Chapter – II

Methodology
2.1. THE HISTORY OF OPERATION RESEARCH

It is generally agreed that operations research came into existence as a discipline during World War II when there was a critical need to manage scarce resources. However, a particular model and technique of OR can be traced back to much earlier times. The term ‘Operations research’ was coined as a result of research on military operations during this war. Since the war involved strategic and tactical problems, which were greatly complicated, to expect adequate solutions from individuals or specialists in a single discipline was unrealistic. The fundamental mathematical foundation on programming problems was first established by Von Neumann in 1932 (published in 1937) and the major use of mathematical programming (MP) to practical problems was presented by Kantorovitch in 1939. The great development in this area was there during the mid 1950s and 1960s of last century by the pioneer researchers in the field of OR and Management Sciences. The mathematical model for programming problems and their constructive solution procedure for optimal allocation of resources were first presented by Hitchcock in 1941. Therefore groups of individuals who collectively specialists in mathematics, economics, statistics and probability theory, engineering, behavioral, and physical science were considered to formed as special units within the armed forces to deal with strategic and tactical problems of various military operations.
Such groups were first formed by the British Air Force and later, the American armed forces formed similar groups. One of the groups in British came to be known as Blackett’s Circus. This group, under the leadership of Prof P M S Blackett was attached to the Radar Operational Research unit and was assigned the problem of analyzing the coordination of radar equipment at gun sites. The effort of such groups, especially in the area of radar detection is considered vital in Britain winning the air battle. Following the success of this group, such a mixed-team approach was also adopted in other allied nations.

After the war ended, scientist who had been active in the military OR groups made efforts to apply the operations research approach to civilian problems, related to business, industry, research and development etc. There were three important factors behind the rapid development in the use of operations research approach.

i) The economic and industrial boom after World War II resulted in continuous mechanization, automation, decentralization of operations and division of management functions. This industrialization also resulted in complex managerial problems, and therefore application of operations research to managerial decision-making problem became popular.

ii) Many operations researchers continued their research after war. Consequently, some important advancement was made in various operations research techniques. A key person in the
post-war development of OR was George B Dantzig. In 1947, he developed the concept of linear programming and its solution by a method known as simplex method. Besides linear programming, many other techniques in OR, such as statistical quality control, dynamic programming, queuing theory and inventory theory developed before the end of the 1950s. Analytic power was made available by high speed computers.

The use of computers made it possible to apply many OR techniques for practical decision analysis.

During the 1950s, there was substantial progress in the application of OR techniques for civilian activities along with a great interest in the professional development and education in OR. Many colleges and universities introduced OR in their curricula. They were generally schools of engineering, public administration, business management, applied mathematics, economics, computer science etc. Today, however, service organization such as banks, hospitals, libraries, airlines, railways etc recognize the usefulness of OR in improving efficiency. In 1948, an OR club was formed in England which later changed its name to the Operational Research Society of UK. Its journal, OR Quarterly, first appeared in 1950. The Operations Research Society of America (ORSA) was founded in 1952 and its journal, Operational Research was first published in 1953. In the same year, The Institute of Management Science (TIMS) was founded as an international society to identify, extend
and unify scientific knowledge pertaining to management. Its journal, Management Science, first appeared in 1954.

In India, operational research came into existence in 1949 when an OR unit was established at Regional Research Laboratory, Hyderabad for planning and organizing research. At the same time Prof. R S Verma also set up an OR team at Defence Science Laboratory to solve problems of store, purchase and planning. In 1953, Prof. P C Mahalanobis established an OR team in the Indian Statistical Institute, Calcutta to solve problems related to national planning and survey. The OR Society of India (ORSI) was formed in 1957 and later started publishing its journal OPSEARCH from 1964. In the same year, India along with Japan became a member of the International Federation of Operational Research Societies (IFORS) with its headquarters in London. The other members of IFORS were UK, USA, France and West Germany.

A year later, project scheduling techniques: Program Evaluation and Review Technique (PERT) and Critical Path Method (CPM) were developed as efficient tools for scheduling and monitoring lengthy, complex and expensive project of that time. By the 1960s OR groups were formed in several organizations. Educational and professional development programmes were expanded at all levels and certain firms specializing in decision analysis were also formed. The American Institute for Decision Sciences came into existence in 1967. It was formed to promote, develop and apply quantitative approach to functional and
behavioral problems of administration. It started publishing a journal, Decision Science, in 1970.

Because of OR’s multi-disciplinary character and application in varied fields, it has a bright future, provided people devoted to OR study can held and meet the needs of society. Some of the problems in the area of hospital management, University management, energy conservation, environmental pollution, etc. have also been solved by OR specialists and this is an indication that OR can also contribute towards the improvement in the social life and areas of global need. However, in order to make the future of OR brighter, its specialists have to make good use of the opportunities available to them.

2.2. MODEL AND MODELING OPERATION RESEARCH:

Both simple and complex systems can easily be studied by concentrating on some portion or key features instead of concentrating on every detail of it. The approximation or abstraction maintaining only the essential elements of the system, which may be constructed in various forms by developing relationship among variables and parameters of the system is called a model. In general models attempt to describe the necessity a situation by abstracting from reality so that the decision maker can study the relationship among the relevant variables models never attempt to duplicate reality in all aspects. Hence, models do not and cannot represent every aspect of reality because of innumerable and changing
characteristics of real life problems to be represented. Instead, they are limited approximation of reality.

From the above discussion we arrived at a conclusion that a model is constructed to analyze and understand the given system for the purpose of improving its performance. The reality of the solution obtained from a model depends on the validity of the model under study. A model, however, allows the opportunity to test the behavioral changes of a system without hampering the ongoing operation.

The key to model building lies in abstracting only the relevant variables that affect the criteria of the measure of performance of the given system and expressing the relationship in a suitable form.

2.3. ADVANTAGES OF MODEL:
Models are used to represent complex problem in a similar way. It also serves the following purposes:

a) A model provides economically the representation of the realities of the system i.e. model helps the decision maker to visualize a system so that he can understand the system structure or operation in better way.

b) The system can be viewed in its entirety, with all the components being considered simultaneously.

c) A Model transmits ideas and visualization among people in understanding the system
d) A model allows us to analyze and experiment in complex situations
to a degree that would be impossible in the actual system and
environment.
e) The models make easier the investigation and provide a powerful
tool for predicting the future state of the process.

2.4 APPLICATION AND SCOPE OF OPERATION RESEARCH

We find the use of operation research in all the fields. Some of the
industrial / government / business problem which can be analyzed by OR
technique which arranged by functional areas as follows:

a) Finance and Accounting.
b) Marketing .
c) Purchasing, Procurement and Exploration.
d) Production Management.
e) Manufacturing, Maintenance and Project Scheduling.
f) Personal Management.
g) Techniques and General Management.

2.5. GENERAL STRUCTURE OF LINEAR PROGRAMMING MODEL:

The General structure of LP Model consists of basic components namely,
Decision variables, Objective function and constraints.
Decision Variable:
Variable which are used to arrive at an optimal solution are called decision variables.

The Objective Function:
The associated function of an LP problem is expressed in terms of decision variables which is to be optimized (Maximized or minimized) is called the objective function.

Constraints:
There is always certain limitation on the use of resources that limit the degree to which an objective can be achieved. Such constraints must be expressed as linear equalities or inequalities in terms of decision variables. The solution of LP model must satisfy these constraints.

2.6. ASSUMPTIONS OF LINEAR PROGRAMMING PROBLEM:
There are basic assumptions, which are necessary for all linear programming models namely, certainty, continuity and linearity.

Certainty:
In all types of LP models it is considered that all model parameters such as availability of resources in the tune of a unit of decision variable and consumption of resources by every unit of decision variable must be
known. In some cases they are constant and in some cases they be either random variable represented by a known distribution.

**Continuity:**

The values of the decision variables are allowed to assume continuous or discrete values for instant if the number of machine is 2.5, then such values are not acceptable and therefore must be assigned integer values. In such situations to get rid of fractional values of decision variables, for which we used integer programming.

**Linearity:**

In the present modeling work we consider the objective function and all the constraints to bear a linear relationship of decision variables and LP model to be deterministic type.

### 2.7. GENERAL MATHEMATICAL MODEL OF LPP:

The General linear programming model with n decision variables and m constraints can be put in the following form.

Calculate the value of decision variables $X_1, X_2, X_3, \ldots, X_n$ so as to

$$\text{Optimize } Z = c_1X_1 + c_2X_2 + \ldots + c_nX_n.$$
Subjected to linear constrains,

\[ a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1 \]
\[ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2 \]
\[ \vdots \]
\[ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m \]

and \( x_1, x_2, \ldots, x_n \geq 0 \)

Here \( c_1, c_2, \ldots, c_n \) are coefficients representing the per unit contribution of respective decision variables \( x_1, x_2, \ldots, x_n \) to the value of objective function. \( a_i \) are called technological coefficients or input-output coefficients. \( b^T = [b_1, b_2, \ldots, b_m] \) is called requirement of \( i^{th} \) constraint. In general LPP, \( x_i \geq 0, i=1,2,\ldots,n \) are called non-negativity of the decision variable out of three inequality means that in a specific problem each constrain may take any one of them.
2.8. Simplex Algorithm

For the solution of any LPP by the simplex algorithm, the existence of an initial basic feasible solution is always assumed. The steps for the computation of an optimum solution are as given below:

i) We check the objective function of the given LPP is to be maximized or minimized. If it is to be minimized then we convert to a problem of maximization by using the result

\[
\text{Minimum } z = - \text{ Maximum } (-z)
\]

ii) We check whether all \( b_i \) (\( i = 1, 2, \ldots, m \)) are non-negative. If any of the \( b_i \) is negative then we multiply the corresponding inequality by \(-1\) to convert all \( b_i \) (\( i = 1, 2, \ldots, m \)) non-negative.

iii) We convert all constraint inequality into equations by introducing slack and/or surplus variable in the constraints and we put the cost of these variable equal to zero in the objective function.

iv) We obtain an initial basic feasible solution to the problem and put it in the first column in the simplex table.

v) Now we calculate the net evaluation \( z_j - c_j \) (\( j = 1, 2, \ldots, n \)) by using the relation \( z_j - c_j = c_ay_j - c_j \) and examine the sign of \( z_j - c_j \).

a) If all \( z_j - c_j \geq 0 \) then initial basic feasible solution is an optimum basic feasible solution.

b) If all \( z_j - c_j \leq 0 \) then we proceed to the next step.
vi) If there are more than one negative $z_i - c_i$, then we choose the most negative to them. Let it be $z_r - c_r$ for some $j = r$.

a) If all $y_i \leq 0$ ($i = 1, 2, \ldots, m$), then there is an unbounded solution of the given problem.

b) If all at least one $y_r > 0$ ($i = 1, 2, \ldots, m$), then the corresponding vector $y_r$ enters the basis $y_B$.

vii) Now we compute the ratios

$$\left\{ \frac{x_{Bi}}{y_{Ir}}, \quad y_{Ir} > 0, \quad i = 1, 2, \ldots, m \right\}$$

and see the minimum of them. Let the minimum of these ratios be $x_{Bk} / y_{kr}$ then $y_k$ will leave the basis $y_B$. The common element $y_{kr}$ which is in the $k^{th}$ row and $r^{th}$ column is known as pivotal element of the table.

viii) Convert the leading element to unity by dividing its row elements by the leading element itself and all other elements in its column to zeros by making use of the relation

$$\hat{y_i} = \frac{y_i - y_{ki}}{y_{kr}} \quad y_{Ir}, \quad i = 1, 2, \ldots, m + 1, \quad i \neq k$$

and $\hat{y_{kj}} = y_{kj} / y_{kr}$

j = 0, 1, 2, \ldots, n

ix) We go to step V and repeat the computational procedure until either an optimum solution is obtained or there is an indication of unbounded solution.
2.9. ARTIFICIAL VARIABLE TECHNIQUE

In the previous situations the slack variables only provided the initial basic feasible solution but there are many situations where slack variables only can not provide such solutions. If any of the constraints of LPP is of the type $\geq$ or $\leq$, then we have to introduce artificial variable which will provide initial basic feasible solutions.

There are two methods of solving this type of problems

a) The “Big-M-Method” or the Method of Penalties.

b) The “Two Phase Method” due to Dantzig, Orden & Wolfs.

2.10. TWO PHASE SIMPLEX METHOD:

The two phase is an alternative approach to solve a given LPP in which some artificial variables are involved. In the first phase the simplex method is applied to a specially constructed LPP leading to a final simplex table that contains basic feasible solutions to the original LPP. In the second phase, then leads from the basic feasible solution determined by first phase to an optimum basic feasible solution, if any, by an application of simplex method.

The procedure of solving a LPP by two-phase method is given below:

i) We ensure that all $b_i$ are non-negative. If any one of them is negative we make them non-negative by multiplying both side by $-1$. 
ii) We add artificial variables to the constraint / constraints of the type \( \geq \) or \( = \).

iii) We express the LPP in the standard form to obtain initial basic feasible solution.

iv) We assign a cost \(-1\) to the artificial variables and a 0 cost to all other variables in the objective function. Then new objective function is

\[
z^* = -A_1 - A_2 - A_3 \ldots - A_p \quad (\text{where } A_i, i=1,2, \ldots, p) \text{ are the artificial variables.}
\]

v) We write down the auxiliary LPP in which new objective function is to be maximized under given certain constraints.

vi) We solve the auxiliary LPP by simplex method until any one of the following cases arises:

a) \( \text{Max } z^* < 0 \) and at least one artificial vector appear in the optimum basis at positive level.

b) \( \text{Max } z^* = 0 \) and at least one artificial vector appear in the optimum basis at 0 level.

c) \( \text{Max } z^* = 0 \) and no artificial vector appear in the optimum basis.

In case (a) given LPP does not provide any feasible solution where as in cases (b) & (c) we proceed on to phase 2.
Phase 2: We use the optimal table of phase 1 containing optimal solution as the starting solution of the original LPP and we assign the actual cost to the original variable and 0 cost to the artificial variables in the objective function. After that we applied the simplex method to solve it.

2.11. POST OPTIMAL ANALYSIS OF LP MODEL:

Even after having the optimal solution of original LPP if we allow slight changes either of the parameter or structure of given LPP study the optimality of the new problem is called post optimal analysis or we may say sensitivity analysis. Under this post optimal analysis we can include the following:

a) Changes in the requirement vector b.

b) Changes in the cost vector c.

c) Change in the technological coefficient.

After the above changes have been effected in LPP we may have the following cases:

h) The optimal solutions remain invariant.

i) Basic variables remain the same but their values undergo changes.

j) Basic solutions get changed entirely.
2.12. CHANGES IN COST VECTOR

Let $X_B$ be the optimum basic feasible solution of LPP,

Maximize $Z = CX$ subjected to constraints $AX = b$ and $X \geq 0$

If $C_B$ be the cost vector associated with optimum basic feasible solution $X_B$ then any changes in $C$ may definitely bring some change in the optimality condition. There are two possibilities

i) $C_K$ is not in $C_B$

ii) $C_K$ is in $B$

In first case condition of optimality is $\Delta C_K \leq Z_K - C_K$, in second case condition of optimality is

$$\begin{align*}
\text{Max.} & \quad - (Z_j - C_j) \\
& \quad \text{Min.} \quad - (Z_j - C_j)
\end{align*}$$

2.13. CHANGE IN THE REQUIREMENT VECTOR

Let $X_B$ be the optimum basic feasible solution of LPP,

Maximize $Z = CX$ subjected to constraints $AX = b$ and $X \geq 0$
The condition of optimality for changes in $b_k$ is

$$\begin{align*}
\text{Max} & \quad \left\{ -x_{B_t} \right\} \leq \Delta b_k \leq \left\{ -x_{B_t} \right\} \\
& \text{for } b_k > 0 \\
& \text{for } b_k < 0
\end{align*}$$

2.14. INTRODUCTION TO REPLACEMENT PROBLEM

The problem of replacement is felt when the job performing units such as men, machine, equipments etc. become less effective or useless due to certain failure or gradual deterioration in their efficiency. By replacing those with new one at frequent intervals, maintenance and other overhead cost can be reduced. However such replacement would increase the need of capital cost for new ones.

Thus the basic problem in such situation to formulate a replacement policy to determine an age at which replacement is most economical keeping in view all possible alternatives.

2.15. REPLACEMENT OF AN ITEM DETORIATE WITH TIME

Normally the cost of maintenance and repair of certain equipments increases with time and a stage may come when this maintenance cost become so high that it is more wise to replace the item by a new one. Here we discuss the case where time ‘t’ is considered as discrete variable.
C: Capital cost of an equipment

S': Scrap value of equipment

n: No. of years that equipment is in use

f(t): The maintenance cost function

A(n): Average total annual cost.

We take the value of t = 1, 2, 3, ................. then

\[ A(n) = \frac{C - S}{n+1} + \frac{1}{n} \sum_{t=1}^{n+1} f(t) \]

Now, we wish to minimize A(n) for that value of n for which

A(n + 1) ≥ A(n) and A(n - 1) ≥ A(n)

For this we write

\[ A(n+1) = \frac{C - S}{n + 1} + \frac{1}{n+1} \sum_{t=1}^{n+1} f(t) \]

\[ = \frac{1}{n+1} \left[ C - S + \sum_{t=1}^{n} f(t) \right] + \frac{1}{n+1} f(n+1) \]

\[ = \frac{1}{n+1} \left[ A(n) + f(n+1) \right] \]
Therefore, \( A(n+1) - A(n) = \frac{1}{n+1} \sum_{i=n+1}^{f(n+1) - f(n)} \)

Thus \( A(n+1) - A(n) \geq 0 \) which implies that, \( f(n+1) \geq A(n) \)

Similarly we can show \( f(n) \leq A(n-1) \)

From the above we can conclude that if the maintenance cost in \((n+1)\)th year is more than average total cost in the \(n\)th year and the \(n\)th year maintenance cost < previous year's average total cost then a machine should be replaced.

**2.16. INTRODUCTION TO INVENTORY CONTROL.**

In our daily life, a shopkeeper can say how much quantity required of his customer /month or /week and accordingly he place order to whole seller.

But this is not the case with the manager of a big organization because stocking in such cases depends upon various factors, for example demand, time of ordering etc. so here real problem is to have a compromise between over stocking and under stocking. The study of such type of problems is term as material management or inventory control.

In broad sense, inventory may be defined as the stock of goods that are kept in order to ensure smooth and efficient running of organization.

**2.17. INVENTORY DECISIONS**

The manager must take three basic decisions in order to accomplish the function of inventory:

(a) When should the order be placed?
(b) How much safety stock should be kept?
(c) How much amount of an item should be ordered?
2.18. INVENTORY TERMINOLOGY

(a) **Lead time.** This is a period of time between ordering and replenishment.

(b) **Buffer stock:** This is a stock allowance to cover the demand of materials during the lead time.

(c) **Maximum level:** This is a stock level calculated as the maximum desirable stock to be maintained and is an indicator to the management to show when stocks has risen too high.

(d) **Reorder level:** This is the level of stock at which a further replenishment order should be placed. This stock level is dependent on lead time and the rate of consumption.

2.19. INTRODUCTION TO LINEAR TRANSPORTATION PROBLEM

The Linear Transportation Problem is well known classified transportation problem and most developed part in the area of transportation planning problem. It is actually a typical situation in the field of linear programming.

Here, the objective is to find the optimum transportation plan so as to minimize the total transportation cost. There are two cases namely,

I. Two Dimensional case.

II. Three Dimensional case.
2.20. SOLUTION OF TRANSPORTATION PROBLEM

The solution consists of the following steps.

Step 1. To find the problem is balanced or unbalanced. If balance goes to step 2.

Step 2. Otherwise first of all make it balance by introducing dummy source / dummy destination.

Step 3. Find initial basic feasible solution.

Step 4. To obtain an optimal solution by making successive improvement to initial basic feasible solution until no further decrease in transportation cost is possible.

2.21 TO FIND AN INITIAL BASIC FEASIBLE SOLUTION

There are several methods for finding the initial basic feasible solution of transportation problem namely

I. North – West corner rule.

II. Least cost method.

III. Vogel Approximation method (VAM).

The most prominent and widely used is the VAM.

2.22 OPTIMALITY TEST

After getting the initial basic feasible solution of transportation problem, we test this solution for optimality. To test the optimality we use MODI method.