CHAPTER II

Magnetohydrodynamic flow between two parallel plates, one in uniform motion and the other at rest with Uniform section at the stationary plate
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Magnetohydrodynamic flow between two parallel plates, one in uniform motion and the other at rest with uniform suction at the stationary plate.

2.1 INTRODUCTION:

The magneto hydro dynamic Couette flow between two parallel plates have been investigated by many workers. Hartman (1937) has studied the steady magnetohydrodynamic channel flow of a conducting fluid under a uniform magnetic field transverse to an electrically insulated channel wall. The problem of porous wall Couette type flow with superimposed axial pressure gradient has been discussed by Cramer (1959) and Lilley (1959). Mehta (1963) has studied the Couette type flow of an incompressible, viscous and infinitely electrically conducting fluid between two equally porous parallel planes under the assumption that the rate of suction at one wall is equal to the rate of injection of the fluid at the other wall. The Couette flow between two plates with small suction at the stationary plate have been studied by Verma and Bansal (1966). They have also obtained an exact solution of the Navier – Stoke’s equations reduced to second and third order non – linear differential equations with appropriate boundary conditions. They have shown that due to suction at the stationary plate an adverse pressure gradient is developed which causes a back – flow at large distance from the mouth of the channel. Recently Ghosh (2007) has discuss the Hydromagnetic fluctuating flow of viscoelastic fluid in a porous channel and has derived an exact periodic solution by assuming that on one plate the fluid is injected with certain velocity and that it is sucked of the other plate with the same velocity. Both plates are oscillating with a known velocity. He has employed perturbation method and discuss the effect of viscoelastic parameter, Cruss flow Reynold number, frequency parameter, and Hartman number on the velocity as well as wall shear stress of the flow.

In this problem are have discussed the Hydromagnetic Couette type flow with small uniform suction at the stationary plate. Exact solutions of the governing equations are obtained. The longitudinal and transverse velocity profiles for \( \lambda \) (Suction parameter) = 0.1, \( R \) (Reynolds number) = 100 and different values of \( M \) (Hartman number) are drawn. It is found that at a distance, depending on the Hartman number, an adverse pressure gradient
develops which causes a back flow. The longitudinal and transverse velocity profiles, the axial pressure increase and the variation of co-efficient of friction at the stationary plate are shown graphically.

2.2 **BASIC EQUATIONS**

We consider the flow of viscous incompressible and electrically conducting fluid between two horizontal parallel plates situated at a distance $y_0$ apart. The upper plate is moving with uniform velocity $U$ and the velocity at the stationary plate is uniform and small. A Cartesian coordinate system is used to represent the fluid motion with $x$-axis along the stationary plate, $y$-axis is perpendicular to it and a horizontal direction perpendicular to $x$-axis is considered as $z$-axis. Here the motion of the fluid does not depend on $z$-axis and the direction of the constant magnetic field is parallel to the axis of $y$. Let $u$ and $v$ be the components of velocity in the direction $x$ and $y$ respectively.

The equations governing the steady flow of fluids with Ohm’s law and Maxwell’s equations are

$$\rho \left( \nabla \cdot \mathbf{V} \right) = - \nabla p + \mu \nabla^2 \mathbf{V} + \mathbf{J} \times \mathbf{B}, \quad (2.2.1)$$

$$\text{curl} \ H = 4 \pi \mathbf{J}, \quad (2.2.2)$$

$$\text{div} \ \mathbf{B} = 0, \quad (2.2.3)$$

$$\text{curl} \ \mathbf{E} = 0, \quad (2.2.4)$$

$$\text{div} \ \mathbf{E} = 0, \quad (2.2.5)$$

$$\mathbf{J} = \sigma \left[ \mathbf{E} + \mathbf{V} \times \mathbf{B} \right], \quad (2.2.6)$$

and

$$\text{div} \ \mathbf{V} = 0, \quad (2.2.7)$$

where

$\mathbf{V}, \ H, \ \mathbf{B}, \ \mathbf{J}, \ \mathbf{E}, \ \rho, \ \sigma, \ \mu, \ and \ p$ denote respectively the velocity, magnetic field, magnetic field induction, electrical current density, electric
field intensity, density of the fluid, electric conductivity, coefficient of viscosity and pressure.

The fluid is assumed to be ionized and however within any small but finite volume the number of particles of positive and negative charges are nearly equal. The total excess charge density \( \theta \) and imposed electric field intensity \( E \) are assumed to be zero.

The equation (2.2.6) thus reduces to

\[
\vec{J} = \sigma (\vec{V} \times \vec{B}) \tag{2.2.8}
\]

and

\[
\vec{B} = B_0 + b.
\]

In this analysis \( b \) is considered as a perturbation on the basic field strength and negligible in comparison with \( B_0 \). Also a constant magnetic field of strength \( H_0 \) is applied perpendicular to the plate and fixed relative to them. We consider the following assumptions:

(i) Electrical conductivity \( \sigma_e \) of the fluid is sufficiently large so that the displacement current is neglected.

(ii) No external electric field is applied.

(iii) The secondary effects of magnetic induction are neglected

We assume as

Velocity = \( \vec{V} = (u, v, 0) \) \tag{2.2.9}

Magnetiv flux = \( \vec{B} = (0, B_0, 0) \) \tag{2.2.10}

Where \( B_0 = \mu_e H_0 \).

Then equations (2.2.1) and (2.2.7) transform to

\[
\oint \left[ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right] = -\frac{\partial \phi}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \sigma_e B_0^2 u \tag{2.2.11}
\]

\[
\oint \left[ \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} \right] = -\frac{\partial \phi}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \tag{2.2.12}
\]
and
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  \hspace{1cm} (2.2.13)

Since we have assumed that there is uniform suction
\[ \frac{\partial v}{\partial x} = 0 \]  \hspace{1cm} (2.2.14)

This shows that \( V \) is independent of \( x \) and a function of \( y \) only.

From equation (2.2.13) we have
\[ \frac{\partial u}{\partial x} = - \frac{\partial v}{\partial y} \]

or,
\[ \frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \left( - \frac{\partial v}{\partial y} \right) = 0 \]  \hspace{1cm} (2.2.15)

Using (2.2.14) and (2.2.15), equation (2.2.11), (2.2.12) and (2.2.13) then reduces to
\[ \frac{\partial}{\partial x} \left[ \frac{u}{\partial x} + \frac{u}{\partial y} \right] = - \frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \sigma_e B_0^2 u \]  \hspace{1cm} (2.2.16)

\[ \frac{\partial}{\partial y} \frac{\partial v}{\partial y} = - \frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial y^2} \]  \hspace{1cm} (2.2.17)

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  \hspace{1cm} (2.2.18)

With the boundary conditions:
\[ \begin{align*}
  u &= 0, \quad v = -v_0 \quad \text{at } y = 0 \\
  u &= U, \quad v = 0 \quad \text{at } y = y_0
\end{align*} \]  \hspace{1cm} (2.2.19)
Where \( v_0 \) is the velocity of suction.

Introducing non-dimensional variable as

\[
\begin{align*}
\chi &= \frac{\chi}{y_0}, \quad y = \frac{y}{y_0}, \quad u = \frac{u}{U}, \\
\tilde{\chi} &= \chi \quad \tilde{y} = y, \quad \tilde{u} = u.
\end{align*}
\]

Equation (2.2.16), (2.2.17) and (2.2.18) becomes

\[
\frac{\partial \tilde{u}}{\partial \tilde{\chi}} + \frac{\lambda}{R} \frac{\partial \tilde{u}}{\partial \tilde{y}} = - \frac{1}{\partial \tilde{\chi}} - \frac{1}{\partial \tilde{y}^2} \frac{\partial^2 \tilde{u}}{\partial \tilde{y} \partial \tilde{y}^2} R^2, \quad \tilde{p} = \tilde{p}(\tilde{u}, \tilde{v}), \quad \tilde{u}(\tilde{x}, \tilde{y}) = u_0 + \tilde{u}(\tilde{x}, \tilde{y}), \quad \tilde{v}(\tilde{y}) = v_1(\tilde{y}).
\]

Where

\[
\begin{align*}
R &= \frac{Uy_0}{v} \quad \text{is the Reynolds number}, \\
M &= \frac{\mu_e h_0}{y_0} \left( \frac{\sigma_e}{\rho} \right)^{1/2} \quad \text{is the Hartmann number}, \\
\lambda &= \frac{v_0 y_0}{v} \quad \text{is the suction parameter}.
\end{align*}
\]

The boundary conditions for velocity which reduces to

\[
\tilde{u} = 0, \quad \tilde{v} = 1 \quad \text{at} \quad \tilde{y} = 0
\]

And

\[
\tilde{u} = 1, \quad \tilde{v} = 0 \quad \text{at} \quad \tilde{y} = 1
\]

Let

\[
\tilde{p}(\tilde{x}, \tilde{y}) = p_0(x) + p_1(x) (\tilde{x}, \tilde{y}), \\
\tilde{u}(\tilde{x}, \tilde{y}) = u_0(x) + u_1(x) (\tilde{x}, \tilde{y}), \\
\tilde{v}(\tilde{y}) = v_1(\tilde{y}).
\]
Where \( p_1, u_1, v_1 \) are the perturbations caused by the suction and \( p_0, u_0 \) are the known quantities for the flow when there is no suction satisfying the following equations

\[
\frac{\partial p_0}{\partial \chi} = 0, \quad \frac{\partial p_0}{\partial y} = 0, \quad \frac{\partial u_0}{\partial \chi} = 0, \quad \frac{\partial^2 u_0}{\partial y^2} - M^2 u_0 = 0
\] (2.2.27a) (2.2.27b) (2.2.27c) (2.2.27d)

Therefore we have

\[ p_0 = \text{constant} \] (2.2.28a)

The solution of equation (2.2.27d) using the boundary condition (2.2.25) is given by

\[ u_0 = \frac{\text{Sinh}M}{\text{Sinh}M} \quad \text{y} \]

(2.2.28b)

Using (2.2.25) - (2.2.28b) equations (2.2.21), (2.2.22) and (2.2.23) on dropping the bars, becomes

\[
\left( \frac{\text{Sinh}M y}{\text{Sinh}M} + u_1 \right) \frac{\partial u_1}{\partial \chi} - \frac{\lambda}{R} \left( \frac{\text{Sinh}M}{\text{Sinh}M} + \frac{\partial u_1}{\partial y} \right) = - \frac{\partial p_1}{\partial \chi} \]

\[
1 \frac{\partial^2 u_1}{\partial y^2} - \frac{M^2}{R} u_1
\]

(2.2.29)
$$\frac{\lambda^2}{R^2} \frac{\partial u_1}{\partial y} + \frac{\partial p_1}{\partial y} = \frac{\lambda}{R^2} \frac{\partial^2 v_1}{\partial y^2}$$  \hspace{1cm} (2.2.30)$$

and

$$\frac{\partial u_1}{\partial \chi} - \frac{\lambda}{R} \frac{\partial v_1}{\partial y} = 0$$  \hspace{1cm} (2.2.31)$$

With boundary conditions

$$u_1 = 0, v_1 = 1 \text{ at } y = 0$$
$$u_1 = 0, v_1 = 0 \text{ at } y = 1$$

(2.2.32)

2.3 METHOD OF SOLUTION:

Let $v_1 = f(y)$  \hspace{1cm} (2.2.33)

Then integrating the equation (2.2.31) with respect to $x$, we get

$$u_1 = \frac{\lambda}{R} \chi f'(y) + F(y)$$  \hspace{1cm} (2.2.34)$$

where $f(y)$ and $F(y)$ are to be determined.

Using (2.2.33) and (2.2.34), equations (2.2.29) and (2.2.30) becomes

$$\begin{align*}
\left[ \frac{\text{Sinh} My}{\text{Sinh} M} + \frac{\lambda}{R} \chi f' + F \right] \frac{\lambda}{R} f' - \frac{\lambda}{R} f & = \left[ \frac{\text{M} \text{Cosh} My}{\text{Sinh} M} + \frac{\lambda}{R} \chi f'' + F' \right] \\
\left[ \frac{\lambda}{R} \chi f'' + F' \right] & = - \frac{\partial p_1}{\partial \chi} + \frac{1}{R} \left[ \frac{\lambda}{R} \chi f''' + F'' \right] - \frac{M^2}{R} \left[ \frac{\lambda}{R} \chi f' + F \right] \\
\lambda^2 ff' & = - \frac{\partial p_1}{\partial y} + \frac{\lambda}{R^2} f'' 
\end{align*}$$

(2.2.35) 

(2.2.36)
Due to absence of external pressure gradient, we have from equation (2.2.35)

\[ F'' - M^2 F + \lambda \left[ \frac{M \text{Cosh} M y}{\text{Sinh} M} f - f' \frac{\text{Sinh} M y}{\text{Sinh} M} \right] = 0 \]  

(2.2.37)

Differentiating the equation (2.2.36) with respect to x, we get

\[ 0 = - \frac{\partial^2 p_1}{\partial x \partial y} \]

or,

\[ \frac{\partial^2 p_1}{\partial x \partial y} = 0 \]  

(2.2.38)

Again differentiating (2.2.35) with respect to y and using equations (2.2.37) and (2.2.38), we obtain

\[ \frac{d}{dy} \left[ f''' - M^2 f' + \lambda \{ f f'' - f'^2 \} \right] = 0 \]  

(2.2.39)

Integrating the equation (2.2.39) we get

\[ f''' - M^2 f' + \lambda \{ f f'' - f'^2 \} = C \]  

(2.2.40)

Where C is the constant of integration to be determined. The boundary conditions for f and F are

\[ \begin{cases} f(0) = 1, f'(0) = 0, F(0) = 0 \\ f(1) = 0, f'(1) = 0, F(1) = 0 \end{cases} \]  

(2.2.41)

**SOLUTION FOR SMALL \( \lambda \):**

The solution for the equation (2.2.40) can be expressed for small values of \( \lambda \) by a power series developed near \( \lambda = 0 \) as follows:

\[ f = f_0 + \lambda f_1 + \lambda^2 f_2 + \ldots + \lambda^n f_n \]  

(2.2.42)

and

\[ C = C_0 + \lambda C_1 + \lambda^2 C_2 + \ldots + \lambda^n C_n \]  

(2.2.43)
Where \( f_n'' \) and \( C_n'' \) are taken independent of \( \lambda \).

Using equations (2.2.42) and (2.2.43) in the equation (2.2.40) and then equating the co-efficients of like powers of \( \lambda \), we obtain

\[ f_0''' - M^2 f_0' = C_0 \]  \hspace{1cm} (2.2.44)

and

\[ f_1''' - M^2 f_1 + f_0 f_0'' - f_0'^2 = C_1 \]  \hspace{1cm} (2.2.45)

The boundary conditions are

\[
\begin{aligned}
  f_0(0) &= 1, & f_n(0) &= 0 & \text{for } n \geq 1 \\
  f_n(1) &= 0, & f_n'(0) &= 0, & f_n'(1) &= 0, & \text{for } n \geq 0 \\
\end{aligned}
\]  \hspace{1cm} (2.2.46)

The solution of the equation (2.2.44) satisfying the boundary conditions (2.2.46) is

\[ f_0(y) = k_1 + k_2 y + k_3 \cosh My + k_4 \sinh My \]  \hspace{1cm} (2.2.47)

Where

\[
\begin{aligned}
  K_1 &= \frac{C_0 M - M^3 \cosh M - C_0 \sinh M}{M^3 (1 - \cosh M)}, \\
  K_2 &= -\frac{C_0}{M^2}, \\
  K_3 &= \frac{M^3 - C_0 M + C_0 \sinh M}{M^3 (1 - \cosh M)}, \\
  K_4 &= -\frac{C_0}{M^3}, \\
\end{aligned}
\]

and

\[
\begin{aligned}
  C_0 &= \frac{M^3 \sinh M}{M \sinh M + 2 (1 - \cosh M)}.
\end{aligned}
\]
Also the solution of the equation (2.2.45) using (2.2.47) and the boundary conditions (2.2.46) is

\[
f_1(y) = \alpha_1 + \alpha_2 \cosh My + \alpha_3 \sinh My \quad \frac{\{C_1 + K_2^2 - M^2 (K_3^2 - K_4^2)\}}{M^2} \quad y + \frac{(7K_2 K_4 - 2M K_1 K_3)}{4M} \quad y \cosh My + \frac{(7K_2 K_3 - 2M K_1 K_4)}{4M} \quad y \sinh My - \frac{K_2 K_3}{4} \quad y^2 \cosh My - \frac{K_2 K_4}{4} \quad y^2 \sinh My \quad (2.2.48)
\]

Where

\[
\begin{align*}
\alpha_1 &= \frac{\{C_1 + K_2^2 - M^2 (K_3^2 - K_4^2)\} - \frac{(7K_2 K_4 - 2M K_1 K_3 - MK_2 K_3)}{M^2 (1 - \cosh M)}}{4M(1 - \cosh M)} - \frac{\sinh M}{M^2} \left[ \frac{K_2 K_3 M^2 - 2K_1 K_4 M^3 - K_2 K_3 M^3 - 7K_2 K_4 M + 2M^2 K_1 K_3 + 4 \{C_1 + K_2^2 - M^2 (K_3^2 - K_4^2)\}}{4M^3 (1 - \cosh M)} \right], \\
\alpha_2 &= -\alpha_1, \\
\alpha_3 &= \frac{4 \{C_1 + K_2^2 - M^2 (K_3^2 - K_4^2)\} - 7M K_2 K_4 + 2M^2 K_1 K_3}{4M^3}
\end{align*}
\]

and

\[
C_1 = \frac{M \sinh M}{4 \left[ M \sinh M + 2(1 - \cosh M) \right]} - \frac{\cosh M (2K_2 K_4 M + 2K_1 K_4 M - 7K_2 K_3)}{4 \left[ M \sinh M + 2(1 - \cosh M) \right]} + \sinh M (2K_1 K_3 M + 2K_2 K_3 M - 7K_2 K_4) + \{7K_2 K_3 - 2K_1 K_4 M +
\]
Thus from (2.2.42), (2.2.43), (2.2.47) and (2.2.48) we have for the first order perturbation,

\[ f(y) = f_0(y) + \lambda f_1(y) \]

\[ = (K_1 + \lambda \alpha_1) + \left[ \frac{K_2 M^2 - \lambda C_1 - \lambda K_2^2 + \lambda M^2 (K_3^2 - K_4^2)}{M^2} \right] y + \]

\[ + (K_3 + \lambda \alpha_2) \cos \lambda My + (K_4 + \lambda \alpha_3) \sin \lambda My + \]

\[ + \frac{7K_2K_4 - 2K_1K_3M}{4M} y \cos \lambda My + \frac{7K_2K_3 - 2K_1K_4M}{4M} y \sin \lambda My \]

\[ + \frac{K_2K_3}{4} y^2 \cos \lambda My - \frac{K_2K_4}{4} y^2 \sin \lambda My \]  \hspace{1cm} (2.2.49)

and

\[ C = C_0 + \lambda C_1 \]  \hspace{1cm} (2.2.50)

Also the solution for the equation (2.2.37) can be expressed for small values of \( \lambda \) by a power series developed near \( \lambda = 0 \) as follows:

\[ F = F_0 + \lambda F_1 + \lambda^2 F_2 + \ldots + \lambda^n F_n \]  \hspace{1cm} (2.2.51)

Substituting the values of \( f \) and \( F \) from (2.2.42) and (2.2.51) in the equation (2.2.37) and then equating the co-efficients of like powers of \( \lambda \), we get
\[ F_0'' - M^2 F_0 = 0 \]  \hspace{2cm} (2.2.52)

and

\[ F_1'' - M^2 F_1 + \frac{MC\cosh M}{\sinh M} f_0 + F_0' f_0 - \frac{\sinh M}{\sinh M} f_0' - F_0 f_0' = 0 \]  \hspace{2cm} (2.2.53)

With the boundary conditions

\[
\begin{align*}
  f_n(0) &= 0, \quad \text{for } n \geq 0 \\
  f_n(1) &= 0, \quad \text{for } n \geq 0
\end{align*}
\]  \hspace{2cm} (2.2.54)

The solution of the equation (2.2.52) using the boundary conditions (2.2.54) is

\[ F_0(y) = 0 \]  \hspace{2cm} (2.2.55)

Also the solution of the equation (2.2.53) using (2.2.55) and the boundary conditions (2.2.54) is

\[ F_1(y) = A_1 \cosh My + A_2 \sinh My - \frac{K_1}{2 \sinh M} y \sinh My + \frac{3K_2}{4 M \sinh M} \cosh My - \frac{K_2}{4 \sinh M} y^2 \sinh My + \frac{K_3}{M \sinh^2 M} \]  \hspace{2cm} (2.2.56)

where

\[ A_1 = - \frac{K_3}{M \sinh M} \]

and

\[ A_2 = \frac{(2K_1 + K_2)}{4 \sinh M} - \frac{(3K_2 - 4K_3)}{4 M \sinh^2 M} \cosh M - \frac{K_3}{M \sinh^2 M} \]

Thus from (2.2.51), (2.2.55) and (2.2.56), we have

\[ F(y) = F_0(y) + \lambda F_1(y) \]
\[
\begin{align*}
\lambda &= A_1 \cosh \lambda y + A_2 \sinh \lambda y - \frac{K_1}{2 \sinh \lambda y} + \\
&\quad + \frac{3K_2}{4 \sinh \lambda y} - \frac{K_2}{4 \sinh \lambda y} - \frac{K_3}{\sinh \lambda y} \\
&= \lambda \left[ A_1 \cosh \lambda y + A_2 \sinh \lambda y - \frac{K_1}{2 \sinh \lambda y} + \frac{3K_2}{4 \sinh \lambda y} - \frac{K_2}{4 \sinh \lambda y} - \frac{K_3}{\sinh \lambda y} \right] 
\end{align*}
\]

Therefore using equations (2.2.28b), (2.2.33), (2.2.34), (2.2.49) and (2.2.56), equation (2.2.26) on dropping the bars becomes

\[
u (x, y) = U_0 + U_1 (x, y)
\]

\[
\text{Or, } u = \frac{\sinh \lambda y}{\sinh \lambda y} + \frac{\lambda}{R} \frac{f'(y)}{y} + \frac{\lambda}{R} F_1 (y)
\]

and \(v = f(y)\)

Where \(f(y)\) and \(F_1(y)\) are obtained from equations (2.2.49) and (2.2.56).

From (2.2.35) and (2.2.36), the pressure distribution is

\[
a(0, 0) - a(x, y) = \frac{\lambda}{R^2} f' - \frac{\lambda C}{2 R^2} \chi^2
\]

Where \(C\) is obtained from the equation (2.2.50)

The pressure increase in the direction of \(x\) is given by

\[
a(x, y) - a(0, y) = \frac{\lambda C}{2 R^2} \chi^2
\]

At the lower plate, the shearing stress is

\[
\tau_0 = \mu \frac{U}{y_0} \left( \begin{array}{c}
\frac{\partial u}{\partial y} \\
\frac{\partial y}{\partial y} = 0
\end{array} \right)
\]
And the co-efficient of skin friction is given by

\[
C_f = \frac{2 T_0}{\rho U^2}
\]

\[
= 2 \frac{\mu}{\rho U y_0} \left( \frac{\partial u}{\partial y} \right)_{y = 0}
\]

\[
= \frac{2}{R} \left( \frac{\partial u}{\partial y} \right)_{y = 0}
\]

\[
= \frac{2}{R} \left[ \frac{M}{\sinh M} + \frac{3k_2 \lambda}{4 M \sinh M} + \frac{k_3 M^2 \lambda}{R} \right]
\]

where \( R = \frac{U y_0}{\nu} \), \( \nu = \frac{\mu}{\rho} \)

4. **RESULTS AND DISCUSSION**:

The longitudinal and transverse velocity profiles for \( \lambda = 0.1 \), \( R = 100 \), \( M = 5 \) at various cross-sections of the channel are shown in figure (1.1) and they are compared with the case without magnetic field i.e. \( M = 0 \). It is also noticed that due to a small suction at the stationary plate a flow from large values of \( x \) towards the mouth of the channel is developed near the stationary plate. In the presence of magnetic field, the back flow occurs right up to the small distances from the mouth of the channel. If the Hartman number is small, viscosity dominates and the velocity profile is similar to the case of non-conducting fluid.
When the Hartmann number increases, the region of back flow is extended from the stationary plate towards the upper plate and it is shown in the figure (1.2) for various values of the Hartmann number and $\lambda = 0.1$, $R = 100$ and $x = 100$. This is due to the fact that viscosity becomes unimportant as the Hartmann number increases except near the upper plate.

In the figure (1.3) we have shown the pressure distribution for $\lambda = 0.1$, $R = 100$ and $M = 0, 1$. The pressure in the main flow direction is parabolic and it is also observed that this pressure increase decreases with the increase of Hartmann number.

The coefficient of skin friction at the stationary plate is shown in the figure (1.4) for $R = 100$ and for different values of $\lambda = 0.1, 0.2, 0.5$ and $M = 0, 1, 2$. We have also found that the coefficient of skin friction decreases with the increase of suction parameter or Hartmann number.
Fig. 11 Longitudinal and transverse velocity profiles plotted against $y$. $x=0^1$, $R=100$, $M=5$ -- and $M=0$ ---
Fig. 12. Longitudinal velocity profile plotted against $y$ ($\lambda = 0.1$, $R = 100$ and $x = 100$).
Fig. 1.3 Axial pressure increase vs. length in flow direction for λ=0.1 and R=100.