Chapter V

Post-Optimal Analysis
Post-Optimal Analysis

Introduction:

In this chapter we are interested to study post-optimality of some of the Linear Programming models which are developed in the previous Chapter. Mainly we will study here the variations in cost and requirement vectors of different models.

5.1. Post-Optimal Analysis of Model- I

Firstly we go for post optimal study of model I dealing with the maximization of profit component of the organization.

The optimum simplex table of model-I is given below:

<table>
<thead>
<tr>
<th>$C_i$</th>
<th>$C_B$</th>
<th>$x_B$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
<th>$Y_4$</th>
<th>$Y_5$</th>
<th>$Y_6$</th>
<th>$Y_7$</th>
<th>$Y_8$</th>
<th>$Y_9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1400</td>
<td>1533</td>
<td>833</td>
<td>1033</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$x_2 = 14$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$x_5 = 4$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_7 = 70$</td>
<td>-3</td>
<td>0</td>
<td>2</td>
<td>7</td>
<td>0</td>
<td>-28</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_8 = 16$</td>
<td>5</td>
<td>0</td>
<td>10</td>
<td>5</td>
<td>0</td>
<td>-110</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Z^* = 21462$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Z - C_i$</td>
<td>133</td>
<td>0</td>
<td>700</td>
<td>500</td>
<td>0</td>
<td>1533</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
At first we investigate the variations in $C$

**Variation in $C_1$**:

Since $C_1 \notin C_8$, the change $\Delta C_1$ in $C_1$, with the solution remaining optimal, is given by

$$\Delta C_1 \leq Z_1 - C_1 \quad \text{giving} \quad \Delta C_1 \leq 133$$

Therefore the range over which $C_1$ can vary with maintaining the optimality of the current optimal solution is given by

$$-\infty \leq C_1 \leq C_1 + \Delta C_1$$

i.e., $-\infty \leq C_1 \leq 1400 + 133$ i.e., $-\infty \leq C_1 \leq 1533$

**Variation in $C_2$**:

Since $C_2 \notin C_8$, the range of $\Delta C_2$ is given by

$$\max \{ - (z_1 - c_i) / y_2 \} \leq \Delta C_2 \leq \min \{ - (z_i - c_i) / y_2 \}$$

$y_2 > 0 \quad y_2 < 0$

i.e., $\max \{ -133/1, -700/1, -500/1, -1533/1 \} \leq \Delta C_2 \leq \infty$

i.e., $-133 \leq \Delta C_2 \leq \infty$

The required range over which $C_2$ can vary maintaining the condition of optimality is

$$1533 - 133 \leq C_2 \leq 1533 + \infty$$

i.e., $1400 \leq C_2 \leq \infty$
Variation in $C_3$:
Since $C_3 \notin C_B$, the allowed change $\Delta C_3$ in $C_3$, so that the solution remains optimal, is given by

$$\Delta C_3 \leq Z_3 - C_3 \quad \text{i.e.,} \quad \Delta C_3 \leq 700$$

Therefore the range over which $C_3$ can vary maintaining the optimality of the solution is given by

$$-\infty \leq C_3 \leq C_3 + \Delta C_3$$

i.e., $-\infty \leq C_3 \leq 833 + 700$ \quad i.e., $-\infty \leq C_3 \leq 1533$

Variation in $C_4$:
Since $C_4 \notin C_B$, the allowed change $\Delta C_4$ in $C_4$, so that the solution remains optimal is given by

$$\Delta C_4 \leq Z_4 - C_4 \quad \text{i.e.,} \quad \Delta C_4 \leq 500$$

Therefore the range over which $C_4$ can vary maintaining the optimality of the solution is given by

$$-\infty \leq C_4 \leq C_4 + \Delta C_4$$

i.e., $-\infty \leq C_4 \leq 1033 + 500$ \quad i.e., $-\infty \leq C_4 \leq 1533$

Variation in $b$:
From the optimum simplex table we have

$$x_B = \begin{bmatrix} 14 & 4 & 70 & 16 \end{bmatrix}$$

$$B = \begin{bmatrix} y_9 & y_6 & y_7 & y_8 \end{bmatrix}$$
The individual effect of \( b \) where \( b = [ b_1 \ b_2 \ b_3 \ b_4 ] \) such that the optimality of the basic feasible solution is not violated is given by

\[
\max \{ -X_{bj} / b_k \} \leq \Delta b_k \leq \min \{ -X_{bj} / b_k \}
\]

For \( b_1 \):

\[
\Delta b_1 \leq \min \{ -4/-1 \} \quad \text{i.e.,} \quad \Delta b_1 \leq 4
\]

Here \( b_1 = 10 \), therefore \( b_1 \leq 4 + 10 \) i.e., \( b_1 \leq 14 \)

For \( b_2 \):

\[
\max \{ -14/-1, -4/-1 \} \leq \Delta b_2 \leq \min \{ -70/-28, -16/-110 \}
\]

\[
i.e., \quad -4 \leq \Delta b_2 \leq 0.15
\]

Here \( b_2 = 14 \), therefore \( 14 - 4 \leq b_2 \leq 14 + 0.15 \) i.e., \( 10 \leq b_2 \leq 14.15 \)

For \( b_3 \):

\[
\max \{ -70/-1 \} \leq \Delta b_3 \quad \text{i.e.,} \quad -70 \leq \Delta b_3
\]

Here \( b_3 = 462 \), therefore \( 462 - 70 \leq b_3 \) i.e., \( 392 \leq b_3 \)

For \( b_4 \):

\[
\max \{ -16/-1 \} \leq \Delta b_4 \quad \text{i.e.,} \quad -16 \leq \Delta b_4
\]

Here \( b_4 = 1680 \), therefore \( 1680 - 16 \leq b_4 \) i.e., \( 1664 \leq b_4 \)
5.2. Post Optimal Analysis of Model-II:

Next we go for post optimal study of model II which has also dealt with the maximization of profit component of the organization.

The optimum simplex table of model-II is given below:

<table>
<thead>
<tr>
<th>C'</th>
<th>C_B</th>
<th>X_B</th>
<th>Y_1</th>
<th>Y_2</th>
<th>Y_3</th>
<th>Y_4</th>
<th>Y_5</th>
<th>Y_6</th>
<th>Y_7</th>
<th>Y_8</th>
<th>Y_9</th>
<th>Y_10</th>
<th>Y_11</th>
</tr>
</thead>
<tbody>
<tr>
<td>1400</td>
<td>1533</td>
<td>833</td>
<td>1033</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1033</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1533</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Z = 18462</td>
<td>34.72</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2272.6</td>
</tr>
</tbody>
</table>

\( Z' - C' = 9.04 \)
Now we investigate the variations in $C_i$.

**Variation in $C_1$:**

Since $C_1 \notin C_B$, the allowed change $\Delta C_1$ in $C_1$, so that the solution remains optimum, is given by

$$\Delta C_1 \leq Z_1 - C_1 \quad \text{i.e.,} \quad \Delta C_1 \leq 9.04$$

Therefore the range over which $C_1$ can vary with maintaining the optimality of the solution is given by

$$-\infty \leq C_1 \leq C_1 + \Delta C_1$$

i.e., $-\infty \leq C_1 \leq 1400 + 9.04$ i.e., $-\infty \leq C_1 \leq 1409.04$

**Variation in $C_2$:**

Since $C_2 \notin C_B$, the range of $\Delta C_2$ is given by

$$\max \left\{ -\frac{(z_i - c_i)}{y_{2i}} \right\} \leq \Delta C_2 \leq \min \left\{ -\frac{(z_i - c_i)}{y_{2i}} \right\}$$

$y_{2i} > 0 \quad y_{2i} < 0$

i.e., $\max \{ -9.04/1, 0/1, 0/1 \} \leq \Delta C_2 \leq \infty$

i.e., $-9.04 \leq \Delta C_2 \leq \infty$

The required range over which $C_2$ can vary with maintaining the condition of optimality is given by

$$1533 - 9.04 \leq C_2 \leq 1533 + \infty$$

i.e., $1523.9 \leq C_2 \leq \infty$
Variation in $C_3$:
Since $C_3 \notin C_B$, the change $\Delta C_3$ in $C_3$, so that the solution remains optimum is given by

$$\Delta C_3 \leq z_3 - C_3 \quad i.e., \quad \Delta C_3 \leq 34.72$$

Therefore the range over which $C_3$ can vary with maintaining the optimality of the solution is given by

$$-\infty \leq C_3 \leq C_3 + \Delta C_3$$

i.e., $-\infty \leq C_3 \leq 833 + 34.72$ i.e., $-\infty \leq C_3 \leq 867.72$

Variation in $C_4$:
Since $C_4 \notin C_B$, the range of $\Delta C_4$ is given by

$$\max \{ -(z_j - c_j) / y_{ij} \} \leq \Delta C_4 \leq \min \{ -(z_j - c_j) / y_{ij} \}$$

$$y_{ij} > 0 \quad y_{ij} < 0$$

i.e., $\max \{ -34.72/0.84, -206.72/0.2, -2272.6/2.2 \} \leq AC_4$

$$\leq \min \{ -9.04/-.12, -.706/-.8 \}$

i.e., $-41.3 \leq \Delta C_4 \leq 75.3$

The required range over which $C_4$ can vary with maintaining the condition of optimality is

$$1033 - 41.3 \leq C_4 \leq 1033 + 75.3$$

i.e., $991.7 \leq C_4 \leq 1108.3$
Variation in $b$:

From the optimum simplex table we have

\[ x_B = [0 \ 4 \ 10 \ 70 \ 4 \ 0] \]

\[ B = [y_1 \ y_6 \ y_7 \ y_8 \ y_9 \ y_{10}] \]

\[
\begin{bmatrix}
-1 & 7 & 1 & 0 & 0 & -0.2 & 0 \\
0.7 & 0 & 0 & 0 & 0.2 & 0 \\
2.2 & 0 & 0.2 & 0 & -0.8 & 0 \\
-1.95 & 0 & 0 & 1 & -18 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
-1.7 & 0 & 0 & 0 & 0.8 & 1
\end{bmatrix}
\]

The individual effect of $b$, where $b = [b_1 \ b_2 \ b_3 \ b_4 \ b_5 \ b_6]$ such that the optimality of the basic feasible solution is not violated is given by

\[
\max \{ -X_B / b_k \} \leq \Delta b_k \leq \min \{ -X_B / b_k \}
\]

\[ b_k > 0 \quad b_k < 0 \]

For $b_1$:

\[
\max \{ -4 / 7, -10/2.2 \} \leq \Delta b_1 \leq \min \{ -70/-195, 0/-1.7 \}
\]

i.e.,

\[
\max \{ -5.7, -4.5 \} \leq \Delta b_1 \leq 0
\]

i.e.,

\[ -4.5 \leq \Delta b_1 \leq 0 \]

Here $b_1 = 10$, therefore

\[ 10 - 4.5 \leq b_1 \leq 10 + 0 \]

i.e.,

\[ 5.5 \leq b_1 \leq 10 \]
For $b_2$: We find

\[
\text{Max } \{-4/1\} \leq \Delta b_2
\]

i.e.,

\[
-4 \leq \Delta b_2
\]

Here $b_2 = 14$, therefore

\[
14 - 4 \leq b_2 \quad \text{i.e.,} \quad 10 \leq b_2
\]

For $b_3$:

\[
\text{max } \{-10/.2\} \leq \Delta b_3
\]

i.e.,

\[
-50 \leq \Delta b_3
\]

Here $b_3 = 462$, therefore

\[
462 - 50 \leq b_3 \quad \text{i.e.,} \quad 412 \leq b_3
\]

For $b_4$:

\[
\text{max } \{-70/1\} \leq \Delta b_4
\]

i.e.,

\[
-70 \leq \Delta b_4
\]

Here $b_4 = 1680$, therefore

\[
1680 - 70 \leq b_4 \quad \text{i.e.,} \quad 1610 \leq b_4
\]

For $b_6$ we have

\[
\text{Max } \{-0/1\} \leq \Delta b_6
\]

i.e.,

\[
0 \leq \Delta b_6
\]

Here $b_6 = 10$, therefore

\[
10 - 0 \leq b_6 \quad \text{i.e.,} \quad 10 \leq b_6
\]

5.2 Post Optimal Analysis of Model-III:

Now we study the sensitivity analysis of model III which has dealt with minimizing the expenditure of the organization running the transport.
system.

The optimum simplex table of model-III is given below:

<table>
<thead>
<tr>
<th>$C_i$</th>
<th>$-4331$</th>
<th>$-4181$</th>
<th>$-4494$</th>
<th>$-4331$</th>
<th>$0$</th>
<th>$0$</th>
<th>$0$</th>
<th>$0$</th>
<th>$0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_B$</td>
<td>$x_8$</td>
<td>$Y_1$</td>
<td>$Y_2$</td>
<td>$Y_3$</td>
<td>$Y_4$</td>
<td>$Y_5$</td>
<td>$Y_6$</td>
<td>$Y_7$</td>
<td>$Y_8$</td>
</tr>
<tr>
<td>0</td>
<td>$x_5$</td>
<td>-3</td>
<td>0</td>
<td>2</td>
<td>7</td>
<td>1</td>
<td>28</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$=382$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-4181</td>
<td>$x_2$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$=10$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$x_7$</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>$x_8$</td>
<td>-133</td>
<td>0</td>
<td>-700</td>
<td>-500</td>
<td>0</td>
<td>1533</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$=2670$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Z=41810$</td>
<td>150</td>
<td>0</td>
<td>313</td>
<td>150</td>
<td>0</td>
<td>4181</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

**Variation in $C_1$:**

Since $C_1 \notin C_B$, the change $\Delta C_1$ in $C_1$, so that the solution remains optimum is given by

$$\Delta C_1 \leq Z_1 - C_1 \quad \text{i.e.,} \quad \Delta C_1 \leq 150$$
Therefore the range over which $C_i$ can vary with maintaining the optimality of the solution is given by

$$-\infty \leq C_i \leq C_i + \Delta C_i$$

i.e., $-\infty \leq C_i \leq -4331 + 150$ i.e., $-\infty \leq C_i \leq -4181$

For actual problem $C_i$ lies between 4181 and 4331

Variation in $C_2$:
Since $C_2 \notin C_B$, the range of $\Delta C_2$ is given by

$$\max \{- \frac{(z_i - c_i)}{y_{2i}}\} \leq \Delta C_2 \leq \min \{- \frac{(z_i - c_i)}{y_{2i}}\}$$

$y_{2i} > 0$ $y_{2i} < 0$

i.e., $\max \{-150/1, 0/1.313/1, -150/1\} \leq \Delta C_2 \leq \min(-4181/-1)$

i.e., $0 \leq \Delta C_2 \leq 4181$

The required range over which $C_2$ can vary maintaining the condition of optimality is therefore

$$-4181 + 0 \leq C_2 \leq -4181 + 4181$$ i.e., $-4181 \leq C_2 \leq 0$

For actual problem $C_2 \leq 4181$

Variation in $C_3$:
Since $C_3 \notin C_B$, the change $\Delta C_3$ in $C_3$, so that the solution remains optimum is given by

$$\Delta C_3 \leq z_3 - C_3$$ i.e., $\Delta C_3 \leq 313$

Therefore the range over which $C_3$ can vary maintaining the optimality of the solution is given by
\[-\infty \leq C_3 \leq C_3 + \Delta C_3\]

i.e., \[-\infty \leq C_3 \leq -4494 + 313\]

i.e., \[-\infty \leq C_3 \leq -4181\]

For actual problem \(C_3\) lies between 4181 and 4494

**Variation in \(C_4\):**

Since \(C_4 \notin \mathbb{B}\), the change \(\Delta C_4\) in \(C_4\), so that the solution remains optimum is given by

\[\Delta C_4 \leq Z_4 - C_4\]

i.e., \(\Delta C_4 \leq 150\)

Therefore the range over which \(C_4\) can vary maintaining the optimality of the solution is given by

\[-\infty \leq C_4 \leq C_4 + \Delta C_4\]

i.e., \[-\infty \leq C_4 \leq -4331 + 150\]

i.e., \[-\infty \leq C_4 \leq -4181\]

For actual problem \(C_4\) lies between 4181 and 4331

**Variation in \(b\):**

From the optimum simplex table we have

\[x_B = [382 \ 10 \ 4 \ 2670]\]

\[B = [y_9 \ y_9 \ y_7 \ y_8]\]

\[
\begin{bmatrix}
1 & -28 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & -1 & 1 & 0 \\
0 & -1533 & 0 & 1 \\
\end{bmatrix}
\]
The individual effect of $b_i$ where $b = \{b_1, b_2, b_3, b_4\}$ such that the optimality of the basic feasible solution is not violated is given by

$$\max \left\{ -\frac{x_B}{b_k} \right\} \leq \Delta b_k \leq \min \left\{ -\frac{x_B}{b_k} \right\}$$

$b_k > 0$  \hspace{1cm} $b_k < 0$

**For $b_1$** we get

$$\max \{ -\frac{382}{1} \} \leq \Delta b_1 \hspace{1cm} \text{i.e.,} \hspace{1cm} -382 \leq \Delta b_1$$

Here $b_1 = 462$, therefore $462 - 382 \leq b_1$, i.e., $80 \leq b_1$

**For $b_2$** we get

$$\max \{ -\frac{10}{1} \} \leq \Delta b_2 \leq \min \{ -\frac{382}{-28}, -\frac{4}{-1}, -\frac{2670}{-1533} \}$$

i.e., $-10 \leq \Delta b_2 \leq 1.7$

Here $b_2 = 10$, therefore, $-10 + 10 \leq b_2 \leq 10 + 1.7$

i.e., $0 \leq b_2 \leq 11.7$

**For $b_3$**:

$$\max \{ -\frac{4}{1} \} \leq \Delta b_3 \hspace{1cm} \text{i.e.,} \hspace{1cm} -4 \leq \Delta b_3$$

Here $b_3 = 14$, therefore $14 - 4 \leq b_3$, i.e., $10 \leq b_3$

**For $b_4$**:

$$\max \{ -\frac{2670}{1} \} \leq \Delta b_4 \hspace{1cm} \text{i.e.,} \hspace{1cm} -2670 \leq \Delta b_4$$

Here $b_4 = 18000$, therefore $18000 - 2670 \leq b_4$

i.e., $15330 \leq b_4$
5.4 Post Optimal Analysis of Model- IV:

The optimum simplex table of model-IV is given below:

<table>
<thead>
<tr>
<th>$C_J$</th>
<th>$C_B$</th>
<th>$X_B$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
<th>$Y_4$</th>
<th>$Y_5$</th>
<th>$Y_6$</th>
<th>$Y_7$</th>
<th>$Y_8$</th>
<th>$Y_9$</th>
<th>$Y_{10}$</th>
<th>$Y_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-38</td>
</tr>
<tr>
<td>129</td>
<td></td>
<td></td>
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<td></td>
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<td></td>
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</tr>
</tbody>
</table>

$x_1 = 4, \quad x_2 = 4, \quad x_3 = 6, \quad x_4 = 6, \quad x_5 = 140, \quad x_6 = 4, \quad x_7 = 163, \quad x_8 = 1033, \quad x_9 = 150, \quad x_{10} = 500, \quad x_{11} = 900.$

$Z^* = \frac{2710}{42710} = 0.0639.$
First we go for the sensitivity analysis of $C$.

**Variation in $C_1$:**

Since $C_1 \notin C_B$, the change $\Delta C_1$ in $C_1$, so that the solution remains optimum is given by

$$\Delta C_1 \leq Z_1 - C_1$$

i.e.,

$$\Delta C_1 \leq 150$$

Therefore the range over which $C_1$ can vary maintaining the optimality of the solution is given by

$$-\infty \leq C_1 \leq C_1 + \Delta C_1$$

i.e.,

$$-\infty \leq C_1 \leq -4331 + 150$$

i.e.,

$$-\infty \leq C_1 \leq -4181$$

For actual problem $C_1$ lies between 4181 and 4331.

**Variation in $C_2$:**

Since $C_2 \in C_B$, the range of $\Delta C_2$ is given by

$$\max \left\{ -\frac{(z_1 - c_1)}{y_2} \right\} \leq \Delta C_2 \leq \min \left\{ -\frac{(z_1 - c_1)}{y_2} \right\}$$

$$y_2 < 0$$

i.e.,

$$\max \left\{ -150 / 1, 0.1 - 150 / 1 \right\} \leq \Delta C_2$$

i.e.,

$$0 \leq \Delta C_2$$

i.e.,

$$0 - 4181 \leq C_2$$

For actual problem $C_2 \leq 4181$.

**Variation in $C_3$:**

Since $C_3 \notin C_B$, the change $\Delta C_3$ in $C_3$, so that the solution remains optimum is given by
\[ \Delta C_3 \leq Z_3 - C_3 \quad \text{i.e.,} \quad \Delta C_3 \leq 163 \]

Therefore the range over which \( C_3 \) can vary maintaining the optimality of the solution is given by

\[-\infty \leq C_3 \leq C_3 + \Delta C_3 \]

\( \text{i.e.,} \quad -\infty \leq C_3 \leq -4494 + 163 \quad \text{i.e.,} \quad -\infty \leq C_3 \leq -4331 \)

For actual problem \( C_3 \) lies between 4331 and 4494

**Variation in \( c_4 \):**

Since \( C_4 \neq C_B \), the range of \( \Delta C_4 \) is given by

\[ \max \left\{ \frac{-Z_j - c_j}{y_{4j}} \right\} \leq \Delta C_4 \leq \min \left\{ \frac{-Z_j - c_j}{y_{4j}} \right\} \quad y_{4j} \neq 0 \]

\( \text{i.e.,} \quad \max \left\{ \frac{-163}{1}, \frac{0}{1} \right\} \leq \Delta C_4 \leq \min \left\{ \frac{-4331}{-1}, \frac{-150}{-1} \right\} \)

\( \text{i.e.,} \quad 0 \leq \Delta C_4 \leq 150 \)

\( \text{i.e.,} \quad 0 - 4331 \leq C_4 \leq -4331 + 150 \)

\( \text{i.e.,} \quad -4331 \leq C_4 \leq -4181 \)

For actual problem \( C_4 \) lies between 4181 and 4331

**Variation in \( b \):**

From the optimum simplex table we have

\[ x_B = [140 \quad 6 \quad 4 \quad 4 \quad 6 \quad 5670] \]

\[ B = [y_5 \quad y_{11} \quad y_7 \quad y_8 \quad y_9 \quad y_{10}] \]
The individual effect of $b_i$ where $b = [b_1, b_2, b_3, b_4]$ such that the optimality of the basic feasible solution is not violated is given by

$$\max \left\{ \frac{-x_B}{b_k} \right\} \leq \Delta b_k \leq \min \left\{ \frac{-x_B}{b_k} \right\}$$

For $b_1$:

$$\max \left\{ \frac{-140}{1} \right\} \leq \Delta b_1 \quad \text{i.e.,} \quad -140 \leq \Delta b_1$$

Here $b_1 = 462$, therefore $462 - 140 \leq b_1 \quad \text{i.e.,} \quad 322 \leq b_1$

For $b_2$:

$$\max \left\{ \frac{-6}{1} \right\} \leq \Delta b_2 \leq \min \left\{ \frac{-140}{1}, \frac{-4}{1}, \frac{-5670}{1}, \frac{-900}{1} \right\}$$

i.e., \[-6 \leq \Delta b_2 \leq 3.7\]

Here $b_2 = 10$ therefore $-6 + 10 \leq b_2 \leq 10 + 3.7 \quad \text{i.e.,} \quad 4 \leq b_2 \leq 13.7$

For $b_3$:

$$\max \left\{ \frac{-4}{1} \right\} \leq \Delta b_3 \quad \text{i.e.,} \quad -4 \leq \Delta b_3$$

Here $b_3 = 14$, therefore $14 - 4 \leq b_3 \quad \text{i.e.,} \quad 10 \leq b_3$
For \( b_4 \):

\[
\text{max} \{ -140 /1, - 4 /1, - 6 /1 \} \leq \Delta b_4 \leq \text{min} \{ - 6 /1 \}
\]

i.e., \(-2 \leq \Delta b_4 \leq 6\)

Here \( b_4 = 4 \), therefore \( 4 - 2 \leq b_4 \leq 6 + 4 \) i.e., \( 2 \leq b_4 \leq 10 \)

For \( b_5 \):

\[
\text{max} \{ -6 /1 \} \leq \Delta b_5 \quad \text{i.e.} \quad -6 \leq \Delta b_5
\]

Here \( b_5 = 10 \), therefore \( 10 - 6 \leq b_5 \) i.e., \( 4 \leq b_5 \)

For \( b_6 \):

\[
\text{max} \{ 5670 /1 \} \leq \Delta b_6 \quad \text{i.e.} \quad -5670 \leq \Delta b_6
\]

Here \( b_6 = 18000 \), therefore \( 18000 - 5670 \leq b_6 \) i.e., \( 12330 \leq b_6 \).

5.5 Post Optimal Analysis of Model -V

Next we go for post optimal analysis of Model V

The optimum simplex table of model V is given here

<table>
<thead>
<tr>
<th>( C_j )</th>
<th>( 500 )</th>
<th>( 350 )</th>
<th>( 0 )</th>
<th>( 0 )</th>
<th>( 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_B )</td>
<td>( x_8 )</td>
<td>( Y_1 )</td>
<td>( Y_2 )</td>
<td>( Y_3 )</td>
<td>( Y_4 )</td>
</tr>
<tr>
<td>500</td>
<td>( x_8 = 10 )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>( x_7 = 20 )</td>
<td>0</td>
<td>-1</td>
<td>-8</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>( x_5 = 50 )</td>
<td>0</td>
<td>5</td>
<td>-20</td>
<td>0</td>
</tr>
<tr>
<td>( Z = 5000 )</td>
<td>0</td>
<td>150</td>
<td>500</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \sum C_j )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
First we study the variation in $C$.

**Variation in $C_1$:**

Since $C_1 \notin C_B$, the range of $\Delta C_1$ is given by

$$\max \left\{ -\frac{z_i - c_i}{y_{ij}} \right\} \leq \Delta C_1 \leq \min \left\{ -\frac{z_i - c_i}{y_{ij}} \right\}$$

where $y_{ij} \geq 0$ and $y_{ij} < 0$.

i.e., $\max \left\{ -0/1, -150/1, -500/1 \right\} \leq \Delta C_1$ i.e., $0 \leq \Delta C_1$

So, $0 + 500 \leq C_1$ i.e., $500 \leq C_1$

**Variation in $C_2$:**

Since $C_2 \notin C_B$, the change $\Delta C_2$ in $C_2$ so that the solution remain optimum is given by

$$\Delta C_2 \leq Z_2 - C_2$$

i.e., $\Delta C_2 \leq 150$

Therefore the range over which $C_2$ can vary maintaining the optimality of the solution is given by

$$-\infty \leq C_2 \leq C_2 + \Delta C_2$$

i.e., $-\infty \leq C_2 \leq 350 + 150$ i.e., $-\infty \leq C_2 \leq 500$.

**Variation in $b$:**

From the optimum simplex table we have

$$x_B = [10 \ 20 \ 50 \ ]$$

$$B = [ y_3 \ y_4 \ y_5 \ ]$$
The individual effect of $b_i$ where $b = [b_1 \ b_2 \ b_3]$ such that the optimality of the basic feasible solution is not violated is given by

$$\text{Max} \ \{-x/b_k\} \leq \Delta b_k \leq \text{min} \ \{-x/b_k\} \quad b_k > 0 \quad b_k < 0$$

**For $b_1$:**

$$\text{max} \ \{-10/1\} \leq \Delta b_1 \leq \text{min} \ \{-20/-8, -50/-20\}$$

i.e., $-10 \leq \Delta b_1 \leq 5/2$

Here $b_1 = 10$, therefore $-10 + 10 \leq b_1 \leq 5/2 + 10$ i.e., $0 \leq b_1 \leq 12.5$

**For $b_2$:**

$$\text{max} \ \{-20/1\} \leq \Delta b_2 \quad \text{i.e.,} \quad -20 \leq \Delta b_2$$

Here $b_2 = 100$, therefore $-20 + 100 \leq b_2 \quad \text{i.e.,} \quad 80 \leq b_2$
5.6 Post Optimal Analysis of Model – VI

Now we study post optimal study of model VI

The optimum simplex table of model VI is given here

<table>
<thead>
<tr>
<th>C4</th>
<th>C3</th>
<th>C8</th>
<th>x8</th>
<th>Y1</th>
<th>Y2</th>
<th>Y3</th>
<th>Y4</th>
<th>Y5</th>
<th>Y6</th>
<th>Y7</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Variation in C4:

Since $C_1 \geq C_8$, the range of $\Delta C_1$ is given by

$$\max \left\{ \frac{-z_i - c_i}{y_{ij}} \right\} \leq \Delta C_1 \leq \min \left\{ \frac{-z_i - c_i}{y_{ij}} \right\}$$

$$y_{ij} > 0 \quad y_{ij} < 0$$
i.e., \[ \max \{-0/1, -200/1, 1100/1\} \leq \Delta C_1 \leq \min \{-1100/-1\} \]

i.e., \[ 0 \leq \Delta C_1 \leq 1100 \]

Therefore the range over which \( C_1 \) can vary maintaining the optimality of the solution is given by

\[ -\infty \leq C_1 \leq C_1 + \Delta C_1 \]

i.e., \[ -\infty \leq C_1 \leq -1100 + 1100 \quad \text{i.e.,} \quad -\infty \leq C_1 \leq 0 \]

Thus \( C_1 \) lies between 0 and 1100.

**Variation in \( C_2 \):**

Since \( C_2 \notin C_B \), the change \( \Delta C_2 \) in \( C_2 \), so that the solution remains optimum is given by

\[ \Delta C_2 \leq Z - C_2 \quad \text{i.e.,} \quad \Delta C_2 \leq 200 \]

Therefore the range over which \( C_2 \) can vary maintaining the optimality of the solution is given by

\[ -\infty \leq C_2 \leq C_2 + \Delta C_2 \]

i.e., \[ -\infty \leq C_2 \leq -1100 + 200 \quad \text{i.e.,} \quad -\infty \leq C_2 \leq -900 \]

For actual problem \( C_2 \) lies between 900 and 1300

**Variation in \( b \):**

Next we study the variation in \( b \).

From the optimum simplex table we have
The individual effect of bi where b = [ b₁, b₂, b₃, b₄ ] such that the optimality of the basic feasible solution is not violated is given by

\[
\text{max } \{ -x_B / b_k \} \leq \Delta b_k \leq \min \{ -x_B / b_k \}
\]

\[
b_k > 0 \quad \quad \quad \quad \quad \quad \quad b_k < 0
\]

For b₁:

\[
\text{max } \{-64/1\} \leq \Delta b_1 \quad \text{i.e., } -64 \leq \Delta b_1
\]

Here \( b_1 = 150 \), therefore \(-64 + 120 \leq b_1 \) \text{ i.e., } -56 \leq b_1

For b₂:

\[
\text{max } \{-7/1\} \leq \Delta b_2 \leq \text{min } \{-64/-8, -5/-1, -500/-500\}
\]

\text{i.e., } -7 \leq \Delta b_2 \leq 1

Here \( b_2 = 7 \), therefore \(-7 + 7 \leq b_2 \leq 1 + 7 \) \text{ i.e., } 0 \leq b_2 \leq 8

For b₃:

\[
\text{Max } \{-5/1\} \leq \Delta b_3 \quad \text{i.e., } -5 \leq \Delta b_3
\]

Here \( b_3 = 12 \), therefore \(-5 + 12 \leq b_3 \) \text{ i.e., } 7 \leq b_3
For $b_4$: \[ \text{Max } \{-500/1\} \leq \Delta b_4 \quad \text{i.e., } -500 \leq \Delta b_4 \]

here $b_4 = 4000$, therefore $-500 + 4000 \leq b_4$

\[ \text{i.e., } 3500 \leq b_4. \]

5.7 Post Optimal Analysis of Model –VIII

Now we for sensitivity analysis of model VIII

The optimum simplex table of model VIII is given here

<table>
<thead>
<tr>
<th>$C_i$</th>
<th>$C_B$</th>
<th>$x_B$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
<th>$Y_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>650</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$650$</td>
<td></td>
<td>$x_i = 8$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>$x_i = 100$</td>
<td>28</td>
<td>650</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z - c_i$</td>
<td>0</td>
<td>38</td>
<td>650</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

First we study the variation in $C$

**Variation in $C_i$:**

Since $C_1 \notin C_B$, the range of $\Delta C_1$ is given by

\[ \max \{ -(z_i - c_i) / y_{i1} \} \leq \Delta C_1 \leq \min \{ -(z_i - c_i) / y_{i1} \} \]

\[ y_{i1} > 0 \quad \text{and } y_{i1} < 0 \]

\[ \text{i.e., } \max \{ -0/1, -38/1, -650/1 \} \leq \Delta C_1 \]

\[ \text{i.e., } 0 \leq \Delta C_1 \]
Therefore the range over which $C_1$ can vary maintaining the optimality is $650 \leq C_1$.

**Variation in $C_2$:**

Since $C_2 \not\in C_B$, the change $\Delta C_2$ in $C_2$, so that the solution remains optimum is given by

$$\Delta C_2 \leq Z_2 - C_2 \quad \text{i.e.,} \quad \Delta C_2 \leq 38$$

Therefore the range over which $C_2$ can vary maintaining the optimality of the solution is given by

$$-\infty \leq C_2 \leq C_2 + \Delta C_2$$

i.e., $-\infty \leq C_2 \leq 612 + 38$ i.e., $-\infty \leq C_2 \leq 650$

**Variation in $b$:**

From the optimum simplex table we have

$$x_B = \begin{bmatrix} 8 & 100 \end{bmatrix}$$

$$B = \begin{bmatrix} y_3 & y_4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -25 & 1 \end{bmatrix}$$

The individual effect of $b$, where $b = [b_1 \ b_2 \ b_3 \ b_4]$ such that the optimality of the basic feasible solution is not violated is given by

$$\max \{-x_{Bi}/b_k\} \leq \Delta b_k \leq \min \{-x_{Bi}/b_k\}$$

$b_k > 0 \quad b_k < 0$
For \( b_1 \):

\[
\max \{-8/1\} \leq \Delta b_1 \leq \min \{-100/25\}
\]

i.e., \(-8 \leq \Delta b_1 \leq 4\)

Here \( b_1 = 8 \), therefore \(-8 + 8 \leq b_1 \leq 4 + 8\) i.e., \( 0 \leq b_1 \leq 12 \)

For \( b_2 \):

\[
\max \{-100/1\} \leq \Delta b_2 \quad \text{i.e.,} \quad -100 \leq \Delta b_2
\]

here \( b_2 = 300 \), therefore \(-100 + 300 \leq b_2\)

i.e., \( 200 \leq b_2 \).

The above post optimal Study gives the range of variability of \( c_i \) and \( b_i \) with maintaining the optimality.