CHAPTER 7

CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

The main purpose of the present work has been to develop the Self Adaptive Finite Element Technique for analyzing magnetic field problems encountered in Electrical Machines, which is simple to implement, flexible, efficient, and for a given accuracy converges to solution faster. To this end ‘self-adaptive finite element method’ has been developed, by incorporating changes, modifications and new techniques in different stages of traditional finite element analysis and the same has been used to analyse the magnetic fields of electric machines.

In Chapter-1, the different types of magnetic field problems encountered in electric machines and the problems encountered to solve them were surveyed and investigated. Finite Element Method is the most successful numerical method which provides a stable sequential methodology to discretise and solve the field problems for models with quite complex shapes.

In Chapter-2, the finite element technique has been described. The normal procedure is to use the differential form of governing equations by applying Variational or other techniques, which in turn lead to an algebraic system of equations characterized by a large sparse matrix. The variational method consists of postulating the partial differential equations of field problems in terms of variational expression called the energy functional. In most engineering application this expression can be identified with the stored energy in systems, and the Euler equation of this functional will yield the original differential equation. The minimization of energy functional is achieved in the finite element method, whereby the potential function approximating the true solution is defined in discretized subregions of the field region.

The size and placement of these elements largely determines the accuracy with which the problem can be solved. Reducing the element size and thereby increasing the number of nodal points usually yields a more accurate solution but at
the cost of increased CPU time and memory requirement. The key of efficient and
economic solution of the problems is not merely the number of points and elements
but also their placement. Regions with large variation gradients need a higher mesh
density, whereas quiescent regions require a comparatively coarser mesh. The
engineering judgment comes from the behaviour of the system and an estimate of the
computation time. The ‘good’ finite element mesh remains very much related to the
‘art’ of the user where experience and familiarity plays a large part. Otherwise, the
mesh model could be in-efficient, leading to inaccurate solution and wastage of
computer resources.

In self-adaptive procedures, the finite element model is generated iteratively,
beginning with a coarse mesh generated automatically using techniques developed in
Chapter-3 and refining successively to minimize the error in the solution.

It follows that the mesh generator associated with an adaptive
procedure must have two overriding features.

i) It must be possible to add points to the mesh recursively.

ii) It must be possible to refine the mesh anywhere without producing
    elements of the distorted shape.

These two conditions eliminate from consideration many standard mesh
generators. In Chapter-3 triangular mesh for a given set of points has been
developed using Delaunay triangulation. Procedure and algorithm for automatic
point creation for generation of triangular mesh have been developed in Section 3.2.
High quality meshes can be obtained using this procedure. In this approach, the
boundary domain is divided into a number of subregions. Each subregion is further
processed independently to generate interior nodes. The distribution of the interior
nodes depends on the boundary node density. This technique gives good quality
meshes in the interior of the domain also and is flexible, easy and efficient to
implement yet requires minimal manual user input. In section 3.3. procedures for
quadrilateral mesh generation has been developed. The main advantage of
quadrilateral mesh generation is that the total number of quadrilateral elements is
less than that of number of triangular elements in the mesh, and so, the computation
time is reduced.
Many approaches can be used to convert an existing triangular mesh to a quadrilateral mesh. The techniques proposed in this work are:

i) An existing triangular element can be divided into three quadrilaterals by joining the centroid of the triangle with the mid points of each of three sides.

ii) The adjacent triangular elements each having their internal angles within the prescribed limits can be combined to form a set of four quadrilaterals. This has been done in this work by introducing a node at the midpoint of the side common to the two triangles, joining it to the midpoints of the all the remaining sides and omitting the common side altogether. Case (i) is used if a triangle cannot be combined with any of its adjacent triangles to form a quadrilateral. Case (ii) is used when two adjacent triangles can be combined to form quadrilateral.

On the mesh thus generated, the given problem has been solved using the Finite Element Method. As Finite Element Method offers only an approximation to the exact solution of the given mathematically posed problem Chapter-4 dealt with:

i) Determining magnitude of the error at a given, finite, stage of subdivision, (a posteriori error estimate)

ii) And how best to refine the approximation to achieve results of a given desired accuracy.

The interaction between (i) and (ii) has been adapted and many other steps have been taken up in Chapter-4 are required to achieve optimal results. In regenerating the mesh, new elements of the type used originally but of smaller size (h-refinement) have been used.

The triangular elements have been divided into two, three or four depending upon the error in them and their location. In electromagnetic problems a commonly used method for mesh refinement is Delaunay triangulation. It has been applied to automatic mesh generation and also to self-adaptive analysis. In general, the extension of the work to deal with electromagnetic field problems is not straightforward, due to the problem of meshing the special introduction of air/material interfaces. Also, there may be sudden change in material properties, such as relative permeability differences between iron and copper. These
considerations add to the algorithmic complexity of effective self-adaptive schemes. For elements close to material boundaries Delaunay method alone will not guarantee the removal of thin elements. It has therefore been suggested that nodes be selectively added to the mesh after the error refinement stage, in conjunction with the Delaunay’s algorithm, so as to maintain the smallest angle above a specified value. This has been achieved by selecting a node referred to as seed node, which corresponds to the mid point of the longest side.

Methodologies for error analysis have been developed in Chapter-4. When deriving an approximation to the total error, various alternative possibilities exist. One of these is, of course, to obtain a complete solution by halving the existing mesh. Another is to introduce an additional complete polynomial to the approximation. The difference between two such consecutive solutions gives an estimate of the local error albeit at a very considerable cost.

When an hierarchical form of approximation is used, this difference can be obtained at a more reasonable cost, utilizing the original approximation. Error Estimates for hierarchal formulation have been developed in section 4.4 and have been used the basis for p-adaptive refinement. An L shaped problem has been taken up. In section 4.3 error estimation strategy based on global gradient smoothing has been developed. For calculating error norm smoothed continuous values of potentials gradients are used for exact values. Smooth values of the potential gradient with higher approximation to the exact solution has been developed. The error is determined and is compared for its all elements with the maximum permissible error in the element and is used to modify the mesh for subsequent analysis. By making use of them, the finite element program developed can determine which region needs refining and automatically adapt the mesh to suit the problem. The error is calculated for each element and is compared with a predefined limit. It is expected that for a given accuracy of the solution each element have approximately the same level or error. Therefore, every element with an error more than the specified has been adjusted to match this error level, the process is repeated, if necessary, with the ultimate aim that every element contains the same prespecified, allowable error, thus yielding an optimal mesh. It resulted in saving some data
preparation efforts by retaining the original mesh and refining locally, where the error in the solution is more than the specified.

In Chapter- 5 the technique developed in Chapters II, III and IV have been integrated so that there is a constant flow between them and have been applied for solving linear steady state magnetic field problems encountered Electrical Machines.

The Magnetic circuit of an electrical machine can be divided into current carrying, air gap and iron regions. For each of these regions the governing Laplace/ Poisson’s field equations have been obtained.

These equations are subjected to Dirichlet and Neumann Boundary Conditions. For the purpose of analysis, the magnetic potential distribution in rotor slot, and transformer window have been computed, under varying load conditions and the results are in good agreement with the available results.

For non-linear analysis, when a material like iron is present, finite element formulation of an energy functional, which takes into account the material properties and varying boundary conditions, has been developed. Solution to the problem has been achieved by using an iterative scheme and self-adaptive methodology. For adaptive mesh refinements lst order quadrilateral have been chosen since the 2nd order elements do not have the compatibility for higher order shape functions of hierarchical elements.

Error Estimation using hierarchical formulation (p-basis) for adaptive refinement has been applied for a L shaped transformer window problem in Chapter- 4, and the results are in good agreement with the available results. The refinement of the mesh has been in the desired locations i.e. the regions where the flux density variation is largest or where flux density changes its direction, such as in the corners.

However, when the Global Gradient Smoothing Error Estimate Technique was tested on an electro magnet with iron, air, copper winding regions the results obtained were not as expected. The Flux density gradients have a sharp change at the air/iron boundary. The normal component of the flux density i.e. $B_n$ remains same ($B_{n1}=B_{n2}$). However, the tangential component undergoes a change in the ratio
of $\mu_{\text{iron}}/\mu_{\text{air}}$ since $H_{t, \text{iron}} = H_{t, \text{air}}$ and the energy concentration being high on the air side, this method give higher value of error in the elements on the air side, inspite of the fact that material property i.e. $\mu$ is linear in air and hence no refinement is required. So the Global Gradient Smoothing Error Estimate Technique is not suitable for Electro-magnetic field problems having different types of materials with varying magnetic material properties.

Therefore, for further analysis of field problems with non-linear material properties only the 1st order elements have been considered in this dissertation with hierarchical interpolation for Error Estimation and Mesh Refinement.

The computer programme has been developed. ‘MAIN’ programme has been written using ‘FORTRAN’ 77 code. The computer programme for Mesh Generation and Mesh refinement has been written using ‘C’-codes. The computer program incorporates all the modifications including new methodologies developed for the analysis and solution of two-dimensional steady state electro-magnetic linear and non-linear field problems including axi-symmetric field problems.

The study of flux distribution in the magnetic circuit of a turboalternator has been carried out. Variation in the values of flux density and magnetic vector potential in the machine, for values different of current loading in the exciting winding have been obtained. This variation follows non-linear characteristics. The developed program has been validated on an electromagnet and applied to compute the magnetic vector potential and flux density distribution in turboalternator.

The results of these problems were compared with those obtained through the basic finite element technique or with those obtained experimentally whichever available. The results obtained are in good agreement. These have been obtained at a reduced cost and reduced user involvement. Even a relatively inexperienced user can handle problems having complicated geometries and unexpected material distribution. Thus the techniques so developed can be successfully applied to analyse the magnetic field problems in Electrical machines.
Scope For Future Work

The self-adaptive finite element techniques developed in this dissertation have been applied successfully for solving some of the two-dimensional steady state linear and non-linear magnetic field problems in Electrical Machines. The programme is flexible and is of general nature as far as the electromagnetic field problems are concerned and has salient features of an efficient package.

It can be used as a computational tool to study similar field problems in system engineering. The technique can be extended to incorporate transient and other complex problems. It can also be used as a step-by-step procedure for the design of electrical machines.

A few more strategies can be incorporated to make this package more efficient, like using a combination of triangular and quadrilateral elements for discretization of domains having curved boundaries. Application of p method to non-linear field problems can be an area of further research. For mesh refinement a combination h and p method may provide better convergence than a pure h or p method and the work can be extended in that direction.