CHAPTER III

TRANSIENT STABILITY INVESTIGATIONS OF MULTIMACHINE SYSTEMS
EQUIPPED WITH THYRISTOR CONTROLLED DYNAMIC BRAKE

In CHAPTER II the results of investigation of the transient stability improvement with a TCDB when the power system is represented on a two machine basis are given. However in some cases it is of interest to split the actual system into more than two groups to obtain a better or more accurate picture regarding the electromechanical transient phenomena in the system. If the transient stability analysis concerns more than two machine groups then it is termed as multimachine system analysis.

The transient stability analysis of a multimachine power system involves the study of relative motion of generators. The stability is determined from the change in relative angles. When these changes become unbounded at least for one relative angle the original system is said to loose stability. Normally the synchronous machines having the largest angle of separation in steady state operation prior to fault constitute the weakest link in the system. Identical changes in all angles indicate frequency variation and not violation of stability. For such transient stability
investigation load flow computation for the system network with transformers, transmission lines and buses is first carried out to determine the system condition prior to a disturbance. The network representation using the bus frame of reference in admittance or impedance form is used for load flow calculations. For transient stability computations the bus admittance or impedance matrix is to be modified to include loads as static admittance or impedance to ground and the equivalent circuit of machines. The system performance equations during transient condition may be written in terms of modified bus admittance or impedance matrix. The solution of these equations provides the information to the voltages and currents during transient conditions. The motion of machine rotors during transient condition is given by the swing equations, numerical solution of which gives the variation of rotor angles. The stability analysis is performed by combining solution of network performance equations and numerical solution of differential equations characterizing machine performance. In case swing curve computation is only of interest and the bus voltages during transient period are not required, then only those buses representing internal emfs of the generators may be retained. The generator power output may be written in terms of the internal emfs of generators and the driving point and transfer admittances or impedances of the reduced
network. The stability analysis then involves only the solution of swing equations by numerical techniques. In such cases the commutation time is very much reduced as it is not necessary to solve network performance equations for each time interval $\Delta t$ of the swing curve computation[61].

The effectiveness of a TCDB in multimachine power system has been investigated by considering two different system configurations. The first system is a ring-type interconnected system. In this system when a fault occurs on one of the transmission lines, it is cleared by opening of the line, and this does not isolate any station from the rest of the system. Alternate path for interconnection between machines always exists. The other system considered for investigation is a radial type of interconnection involving double-circuit long distance bulk power transmission scheme. In this system a fault on interconnecting transmission line affects the power transfer considerably as the faulted line is opened to clear the fault. The healthy line is overloaded and may affect the stability of the system. The effect of high ceiling rapid response excitation systems in addition to the TCDB is also investigated for this system. The performance of the TCDB has been compared with that of an optimally switched dynamic brake for the ring-type system configuration and with conventional dynamic brake for the radial type interconnected system.
While carrying out the transient stability investigations the mechanical powers of the turbines are assumed to be constant, and all the loads are represented as constant shunt impedances to ground. When air-gap flux linkages are assumed to be constant a synchronous machine is represented by the constant internal emf \( E' \) behind direct axis transient reactance in series with transient reactance \( x'_d \) of the machine. When variation of air-gap flux linkages are taken into account then a salient-pole synchronous machine is represented by the internal emf \( E_Q \) (behind quadrature axis synchronous reactance) in series with the quadrature axis synchronous reactance \( x_q \), and a round rotor synchronous machine is represented by emf \( E_d \) (behind direct axis synchronous reactance) in series with direct-axis synchronous reactance \( x_d \). The effect of excitation system is taken into account by incorporating change in the emf \( E_{ex} \), the exciter voltage referred to the stator side, as a function of the generator terminal voltage, \( V_q \).

3.1 SYNCHRONOUS MACHINE REPRESENTATION WITH \( E' \)'S CONSTANT

In transient stability investigations when internal emfs \( E' \) of all the synchronous machines are held constant in magnitude but their angular positions are allowed to change
the system network configuration may be simplified by eliminating all the buses except the machine internal emf buses by network reduction technique. The driving point and transfer admittance of the reduced network may be calculated for the different operating conditions of the system. The electric power output of the generator then may be obtained from the equation

\[ P_{Gi} = (E_i')^2 Y_{ii} \cos \theta_{ii} + \sum_{i=1}^{n} E_i' E_j' Y_{ij} \cos(\theta_{ii} - \theta_{ij}) \]  

(3.1)

where

- \( P_{Gi} \) = electrical power output of i-th generator in per-unit
- \( Y_{ii} \) = magnitude of the driving point admittance of the internal bus of the i-th machine in per unit
- \( Y_{ij} \) = magnitude of the transfer admittance between i-th and j-th machine internal buses in per-unit;
- \( \theta_{ii} \) = admittance angle associated with \( Y_{ii} \) in radians;
- \( \theta_{ij} \) = admittance angle associated with \( Y_{ij} \) in radians;
\[ \delta_i = \delta_i' - \delta_j' \] = relative angle between \( i \)-th and \( j \)-th machine in radians;

\[ \delta_i' \] = \textit{phase} angle between the internal emf \( E_i' \) of \( i \)-th machine with respect to the reference phasor which rotates at synchronous \textit{speed}, in radians.

The change in rotor angles may be obtained from solution of swing equations. The swing equation of the \( i \)-th machine may be split up into two first order differential equations as follows:

\[ \dot{\delta}_i = \Delta \omega_i = \omega_i - \omega_0 \] (3.2)

\[ \dot{\omega}_i = \frac{p f_o}{H_i} (P_i - P_{Gi}) \] (3.3)

for \( i = 1, 2, \ldots, n \);

where

\[ \Delta \omega_i \] = rotor relative angular velocity of the \( i \)-th machine in radians per second;

\[ \omega_i \] = absolute rotor angular velocity of the \( i \)-th machine in radians per second;

\[ \omega_0 = 2\pi f_o \] = synchronous angular velocity in radians per second;

\[ f_o \] = nominal system frequency in Hz;

\[ H_i \] = per-unit inertia constant of \( i \)-th machine in seconds;
\[ P_{Ti} \] = turbine power in \( i \)-th machine in per-unit;
\[ n \] = total number of synchronous generators.

The swing curves may be obtained by solution of eqs. (3.2) and (3.3) by a suitable numerical technique.

Illustrative Example I

Single line diagram of a ring-connected multi-machine system is shown in Fig. 3.1. The line constants, machine details and the initial operating conditions are given in Tables 3.1 - 3.3 [62].

<table>
<thead>
<tr>
<th>Table 3.1</th>
<th>TRANSMISSION LINE CONSTANTS FOR SYSTEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>From Bus</td>
<td>To Bus</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>
Fig. 3.1 Schematic diagram of ring-connected multimachine system (Example I).
### Table 3.2

<table>
<thead>
<tr>
<th>Bus</th>
<th>Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p in MW</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
</tr>
</tbody>
</table>

### Table 3.3

SYNCHRONOUS MACHINE DATA

<table>
<thead>
<tr>
<th>Generator Number and Bus Number same</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>MVA rating</td>
</tr>
<tr>
<td>H Sec.</td>
</tr>
<tr>
<td>X'd', p.u.</td>
</tr>
<tr>
<td>X'd, p.u.</td>
</tr>
<tr>
<td>X'q, p.u.</td>
</tr>
<tr>
<td>T'do, sec.</td>
</tr>
<tr>
<td>D, p.u.</td>
</tr>
<tr>
<td>E', p.u.</td>
</tr>
<tr>
<td>δ', deg.</td>
</tr>
<tr>
<td>P_G, MW</td>
</tr>
<tr>
<td>Q_G, MVAr</td>
</tr>
</tbody>
</table>
A three-phase short-circuit is assumed near bus-3 on the line connecting buses 3 and 4, and is assumed to clear in a time $TFC=0.411$ second by opening the line 3-4. In this system, the fastest and slowest machines happened to be machine-3 and machine-1 respectively. A dynamic brake with braking resistance $R=5.0$ per-unit is assumed to be connected to the terminals of machine-3 as soon as the fault is cleared and is disconnected after an optimum time interval as determined in reference[33]. The swing curves are computed neglecting the circuit conductances. The swing curves without dynamic brake and with optimally switched dynamic brake correspond to curve-a and curve-b respectively in Fig.3.2. A TCDB with the same value of braking resistance $R=5.0$ per unit is connected to the terminals of the synchronous generator-3 immediately after fault clearance and the triggering of the thyristor bridge is controlled as a function of positive values of rotor relative angular velocity of machine-3 with respect to synchronous speed, $\omega_3$, according to the following law:

$$\alpha = 2\pi/3 - \kappa \omega_3 \Delta \omega_3 \quad \text{for } \Delta \omega_3 > 0$$

(3.4)

$$\alpha = 2\pi/3 \quad \text{for } \Delta \omega_3 \leq 0$$

The corresponding swing curve is shown by curve-c in Fig.3.2. It may be observed from the swing curves of Fig.3.2 that the
Fig. 3.2 Swing curves for example I neglecting circuit conductances.
TCDB is more effective than the optimally switched dynamic brake in damming transient swings.

The effect of variation of gain constant $K_\omega$ of the TCDB is shown in Fig. 3.3. It may be observed from the swing curves that the magnitude of swing of relative angle $\delta_3^1$ decreases as the value of $K_\omega$ is increased unto 3. Further increase in $K_\omega$ has not resulted in significant improvement. Swings of other relative angles $\delta_2^1$ and $\delta_4^1$ for different $K_\omega$ values are shown in Fig. 3.4. Increase of $K_\omega$ decreases the swing of $\delta_2^1$ and $\delta_4^1$ also.

The effectiveness of TCDB for an increased fault clearing time $TFC = 0.6244$ is shown in Fig. 3.5. In this case the circuit conductances are also taken into account. It may be seen from Fig. 3.5 that the TCDB provides better damping than the optimally switched dynamic brake. As the TCDB is a function of rotor relative speeds, and as such speeds are more than one in multimachine systems, different control laws for triggering of the TCDB have been tried with the aim of determining the best from the point of view of damping of transient swings. The following regulation laws have been tried:

Case A. $\alpha = 2\pi/3 - K_\omega \Delta \omega_3^1$, for $\Delta \omega_3^1 > 0$,

$\alpha = 2\pi/3$ for $\Delta \omega_3^1 \leq 0$. 

(3.5)
Fig. 3.3 Effect of variation of $K_\omega$ on transient swing of $\delta_{31}'$. 

1. $K_\omega = 0.1$
2. $K_\omega = 3.0$
3. $K_\omega = 10.0$
Fig. 3.4 Effect of variation of $K_\omega$ on transient swings of $\delta'_{21}$ and $\delta'_{41}$.
Fig. 3-5 Swing curves for Example I including circuit conductances.
where
\[ \Delta \omega_{31} = \omega_3 - \omega_1. \]

**Case B.**
\[ \alpha = \begin{cases} 2\pi/3 - \kappa_\omega (\Delta \omega_{31} + \Delta \omega_{32}), & \text{for } (\Delta \omega_{31} + \Delta \omega_{32}) > 0, \\ 2\pi/3, & \text{for } (\Delta \omega_{31} + \Delta \omega_{32}) \leq 0. \end{cases} \]

where
\[ \Delta \omega_{32} = \omega_3 - \omega_2. \]

**Case C.**
\[ \alpha = \begin{cases} 2\pi/3 - \kappa_\omega (\Delta \omega_{31} + \Delta \omega_{32} + \Delta \omega_{34}), & \text{for } (\Delta \omega_{31} + \Delta \omega_{32} + \Delta \omega_{34}) > 0, \\ 2\pi/3, & \text{for } (\Delta \omega_{31} + \Delta \omega_{32} + \Delta \omega_{34}) \leq 0. \end{cases} \]

where
\[ \Delta \omega_{34} = \omega_3 - \omega_4. \]

The swing curves with TCDB regulated according to the laws mentioned above in Case A, Case B and Case C are shown by curve-A, curve-B and curve-C respectively in Fig.3.6.

In Fig.3.6 the curve-D corresponds to the case when regulation of TCDB is done according to eq.(3.4) using only the rotor relative angular velocity of machine-3 as the control signal. It may be observed that for the first few swings the swing curve is same irrespective of the law of regulation. However
Fig 3.6 Effect of different control laws on transient swing of $\delta_{31}$. 

A—case A
B—case B
C—case C
D—Regulation with $\Delta \omega_3$ signal
in later swings the control law according to eq.\((3.7)\) gives better damping of transient swings.

As the magnitude of braking action on the synchronous generator is a function of braking resistance \(R\) besides the regulation coefficient \(K\), the effect of variation of \(R\)
for a fixed value of regulation coefficient \(K\) = 3.0 is brought out in Fig.\(3.7\). It may be seen that maximum damping of swing is obtained for a value of \(R\) around 2.0 per-unit. These investigations show that TCDB may be regulated with respect to any of the control signals proposed as the main objective is to control the first few swings.

3.2 ILLUSTRATIVE Example II

The Trans-Siberian Transmission system of Unified Power Grid of Soviet Union has been chosen for investigations. Single line diagram of the radially connected multi-machine system is shown in Fig.\(3.8\). The line constants, machine details and the initial operating conditions are given in Tables 3.4 - 3.6 [42].

A three phase short-circuit is assumed near bus-7 on one of the double circuit lines connecting buses 6 and 7.
Fig. 3.7 Selection of best value of $R$ for TCDB in case of Example I.

- $a = 10.0$ p.u.
- $b = 3.0$ p.u.
- $c = 2.0$ p.u.
- $d = 1.0$ p.u.

Graph showing the variation of $\delta_3$, degrees with time, seconds.
Fig. 3-8 Schematic diagram of radially-connected multimachine system (Example II).
### Table 3.4
TRANSMISSION LINE CONSTANTS FOR SYSTEM

<table>
<thead>
<tr>
<th>Line</th>
<th>Bus To Bus</th>
<th>p.u. impedance</th>
<th>Ysh p.u.</th>
</tr>
</thead>
<tbody>
<tr>
<td>From Bus</td>
<td>To Bus</td>
<td>R1</td>
<td>X1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0.0078</td>
<td>0.0780</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.0110</td>
<td>0.1150</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>0.0014</td>
<td>0.0180</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.0051</td>
<td>0.0765</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>0.0045</td>
<td>0.0645</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>0.0024</td>
<td>0.0720</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>0.0011</td>
<td>0.0170</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>0.0005</td>
<td>0.0075</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>0.0030</td>
<td>0.0450</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td>0.0030</td>
<td>0.0600</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
<td>0.0060</td>
<td>0.0904</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>0.0080</td>
<td>0.0800</td>
</tr>
</tbody>
</table>

### Table 3.5
LOADS ON THE SYSTEM

<table>
<thead>
<tr>
<th>Bus</th>
<th>Loads</th>
<th>it Bus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P in MW</td>
<td>Q in MVAR</td>
</tr>
<tr>
<td>1</td>
<td>6960</td>
<td>2960</td>
</tr>
<tr>
<td>4</td>
<td>2060</td>
<td>876</td>
</tr>
<tr>
<td>5</td>
<td>220</td>
<td>352</td>
</tr>
<tr>
<td>6</td>
<td>630</td>
<td>61</td>
</tr>
<tr>
<td>8</td>
<td>1930</td>
<td>820</td>
</tr>
<tr>
<td>11</td>
<td>1430</td>
<td>610</td>
</tr>
<tr>
<td>13</td>
<td>8410</td>
<td>3580</td>
</tr>
</tbody>
</table>
Table 3.6
SYNCHRONOUS MACHINE DATA

<table>
<thead>
<tr>
<th>Generator Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>At Bus</td>
<td>8</td>
<td>4</td>
<td>1</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>MVA Rating</td>
<td>6000</td>
<td>4500</td>
<td>5500</td>
<td>1980</td>
<td>8150</td>
</tr>
<tr>
<td>$H$ Sec.</td>
<td>6.65</td>
<td>46.00</td>
<td>53.00</td>
<td>14.00</td>
<td>91.00</td>
</tr>
<tr>
<td>$x_d'$ p.u.</td>
<td>0.0715</td>
<td>0.0750</td>
<td>0.0750</td>
<td>0.2120</td>
<td>0.0430</td>
</tr>
<tr>
<td>$x_d$ p.u.</td>
<td>0.216</td>
<td>0.212</td>
<td>0.212</td>
<td>0.550</td>
<td>0.150</td>
</tr>
<tr>
<td>$x_q$ p.u.</td>
<td>0.131</td>
<td>0.141</td>
<td>0.141</td>
<td>0.410</td>
<td>0.080</td>
</tr>
<tr>
<td>$T_{do}$ sec.</td>
<td>6.20</td>
<td>9.45</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
</tr>
<tr>
<td>$T_{ex}$ sec</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$T_d$ sec.</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$E_q'$ p.u.</td>
<td>1.0510</td>
<td>1.1128</td>
<td>1.3429</td>
<td>1.0629</td>
<td>1.1675</td>
</tr>
<tr>
<td>$\delta'$ deg.</td>
<td>34.59</td>
<td>1.68</td>
<td>60.27</td>
<td>37.45</td>
<td>17.83</td>
</tr>
<tr>
<td>$P_G$ MW</td>
<td>5373</td>
<td>3940</td>
<td>3200</td>
<td>1910</td>
<td>7640</td>
</tr>
<tr>
<td>$Q_G$ MVAR</td>
<td>-61.2</td>
<td>1136.2</td>
<td>4350.7</td>
<td>115.4</td>
<td>3220.0</td>
</tr>
</tbody>
</table>
The fault is cleared after a definite time interval $TFC=0.012$ second by opening of circuit breakers at the two ends of the line. The system is unstable for this time of fault clearance.

The fastest machine is machine-1. A TCDB is connected to the terminals of machine-1 with braking resistance $0.5$ per unit. The TCDB is regulated according to the following law:

$$\alpha = \frac{2\pi}{3} - \frac{K}{\omega_1} \Delta \omega_1, \quad \text{for } \Delta \omega_1 > 0,$$

$$\alpha = \frac{2\pi}{3}, \quad \text{for } \Delta \omega_1 \leq 0$$

where $\Delta \omega_1$ is the rotor relative angular velocity of machine-1 with respect to the synchronous angular velocity in radians per second.

The swing curves showing the variation of $\delta_{13}^*$, the relative angle between machine-1 and machine-3, for different values of regulation coefficient are shown in Fig.3.9. The plot of angle $\delta_{13}^*$ is only shown as the variation of this angle is maximum among all the relative angles between machines. It may be observed that maximum damping is obtained for a value of $k_\omega$ around 1.0.

The magnitude of braking action on the synchronous generator depends on braking resistance value $R$ of the TCDB besides the regulation coefficient $k_\omega$. The effect of variation of $R$ on transient swings is shown in Fig.3.10. It may be noted from the swing curves that the best value of $R$ is
Fig. 3-9 Effect of variation of $K_\omega$ on transient swings of $\delta_{13}$. 

Symbols:

- a 0.01
- b 0.10
- c 1.0
- d 3.0, 10.0

θ degrees

0 50 100 150 200

1 2 3 4 5
Fig. 3.10 Selection of best value of $R$ for TCDB in case of example II.
around 2.0 per-unit as maximum damping of swings is obtained at this value of $R$.

The various combinations of rotor relative angular velocity have been tried for the regulation of triggering of thyristor bridge, and it has been observed, as in the case of ring-connected multimachine system, that the damping of transient swings for the first few swings is same. Hence the TCDB may be controlled by the relative angular velocity of the machine to which it is connected.

3.3 EFFECT OF VARIATION OF AIR GAP FLUX LINKAGES OF SYNCHRONOUS MACHINE

In this case the equation relating emf's $E_{Qi}$ and $E_{qi}'$ of the $i$-th machine may be written as:

$$\tilde{h}_i E_{qi}' = a_{ii} E_{Qi} + \sum_{k=1}^{n} a_{ik} E_{Qk}$$

$$i = 1, 2, \ldots, n$$

where

$$\tilde{h}_i = \frac{1}{x_{qi} - x_{di}}$$

$$a_{ii} = \tilde{h}_i + y_{ii} \sin \theta_{ii}$$

$$a_{ik} = y_{ik} \sin (\theta_{ik} - \delta_{ik})$$
Where

\[ E'_{qi} = \text{quadrature-axis component of the voltage behind direct-axis transient reactance of the } i\text{-th machine in per-unit;} \]

\[ F_{Qi} = \text{voltage behind quadrature axis synchronous reactance of the } i\text{-th machine in per-unit;} \]

\[ x'_{di} = \text{direct-axis transient reactance of the } i\text{-th machine in per-unit;} \]

\[ x_{qi} = \text{quadrature-axis synchronous reactance of the } i\text{-th machine in per-unit.} \]

With the change in operating condition the system parameters change, thus the parameters \( a_{ii}, a_{ij} \) and \( \beta_i \) also change. Substituting the pre-fault values of \( F'_{qi} \) in Eq.(3.9) and parameters \( a_{ij} \) and \( \beta_i \) corresponding to faulted condition, it is possible to find out the values of all the emfs \( E_{Qi'} \). The eq.(3.9) should be solved in each time interval \( \Delta t \) since \( F'_{q} \) and the coefficients \( a_{ij} \) vary with time. The eq.(3.9) may be rewritten as

\[ E_{Qi} = \frac{a_i}{\beta_{ii}} F'_{qi} - \frac{1}{a_{ii}} \sum_{j=1}^{n} a_{ij} E_{Qj} \quad (3.10) \]

The set of equation (3.10) may be solved by iterative method to determine emfs \( E_{Qi} \):
The internal induced emf $E_{qi}$ of the $i$-th synchronous machine may be determined from the following equation in terms of $E_{qi}$ and $E_{qj}$ as

$$E_{qi} = E_{qi} \frac{x_{di}-x_{d}'}{x_{di}-x_{di}'} - E_{qj} \frac{x_{di}-x_{qi}'}{x_{qi}-x_{di}'}$$  \hspace{1cm} (3.11)$$

The change in $E_{qi}$ during an interval $t$ is given by

$$\Delta E'_{qi} = \frac{E_{exi}(k) - E_{qi}(k-1)}{t_{doi}} \Delta t$$  \hspace{1cm} (3.12)$$

$i = 1,2,3,...,n.$

$k = \text{number of the interval.}$

where

$E_{exi}(k) = \text{excitation voltage referred to stator side}$

of $i$-th machine for $k$-th interval;

$E_{qi}(k-1) = \text{internal emf induced of the } i\text{-th machine}$

in $(k-1)$-th interval.

The electric power output of the machines can be obtained from

$$P_{Gi} = E_{qi}^2 Y_{ii} \cos \theta_{ii} + E_{qi} E_{qj} Y_{ij} \cos(\theta_{ij} - \delta_{ij})$$  \hspace{1cm} (3.13)$$

where $Y_{ii}$ and $Y_{ij}$ are the magnitudes of driving point and transfer admittances of the reduced network with only internal buses of the machines, and $\theta_{ii}$ and $\theta_{ij}$ are the corresponding admittance angles.
The change in rotor angles may be obtained from the solution of swing equations which are given below

\[ p \delta_i' = \Delta \omega_i = \omega_i - \omega_o \]

\[ p \omega_i = \frac{\pi f}{H_i} (D_i - G_i) \quad (3.14) \]

where the variables have their usual meaning.

Illustrative Example:

The same ring connected multimachine system used in section 3.1 has been chosen for stability investigations taking into account variation in air-gap flux linkages. A three phase short circuit is assumed near bus-3 on the line connecting buses 3 and 4. The fault is cleared by opening the line 3-4. Different fault clearing instants are considered to determine the critical fault clearing time. The swing curves for two values of TFC, 0.35 and 0.36674 seconds, are shown in Fig.3.11. From these swing curves it may be observed that the critical fault clearing time is 0.35 second.

A TCDB is assumed to be connected to the terminals of machine-3 just after the fault is cleared in 0.36674 second. The triggering of the TCDB is made a function rotor relative angular velocity of machine-3 and regulated according to the law given by eq.(3.4).
Fig. 3.11 Determination of critical clearing time taking into account variation of air-gap flux linkages of the machines.
The corresponding swing curve is shown by curve-c of Fig. 3.12. In Fig. 3.12 the swing curve without dynamic brake is shown by curve-a while curve-b represents the swing curve when an optimally switched dynamic brake is used. The magnitude of braking resistance used both for TCDB and optimally switched dynamic brake is same. From the swing curves of Fig. 3.12 the relative effectiveness of the TCDB and optimally switched dynamic brake in restoring the system stability is clearly illustrated.

3.4 CONSIDERATION OF TRANSIENTS IN EXCITATION REGULATORS

When a synchronous machine is equipped with excitation regulators then the magnitude of \( E_{ex} \), the excitation voltage, changes at each interval of time \( \Delta t \) of the swing curve computation. The computations are simplified if the regulator characteristics for a given operating condition are assumed to be linear during time interval \( \Delta t \). If the regulator input characteristic is assumed to increase in steps after each interval \( \Delta t \), then the output can be calculated by the method given below [1].

The transient process in a regulator can be written
\[ R = 5.0 \text{ p.u.} \]

1. Without dynamic brake
2. With constant dynamic brake
3. Thyristor controlled dynamic brake in action.

Fig. 3-12 Swing curves comparison of optimally switched dynamic brake and TCDB when variation of flux linkages are taken into account.
\[ D(p) \cdot Y = F(p) \cdot X \]  

\[ X, Y = \text{Laplace transform of input and output variables respectively.} \]

\[ D(p), F(p) = \text{functions of operator } p. \]

If \( F(p) \) \( X \) is a constant during each interval, then it may be assumed to change in steps as shown in Fig.3.13. In this case for the \( k \)-th interval the following expression may be written for regulator output as a time function \( V(t) \) as

\[ V_n = V_o + a_1 g(k\Delta t) + a_2 g(k-1\Delta t) + \ldots + a_{k-1} g(2\Delta t) + a_k g(\Delta t) \]  

(3.16)

where \( V_o \) is the value of regulator output before the transient disturbance and \( g(t) \) is time domain representation of the operator relation given by

\[ D(p)Y = 1 \]  

(3.17)

The precalculated characteristics \( g(t) \) makes it possible to determine according to (3.16) the output from regulator at each interval.

Now if the excitation characteristic is assumed to be of the form

\[ (1+sT_d)(1+sT_{ex}) F_{ex}(s) = K_V \Delta V(s) \]  

(3.18)
Fig. 3-13 Approximation of function $F(P)X$ by a stepped curve.
where

\[ T_d \] = time constant representing delay in the exciter regulator channel in seconds,

\[ T_{ex} \] = time constant of the exciter circuit in seconds,

\[ \frac{\Delta V(s)}{s} \] = Laplace transform of step change of error voltage in per-unit,

\[ K_V \] = gain constant for the voltage channel in per unit excitation per-unit voltage change,

\[ E_{ex}(s) \] = Laplace transform of the excitation signal variation in per-unit.

Then,

\[ E_{ex}(s) = \frac{K_V \frac{\Delta V(s)}{s}}{s(1+sT_d)(1+sT_{ex})} \] \hspace{1cm} (3.19)

Assuming \( K_V \frac{\Delta V(s)}{s} = 1 \), for simplicity and taking inverse Laplace transform of eq.(3.19), the change in excitation voltage in time domain is given by

\[ E_{ex}(t) = 1 - \frac{T_{ex}}{T_{ex} - T_d} e^{-t/T_{ex}} + \frac{T_d}{T_{ex} - T_d} e^{-t/T_d} \] \hspace{1cm} (3.20)

Then the excitation voltage at the k-th interval may be expressed as
\[ E_{ex,k} = E_{ex,(k-1)} + [a_k - E_{ex,(k-1)}] \cdot \left[ 1 - \frac{e^{-\frac{\Delta t}{T_{ex-Td}}}}{\frac{T_{ex}}{T_{ex-Td}}} - \frac{T_d}{T_{ex-Td}} e^{-\frac{\Delta t}{T_d}} \right] \] (3.21)

where,

\[ a_k = e^{exo} + \sum_{i=1}^{k} \Delta a_i \]

Eq(3.21) may be used along with eq.(3.10) - (3.12) to calculate change in \( E_q \) at each interval.

### 3.4.1 ILLUSTRATIVE EXAMPLE

The system chosen for investigations is the radially interconnected multimachine system used in section 3.2.

A three phase fault is assumed near bus-7 on one of the double circuit lines connecting buses 6 and 7. The generators 1 and 2 are considered with high ceiling rapid response excitation systems. Ceiling values selected for the excitation systems are +7.0 per-unit and -5.0 per-unit. Besides for generator-1 relay boosting of excitation to the positive ceiling value for a period of 0.3 second immediately after the fault is considered. The rest of the machines are assumed to be equipped with proportional
type of excitation regulators. The excitation variation of
generators 1 and 2 has been assumed to be of the following
form:

\[(1 + pT_{d1}) (1 + pT_{ex1}) \Delta E_{ex1} = K_{V1} \Delta V_1 + K'_{\delta_{12}} \delta_{12} + K'_{\delta_{13}} \delta_{13} + K''_{\delta_{12}} \delta_{12}^2 + K''_{\delta_{13}} \delta_{13}^2\]

and

\[(1 + pT_{d2}) (1 + pT_{ex2}) \Delta E_{ex2} = K_{V2} \Delta V_2 + K'_{\delta_{21}} \delta_{21} + K'_{\delta_{23}} \delta_{23} + K''_{\delta_{21}} \delta_{21}^2 + K''_{\delta_{23}} \delta_{23}^2\]

(3.22)

where \(\Delta V_1\) and \(\Delta V_2\) are the deviations in the terminal voltages
of generator 1 and 2 respectively. The regulation gains
have the following values:

\[K_{V1} = K_{V2} = 50 \text{ per-unit excitation/per-unit voltage;}\]
\[K_{V3} = K_{V4} = K_{V5} = 30 \text{ per-unit excitation/per-unit voltage;}\]
\[K'_{\delta_{12}} = K'_{\delta_{13}} = 1.4 \text{ per-unit excitation/radians/second;}\]
\[K'_{\delta_{21}} = K'_{\delta_{23}} = 0.8 \text{ per-unit excitation/radians/second;}\]
\[K''_{\delta_{12}} = K''_{\delta_{13}} = 0.3 \text{ per-unit excitation/radians/second};\]
\[K''_{\delta_{21}} = K''_{\delta_{23}} = 0.48 \text{ per-unit excitation/radians/second}.\]
The firing delay angle of the thyristor controlled dynamic brake connected to the fastest outgoing machine i.e. machine-1 is controlled according to the following expression:

$$\alpha = \frac{2\pi}{3} - \frac{v}{\omega} (\omega_{12} + \omega_{13}), \quad \text{for } (\omega_{12} + \omega_{13}) > 0,$$

$$= \frac{2\pi}{3}, \quad \text{for } (\omega_{12} + \omega_{13}) \leq 0.$$ 

Swing curves given in Fig.3.14a correspond to the case when the stability is lost when the only transient stability improvement measure in the system is the high ceiling rapid response excitation regulation on machines 1 and 2. It may be seen that the system stability is not lost in the first swing but in subsequent swings. The nature of variation of rapid response excitation system considered for machines 1 and 2 is also shown in Fig.3.14b. Fig.3.15 corresponds to the case when a conventional dynamic brake is applied to machine-1 in order to improve the transient stability. A value of shunt resistance R of 0.5 per-unit is chosen for this purpose, and the time of its connection being 0.3 seconds. This is found to be ineffective in restoring the system stability though it delays loss of synchronism by a few swings. A TCDB having the same value of shunt braking resistance and gain $\kappa_w = 3.0$, applied to machine-1 gives rise to the swing curves brought out in Fig.3.16. It may be seen that the TCDB very effectively restores the system stability and controls the transient swings to a large extent.
Fig. 3.14 (a) Swing curves for radially connected multimachine systems taking into account forced excitation regulation.

(b) Variation of excitation voltages for machines 1 and 2.
Fig. 3-15 Swing curves showing the effect of conventional dynamic brake when applied along with forced excitation regulation.

TFC 0.0925 sec.
TBR 0.3925 sec.
Fig. 3.16 Swing curves showing the effect TCDB when applied in conjunction with forced excitation regulation.

\( R = 0.5\) p.u.
3.5 CONCLUSIONS

The dynamic brake is one of the effective discrete supplementary controls in improving multimachine transient stability. In the present chapter with reference to two different configurations of multimachine systems it is shown that a TCDB is the best in performance of all such dynamic brakes. The better performance of a TCDB is due to the fact that where an optimally switched dynamic brake acts for a brief time and is successful in controlling first few swings only, the TCDB acts intermittently at the right moments to control every swing during the entire transient period. Thus in one way it is like repeated electric braking with the difference that the braking power exerted instead of remaining same throughout is regulated proportional to the rotor relative angular velocity during each swing of the machines besides making the multiple switching of the dynamic brake automatic. The investigations have also shown that in certain situations as in the case of radial type multimachine system discussed provision of usual transient stability improvement measures such as high ceiling rapid response excitation is alone not sufficient sometimes to retain the system stability especially if the fault is of a severe type. In these cases the use of TCDB alone, or in conjunction with high ceiling rapid response excitation system succeeded in damping out the transient swings considerably and is found to be very effective in maintaining the system stability.