Summary and Conclusion

The Newton’s law is one of the greatest discoveries which scientific investigation ever yielded to mankind. According to this law, every matter of particle attracts every other particle with a force which is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them.

The astronomical observations of the Danish astronomer Tycho Brahe on the motion of Mars led Kepler to formulate the law of planetary motion. Newton gave us an explanation of planetary motion
in terms of the force of gravity which exists between the sun and the planet. Thus, Newton’s law of gravitation hold even when \( r \) is large (except in the case, where relativistic corrections are appreciable). It should also be noted that Newton’s law is valid even when \( r \) is very small but ceases to hold when \( r \) is of the order of \( 10^{-7} \) cm. In this case, instead of attraction, there is repulsion, because if there is attraction between two atoms separated by a distance of the order of \( 10^{-7} \) cm, the atoms will superimpose on each other which is practically impossible. Actually the force in this case is of electrical nature.

Thus we see that the law is applicable to all distances ranging from zero to infinity.

For example, the law of gravitation is true at even very large distances like the distance between galaxies. Gravity appears to exist at even distances of tens of millions of light years. In a galaxy we have a scale of 50,000 to 100,000 light years for measuring distances between heavenly bodies. The earth’s distance from the sun is \( 8\frac{1}{2} \) light minutes, so we see how large these dimensions are. It should be noted that the law, unlike Inverse Square law; as in the case of electrostatics and magnetism, is not influenced by the medium in which the force acts. It is also independent of chemical composition of the attracting masses, and their temperatures. Thus we find that the law is universal.
In a gravitational field, to move a unit mass from one point to another point requires an expenditure of work. This amount of work done may be negative or positive according as the body is moved in the direction or against the direction of the force of attraction. This amount of work done is a measure of the potential difference between the two points between which the unit mass is moved. The gravitational potential at a point in a gravitational field is measured by the amount of work done in bringing a unit mass from infinity to that point.

All the calculation on Gravitational Potential at points outside non-spherical masses like disc or cylinder, available in existing literature, are done considering the points either on the axis or on the plane of the disc/cylinder. However, it is required to be studied at any point outside such bodies.

In this context in Chapter-1, we have briefly discussed the discovery of the laws of Gravitation and its consequences, its history and refinements of Gravitational laws by Einstein.

In Chapter-2, we have briefly discussed about the Gravitational Potential, its physical meaning, equipotential surface, Laplace’s and Poisson’s equations for potential.

In Chapter-3, we have discussed the methods available in literature and textbooks of finding Gravitational Potential at any point on the axis or on the plane of the non-spherical masses like disc and cylinder.
In Chapter-4 we have calculated the gravitational potential of a solid cylinder at any point not required to be only on the plane or on axis, outside the body (Figure 4.1). In figure 4.2, $\psi$ vs $R$ is plotted. Figure 4.3 and 4.4 show the plot of $\psi$ vs $\phi$. These figures (using FORTRAN – LF9556 Compiler) show the variation of gravitational potential with respect to angle $\phi$ and distance $R$.

Similarly, in Chapter-5, we have calculated the gravitational potential of an ellipsoidal mass of Prolate shape at any point outside the Prolate (Figure 5.1). In figure 5.2, $\psi$ vs $R$ is plotted. Figure 5.3 shows the plot of $\psi$ vs $\phi$. These figures (using FORTRAN – LF9556 Compiler) show the variation of gravitational potential with respect to angle $\phi$ and distance $R$.

In Chapter-6, we have calculated the gravitational potential of an ellipsoidal mass of Oblate shape at any point outside the Oblate (Figure 6.1). In figure 6.2, $\psi$ vs $R$ is plotted. Figure 6.3 shows the plot of $\psi$ vs $\phi$. These figures (using FORTRAN – LF9556 Compiler) show the variation of gravitational potential with respect to angle $\phi$ and distance $R$.

Equations (4.14), (5.15) and (6.15) are the expressions for Gravitational Potential of a Cylindrical mass, an elliptical mass of Prolate shape and an elliptical mass of Oblate shape at any point outside the Cylinder, Prolate and Oblate respectively. In Chapter-3 (Sec. 3.3), in case of the calculation of Gravitational Potential of thin uniform straight rod at any point, we have seen that if we draw an ellipse with $A$ and $B$ as foci then for all points on the ellipse, the sum of the focal distances will be the same [where for the point
P on the surface of the ellipse the sum of the focal distances is \((r_1 + r_2)\) and for all points over the surface of the ellipse the potential is constant. The potential is also constant over the ellipsoid obtained by revolving this ellipse about the line joining the foci, i.e., about AB.

In Figure 5.3 and 6.3, we see that the value of \(\psi\) drops to a minimum and then rises again. This may be because of the fact that the potential remains constant only in some ellipsoidal surface like in case of an uniform rod as discussed in Chapter-3 (Sec. 3.3), whereas in this case the distance \(R\) is kept constant and it will generate a spherical surface.

Thus we find that the expressions for Gravitational Potential of an elliptical mass of Prolate shape and an elliptical mass of Oblate shape at any point outside the Prolate (equation (5.15) and Oblate (equation (6.15) are correct.

A knowledge of the Gravitational Potential due to such bodies will help in understanding the dynamics of cluster of galaxies and Gravitational bending of light due to galaxies of various shapes.