CHAPTER - 5

SELF-ADAPTIVE FINITE ELEMENT SOLUTION OF STEADY STATE LINEAR MAGNETIC FIELD PROBLEMS

5.1 Introduction

The past two decades have witnessed phenomenal growth in electrical power system and consequently ratings and sizes of electrical power apparatus such as transformers, turbo-generators, dc machines and a host of other electrical devices has also increased. An accurate prediction of their performance has, therefore, become increasingly important to meet stringent specifications, to effect economy in design, and to ensure reliability of operation. [92]

In order to evaluate these, it is imperative that the electric and magnetic fields under operating conditions are predicted accurately. In this chapter the Self-adaptive Finite Element techniques developed in the preceding chapters have been applied for the analysis of linear magnetic field problems. In the study following assumptions have been made:

a) The electromagnetic field is quasi-stationary and at power frequencies the displacement currents are negligible.

b) The field is assumed not to vary in the axial direction thus constituting a two-dimensional field. In such fields the currents are all in the direction perpendicular to the field and, what is more significant, is that the vector potential has only one component, $A_z$, parallel to the current. Furthermore this vector potential can be identified with the flux function defining the lines of induction.

c) In the analysis of linear field problems, the permeability of iron is assumed to be infinite.

5.2 Formulation of Steady State Linear Field Problems

A number of field problems in electrical power apparatus require determination of potential distribution between surfaces bounded by potentials or their normal derivatives. Most of these problems are governed by the following
types of field equations, subject to specified boundary conditions. The potential distribution in the air gap region can be defined by the following Laplace's equation:

\[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \]  

(5.1)

It is subjected to both Dirichlet and Neumann boundary conditions.

In the region of the problem, the stored energy can be defined by the following expression:

\[ W(\phi) = \frac{1}{2} \int \left( \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} \right)^2 dS \]  

(5.2)

In F.E. method, the solution of Laplace's equation is obtained by creating an approximate expression for the stored energy \( W(\phi) \) associated with the potential function \( \phi(x,y) \). The potential \( \phi \) is assumed to have functions with undetermined coefficients. Minimization of the energy functional determines the coefficients, thereby obtaining the approximation to the potential distribution.

In electrical machines, a number of current sources occur within the region of the problem. The effect due to the presence of these sources, must be explicitly included in the formulation of the model. In a slot having a current carrying conductor, it is seen that the magnetic field in the slot can be defined by \( 'A' \), the magnetic vector potential (MVP) satisfying the vector form of the Poisson's equation. When the slot and the conductor are assumed to be infinitely long, both the current density vector \( A \) and the MVP possesses only longitudinally directed components. The vector form of Poisson's equation, then degenerates into the following form of scalar equation:

\[ \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} = -\mu \mu_0 J \]  

(5.3)
When the iron of the machine is assumed to have infinite permeability, normal potential derivatives at all the iron surfaces and at the center line of the slot must be zero. Thus, the boundary conditions for the problem are clearly defined. In F.E. method, the desired variational formulation of the problem will be to minimize the following energy functional:

\[
W(A) = \frac{1}{2} \int \left( \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} \right)^2 dS - \mu_0 \int A J dS
\]  

(5.4)

5.3 Organization of Computer Program

The organization of the computer programme is described in this section and can be used for two-dimensional field problems. Programme has been developed to suit the requirements of Personal Computer. The program can handle even more than 10,000 nodes and has the capacity to take into account, twenty regions of different properties i.e different permeabilities and conductivities. It consists of basic three parts:

I. Preprocessor
II. Processor
III. Postprocessor

The preprocessor part of the program is further divided into two parts. In the first part the input data of the problem is read in and initial mesh generated. This includes the finite element mesh information (e.g number of elements, type of elements, number of nodes, their x,y co-ordinates), geometry (e.g length of the domain, boundary conditions applicable on the nodes, the node numbers), Load data (e.g material number, source term, magnetization characteristics etc.) and indicators for various options (e.g print, linear analysis, nonlinear analysis, type of algorithm, iterations etc.).

In the second part of the preprocessor, the prepared data of part one is written on a data file and is used as input file for Processor. The Processor, where typically large amount of computing time is spent, consists of a MAIN routine and several subroutines, each having a special purpose. The different subroutines are:

- MAIN reads in the control data and controls the calling in order of the various subroutines. It also recycles the solution for the accuracy specified and prints execution time at the end of each solution.
GDATA reads the geometrical data and boundary conditions of the problem. It generates the mesh and necessary element data if required. The ability of mesh generation is limited to either normal linear, parabolic or cubic elements in any area taking one type of element at a time. All the input and generated data is printed out for checking.

LDATA reads the load data. Distributed current density, concentrated and distributed flux density on any boundary, types of load data is read and equivalent nodal currents are calculated and stored for use in SOLVE and BSUB routines.

EDET scans the element array and its type i.e whether the defined element is triangular or quadrilateral, linear or parabolic. Then each element is associated with an index number to be used in other routines and the number of Gauss integration points. This index number indicates the type of the element. SFR is the shape function routine for two dimensional quadrilateral elements. It forms the shape functions and their local co-ordinate derivatives for each integrating points of all types of elements.

SFRT is the shape function routine for two-dimensional triangular elements. It forms the shape functions and their local co-ordinate derivatives for each integrating points of all types of elements.

AUX calculates the global co-ordinate derivatives of the shape functions by forming the Jacobian matrix, \([J_c]\) and then forms the gradient matrix. If any modification, due to change in element type or mesh refinement gradient matrix is required, it is also carried out in this subroutine.

MODVEC scans the node array for mesh grading and arranges the terms of shape functions and their derivatives.

SFL is a shape function routine for one-dimensional elements.

SET scans the element array and each element is associated with the lowest nodal point number to which it is connected.

STIFM forms the reluctance and conductance matrices of all the elements in an order arranged in SET. These are all stored on a data set on the disc.

JTSTIF and INFSTF routines calculate the reluctance and conductance matrices of interface and infinite elements respectively.
AUXID calculates the global derivatives and Jacobian matrix for the nodes.

SOLVE takes the element reluctance matrix from the data set, assembles the global reluctance matrix, introduces the boundary conditions, and performs Gaussian elimination at every node as soon as its data is complete and then writes the reduced form of equations on to another data set.

BSUB carries out the back substitution and the values of nodal potentials and reactions are printed out.

ERRESTG performs error estimation calculations using global gradient smoothing. ERRESTH performs error estimation calculations using Hierarchic elements. Both these subroutines reside in the MAIN and a code is assigned to each of them. By specifying the code any one of them can be activated.

MODIFY modifies the elements by dividing them. If no error in the solution, then the output is printed by the OUTPUT subroutine otherwise it forms the new nodal arrays and adds new triangles and nodal points.

SETFRONT minimises the frontwidth by renumbering the order of elements. It scans the element array and associates each element with the lowest nodal point number to which it is connected. The maximum bandwidth is also calculated and compared with the maximum allowed.

ARRANGE renumbers the nodes by taking node 1 as the base node and distance as the criterion.

RESOLV carries out the resolution for the new set of load conditions and replaces the operation of SOLVE.

EMFIND is the output routine. It calculates the field intensity and density at node points or at integrating points usually known as Gaussian points or at both as required, and the energy stored in the field region.

FLAUX is the auxiliary subroutine of EMFIND, and used for calculating and printing the data at node points.

Flow sequence for the program is shown in Fig.5.1.

5.4 Test Problem

A two-dimensional problem has been solved to test the validity of the formulation of Self Adaptive Finite Element Technique for the solution of linear
field problems. An exact solution of Laplace’s equation under Dirichlet and
Neumann boundary conditions within a unit square is available [93] and the same
has been used as a test problem. The results obtained by using the normal
parabolic elements of various types are shown in Fig.(5.2). In Fig.(5.2) it can be
noticed that the potential variation in case of normal parabolic elements leaving the
upper boundary free. Fig.(5.3) is in good agreement with the exact solution.

5.5. Sample Problems

5.5.1 Finite Element Formulation Of Axi-Symmetric Problem.

Many interesting transformer problems have been analysed by Rabins [94],
Andersen [95] and Silvester [96] have used the Rabins Case 2 to illustrate their
finite element analysis. The same case is taken up to illustrate the various aspects
of the present finite element analysis, which can be used to solve two dimensional
and axi-symmetric problems. In the analysis numerically integrated general
quadrilateral isoperimetric elements with or without curved sides have been used.
For comparison the solution domain was divided into elements so that the total
number of nodes are approximately same for various type of elements. The results
obtained for approximately 100 nodes and 180 nodes are tabulated in Table 5.1.
For the purpose of analysis, the transformer is assumed rotationally symmetric
about a core leg. It is assumed that all iron is infinitely permeable. The Poisson’s
equation in cylindrical coordinates for axi-symmetric problems can be written as

\[
\frac{\partial}{\partial r} r \frac{\partial (Ar)}{\partial r} + \frac{\partial}{\partial z} v \frac{\partial A}{\partial z} = J \tag{5.5}
\]

subject to the boundary conditions

Vector Potential \(A\) is specified on the part of boundary S1.

\(\frac{\partial A}{\partial n}\) on the part of the boundary S2.

where \(v\) is the reluctivity of the medium and \(J\) is the current density.
In the present formulation the given region is divided into a mesh of finite elements, and the vector potential $A$ is approximated in an element. The mesh is a set of rings, each ring having a general curvilinear quadrilateral cross-section that has been revolved around the $Z$-axis. The current density, $J$ is assumed to be directed in the peripheral direction, and thus the vector potential, $A$ is also peripherally directed. Within each element the vector potential $A$ is assumed to vary according to the equation

$$A = \sum N_i A_i$$

where $N_i$ are the usual shape functions and $A_i$ are the nodal values of vector potentials.

The shape functions for the curved para-linear element can be written as

$$N_1 = \eta_1, \quad N_2 = \eta_2, \quad N_3 = \eta_3, \quad N_4 = \eta_4, \quad N_5 = \eta_5, \quad N_6 = \eta_6$$

where

$$n_1 = \frac{1}{2} \xi (\xi-1), \quad n_2 = (1-\xi^2), \quad n_3 = \frac{1}{2} \xi (\xi+1),$$

$$\eta_1 = \frac{1}{2} (1-\eta) \quad \text{and} \quad \eta_2 = \frac{1}{2} (1+\eta)$$

The potential at any point in para-linear element is given by

$$A = [N] \{A\}^e$$

Reactance Calculation

The leakage reactance can be calculated as pointed out by Andersen and Sylvester, by calculating the stored energy in the leakage field. The leakage reactance $X$, using the stored energy $W$, is given by

$$X = \frac{2\omega W}{I^2}$$

where

$$W = \frac{1}{2} \int_J \text{Adv}$$
is supply frequency, and \( I \) is the current peak or effective, for which the energy \( W \) was evaluated. The integration in Eq. (5.12) extends over all the space occupied by the windings. The short circuit electromagnetic forces acting on the windings are calculated by integration of the volume force density over the volume occupied by the windings. Thus for the axi-symmetric problems the radial and axial components of the force can be written as

\[
\begin{bmatrix}
F_r \\
F_z
\end{bmatrix} = \int \begin{bmatrix}
B_z \\
-B_r
\end{bmatrix} \, dv
\]

(5.13)

where \( B_z \) and \( B_r \) are the flux densities in axial and radial directions respectively. Using the Finite Element technique the force components can be rewritten as

\[
\begin{bmatrix}
F_r \\
F_z
\end{bmatrix} = \int [G] \begin{bmatrix}
A_r \\
A_z
\end{bmatrix} \, dv
\]

(5.14)

The evaluation of stored energy, Eq. (5.12), and force vector, Eq.(5.14), are carried out numerically.

5.5.2 Problem-2 Magnetic Potential distribution in Rotor Slot

The slot considered for the purpose of analysis is shown in Fig.5.5. The smooth stator has been taken to have a fixed value of the magnetic potential. The boundary of the slotted rotor also have a fixed value of magnetic potential. The rest of the magnetic potential drops across the air gap and slot of the rotor. These fixed values of the magnetic potential at the boundaries have values as 100 and 0. These are indicated by heavy lines and are called Dirichlet Boundaries. The remaining boundaries are plane of symmetry and they require vanishing normal potential derivatives. These are called Neumann's boundaries. For computation of magnetic vector potential values, the problem region has been discretized into 95 nodes and 144 elements (Fig.5.5). The equipotential lines have been plotted (Fig.5.6).
To study the effect of current sources, a slot has been considered to contain a conductor carrying a uniform current density of $J = 1$ per unit (p.u.). The bottom of the slot has been assumed to have a zero value of MVP and is specified as Dirichlet boundary condition. Other boundary surfaces can be assumed to have Neumann Boundary conditions. For the purpose of finite element analysis, a slot having p.u. dimension $2.0 \times 1.0$ and $3.0 \times 1.0$ p.u. has been discretized as shown in Fig. 5.7. The corresponding equipotentials are shown in Fig. 5.8 and 5.9. The computed values of the MVP are plotted in Fig. 5.10.

5.6 Conclusions

The use of self-adaptive technique developed allows user to generate optimally refined meshes that enhances convergence to the solution. The program developed incorporating methodologies developed in previous chapters have been used for analysis of linear steady state magnetic field problems.

In Section 5.5, Transformer Leakage Field Analysis is taken up. The agreement with the tested result for reactance is very good. The flux plots for 180 nodes cases are shown in Fig (5.3). The agreement with Anderson and Silvester is good. The flux plots for 100 nodes case are similar. When the number of integrating points for linear, parabolic and cubic elements are 4, 9 and 16 respectively, the present study indicates an approximate time ratio of 1:5:15 for the formation of reluctance matrix of an element and a total problem time ratio of 3:4:7, on the basis of equal number of nodes, for an analysis by linear, parabolic and cubic elements. However, if the number of integrating points in each parabolic element are chosen to be 4, then a time saving of approximately 30% in the formation of reluctance matrix, is achieved and the total problem time is of the same order as for linear elements for a problem having same number of nodes.

The magnetic potential distribution in the slot of electrical machine, subjected to specified boundary conditions have been computed.

In all these problems the input data plus an accuracy parameter has been supplied and the programme generates adaptively an optimal mesh that achieves the desired accuracy.
### TABLE 5.1

**Computed Value of Leakage Reactance**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>No. of Nodes</th>
<th>Types of Elements</th>
<th>No. of Elements</th>
<th>Reactance p.u.</th>
<th>No. of integrating Points per Element</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>100</td>
<td>L.Q.</td>
<td>81</td>
<td>0.56046</td>
<td>4</td>
</tr>
<tr>
<td>2.</td>
<td>96</td>
<td>P.Q.</td>
<td>25</td>
<td>0.58683</td>
<td>9</td>
</tr>
<tr>
<td>3.</td>
<td>100</td>
<td>C.Q.</td>
<td>15</td>
<td>0.58620</td>
<td>16</td>
</tr>
<tr>
<td>4.</td>
<td>180</td>
<td>L.Q.</td>
<td>154</td>
<td>0.56815</td>
<td>4</td>
</tr>
<tr>
<td>5.</td>
<td>176</td>
<td>P.Q.</td>
<td>49</td>
<td>0.59315</td>
<td>9</td>
</tr>
<tr>
<td>6.</td>
<td>184</td>
<td>C.Q.</td>
<td>30</td>
<td>0.59220</td>
<td>16</td>
</tr>
<tr>
<td>7.</td>
<td>96</td>
<td>P.Q.</td>
<td>25</td>
<td>0.59167</td>
<td>4</td>
</tr>
<tr>
<td>8.</td>
<td>176</td>
<td>P.Q.</td>
<td>49</td>
<td>0.59380</td>
<td>4</td>
</tr>
<tr>
<td>9.</td>
<td>96</td>
<td>P.Q.</td>
<td>25</td>
<td>0.57480</td>
<td>9</td>
</tr>
<tr>
<td>10.</td>
<td>96</td>
<td>P.Q.</td>
<td>25</td>
<td>0.57668</td>
<td>4</td>
</tr>
</tbody>
</table>

Tested value of reactance = 0.587 p.u.
Rabins Calculated value = 0.5829 p.u.
Andersen’s calculated value using 3496 First order triangular elements
1833 nodes = 0.5886 p.u.
FIG. 5.1 ORGANIZATION OF THE COMPUTER PROGRAMME

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FIG. 5.2(a) LEAKAGE FIELD OF THE TRANSFORMER

NO. OF NODES = 176, NO. OF PARABOLIC ELEMENTS = 49
FIG. 5.2(b) LEAKAGE FIELD OF THE TRANSFORMER

--- NO. OF NODES = 104, NO. OF CUBIC ELEMENTS = 30

--- NO. OF NODES = 184, NO. OF LINEAR ELEMENTS = 154
FIG. 5.3 RELATIVE POTENTIALS ALONG THE LINE MIDWAY BETWEEN THE TWO WINDINGS (LINE CD)
FIG. 5.4 RELATIVE POTENTIALS IN TRANSFORMER WINDOW (LINE AB)
FIG. 5.5 FINITE ELEMENTS REPRESENTATION OF SLOT.

NODES = 95
ELEMENTS = 144
FIG. 5.6 EQUIPOTENTIAL LINES.
FIG. 5.7 FINITE ELEMENT REPRESENTATION OF SLOTS HAVING CURRENT CARRYING CONDUCTOR.
MVPD BETWEEN LINES = 0.2 p.u.
A = 2.0

FIG. 5.8 EQUIPOTENTIAL FOR SLOT 2x1 p.u.
FIG. 5.9 EQUIPOTENTIAL LINES FOR SLOT 3x1 p.u.
FIG. 5.10 MVP VARIATION IN SLOTS.