability of porous medium, $g$ the acceleration due to gravity, $u'$ and $v'$ the velocity components along $x$-axis and $y$-axis, $\Omega'$ the rotation parameter and the $\sigma$ the electrically conductivity.

Now, we define a complex function $f'$ by

$$f' = u' + i v' \tag{3(b)2.3}$$

where $i = \sqrt{-1}$

Using 3(b)2.3 the equations 3(b)2.1 and 3(b)2.2 can be reduced to a single equation.
\[
\frac{\partial f'}{\partial t'} + v \frac{\partial f'}{\partial y'} + 2i\Omega f' = -\frac{1}{\rho} \frac{\partial p}{\partial x'} - g + v \frac{\partial^2 f'}{\partial y'^2} - \frac{v}{k'} f' - \frac{\sigma B^2_0 f'}{\rho} \tag{3(b).2.4}
\]

In the free stream, the equation 3(b)2.4 takes the form
\[
\frac{dU'}{dt'} + 2i\Omega U' = -\frac{1}{\rho} \frac{\partial p}{\partial x'} - \frac{\rho_u}{\rho} g - \frac{\nu U'}{k'} - \frac{\sigma B^2_0 U'}{\rho} \tag{3(b)2.5}
\]

where \(U'\) is the stream velocity and \(\rho_u\) is the density in the free stream.

\[
\therefore\ \text{Eliminating } \frac{\partial p}{\partial x'}\text{ from 3(b)2.4 and 3(b)2.5, we get}
\]
\[
\frac{\partial f'}{\partial t'} + v \frac{\partial f'}{\partial y'} + 2i\Omega f' = \frac{dU'}{dt'} + 2i\Omega U' + \frac{\rho_u - \rho}{\rho} g + \frac{v}{k'} (U' - f') + \frac{\nu B^2_0}{\rho} (U' - f') + v \frac{\partial^2 f'}{\partial y'^2} \tag{3(b)2.6}
\]

The volume expansion is given by
\[
\rho = \rho_u \left[1 - \beta (T' - T'_{\infty})\right] \tag{3(b)2.7}
\]

where \(T'\) is the temperature of the fluid in the mean stream and \(\beta\) the coefficient of volume expansion.

On using 3(b)2.7 in 3(b)2.6, we get
\[
\frac{\partial f'}{\partial t'} - \frac{dU'}{dt'} + v \frac{\partial f'}{\partial y'} + 2i\Omega (f' - U') \]
\[
= g\beta (T' - T'_{\infty}) + \frac{\nu \partial^2 f'}{\partial y'^2} - \frac{\sigma B^2_0}{\rho} (f' - v') - \frac{v}{k'} (f' - U') \tag{3(b)2.8}
\]
The energy equation is

\[
\frac{\partial T'}{\partial \tau'} + v' \frac{\partial T'}{\partial y'} = \frac{K}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2}
\]

where \(K\) the thermal conductivity and \(c_p\) the specific heat.

The relevant boundary conditions are:

at \(y' = 0\) : \(u' = -v_0 (1 + e^{Ae^{\omega i}})\), \(T' = T'_w\) \hspace{1cm} (3(b)2.10)

at \(y' \to \infty\) : \(u' = U' = U_0 (1 + e^{Ae^{\omega i}})\), \(T' \to T'_\infty\) \hspace{1cm} (3(b)2.11)

where

\(U_0\) is the mean stream velocity.

We introduce the following non-dimensional quantities:

\[
u = \frac{u'}{U_0}, \quad v = \frac{v'}{U_0}, \quad t = \frac{\tau' v_0^2}{v}, \quad y = \frac{y' v_0^2}{v}, \quad U = \frac{U'}{U_0}, \quad \omega = \frac{v_0}{v_0}, \quad \Omega = \frac{2\Omega' v}{v_0},
\]

\[
\theta = \frac{T' - T'_w}{T'_w - T'_\infty}, \quad p = \frac{\mu C_p}{k}, \quad G = \frac{v g \beta (T'_w - T'_\infty)}{U_0 v^2_0}.
\]

\[
K = \frac{v^2_0 k'}{v^2}, \quad M = \frac{\sigma B^2 v^2}{\rho v^2_0}, \quad q = \frac{v'}{U_0}, \quad f = \frac{f'}{U_0'}
\]

The nondimensional forms of the equations 3(b)2.8 and 3(b)2.9 are.

\[
\frac{\partial f}{\partial t} - \frac{dU}{\partial t} - \frac{1}{q} \frac{\partial f}{\partial y} + i\Omega (f - U) = G\theta + \frac{\partial^2 f}{\partial y'^2} - (f - U) \left( M + \frac{1}{k} \right)
\]

\[
P \left[ \frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} \right] = \frac{\partial^2 \theta}{\partial y'^2}
\]

The nondimensional boundary conditions are

at \(y = 0\) : \(f = -i q [1 + e^{Ae^{\omega i}}], \theta = 1\) \hspace{1cm} (3(b)2.14)

at \(y \to \infty\) : \(f \to 1 \to 1 + e^{Ae^{\omega i}}, \theta \to 0\) \hspace{1cm} (3(b)2.15)
3 (b)3. METHOD OF SOLUTIONS

To solve the equations 3(b)2.12 and 3(b)2.13 subject to the boundary conditions 3(b)2.14 and 3(b)2.15, we assume

\[ f = f_0(y) + e^{i\omega y} f_1(y) \]
\[ \theta = \theta_0(y) + e^{i\omega y} \theta_1(y) \]  

(3(b)3.1)

Substituting 3(b)3.1 in the equations 3(b)2.12 and 3(b)2.13 and equating the harmonic terms, we get

\[ f''_0 + f'_0 - Sf_0 = -G\theta_0 - S \]  
\[ f''_1 + f'_1 - (S+i\omega) f_1 = -(S+i\omega) - A f_0 - G \theta_0 \]  
\[ \theta''_0 + P\theta'_0 = 0 \]  
\[ \theta''_1 + P\theta'_1 - i\omega \theta_1 = 0 \]  

(3(b)3.2)  
(3(b)3.3)  
(3(b)3.4)  
(3(b)3.5)

where

\[ S = M + \frac{1}{k} + i\Omega \]

The boundary conditions 3(b)2.14 and 3(b)2.15 reduce to the following

at \( y = 0 \): \( f_0 = -i q \), \( f_1 = -i q A \), \( \theta_0 = 1 \), \( \theta_1 = 0 \)  

(3.3.6)

at \( y \rightarrow \infty \): \( f_0 = 1 \), \( f_1 = 1 \), \( \theta_0 = 0 \), \( \theta_1 = 0 \)

The solutions of the equations 3(b)3.2 to 3(b)2.5 subject to the boundary conditions 3(b)3.6 are

\[ \theta_0 = e^{-\gamma y} \]  
\[ \theta_1 = 0 \]  

(3(b)3.7)  
(3(b)3.8)

\[ f_0 = 1 - \frac{Ge^{-\gamma y}}{P^2 - P - S} + \left[ \frac{G}{P^2 - P - S} - i q \right] e^{\gamma y} \]

(3(b)3.9)
(65)

\[ f_1 = L \left[ 1 - e^{-\lambda_1 y} \right] + M' \left( e^{-\lambda_2 y} - e^{-\lambda_3 y} \right) + N \left( e^{-\lambda_4 y} - e^{-\lambda_5 y} \right) \]  

(3b.3.10)

where,

\[ L = 1 + \frac{A}{S + i\omega} \]

\[ M^* = \frac{AG}{(P^2 - P - s) (P^2 - P - s - i\omega)} \]

\[ N = \frac{A_i}{\omega} \left( \frac{G}{P^2 - P - s} - iq - 1 \right) \]

\[ \lambda_1 = \frac{-1 + \sqrt{1 + 4\omega}}{2}, \quad \lambda_2 = \frac{-1 - \sqrt{1 + 4\omega}}{2} \]

The skin friction at \( y = 0 \) is given by

\[ \tau = \tau_0 + \epsilon |B| \cos(\omega t + \alpha) \]  

(3b.3.11)

where

\[ \tau = \Gamma f'(0) \]  

(3b.3.12)

\[ |B| = \sqrt{B_r^2 + B_i^2} \]  

(3b.3.13)

\[ \tan \alpha = \frac{B_i}{B_r}, \quad Br = \Gamma f'(0) \]  

(3b.3.14)

\[ Bi = \text{Im} f'(0) \]
3(b) 4. DISCUSSION

The graphs for the skinfriction amplitude $|B|$ and the skinfriction phase $\tan \alpha$ given by the equations 3(b)3.13 and 3(b)3.14 respectively are presented in the figures 1, 2 and 3.

From Fig. 1. it is observed that $|B|$ increases as the suction parameter $A$. Same figure also indicates that for large values of $\omega$ the effect of $A$ on $|B|$ is negligible. We can conclude from fig. 1. that for small values of $\omega$, $|B|$ decreases whereas it increases steadily for large values of $\omega$.

It follows from fig. 2 that $|B|$ increases as the rotation parameter $\Omega$. It is also observed that $|B|$ tends to a finite values as $\omega \to \infty$. The effect of $\Omega$ on $|B|$ is negligible for large values of $\omega$. For small values of $\omega$, $|B|$ decreases as $\omega$ increases.

Fig. 3. indicates that the skinfrication phases $\tan \alpha$ increases as $A$ and the effect of $A$ is seen to be negligible for large values of $\omega$. For small values of $\omega$, $\tan \alpha$ decreases and it increases steadily for large values of $\omega$. 
Fig. 1: The skin friction amplitude $|B|$ against frequency parameter $w$ for the parameters

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Fig. 2: The skin friction amplitude $|B|$ against frequency parameter $w$ for the parameters
Fig. 3: The skin friction phase $\tan \alpha$ against frequency parameter $w$ for the parameters