CHAPTER - 4

TOPOLOGICAL DESIGN OF A COMPUTER COMMUNICATION SYSTEM
FOR OPTIMIZING TERMINAL RELIABILITY

A computer communication system is either a set of several interconnected computers or a set of terminals connected to one or more computers [22]. One of the main reasons for the great interest shown in the computer communication networks is the considerable economy that can be achieved through resource sharing [15]. These computer networks made their noticeable appearance in the form of packet switching with the ARPANET [17] developed by the Advanced Research Project Agency in America. 'NASDAQ' (National Association of Securities Dealers Automated Quotation) system is a system of computers, communication links and terminal devices, designed primarily for the collection and distribution of real-time quotations for the over-the-counter securities market. The NASDAQ system is composed of low-medium- and high-speed communication lines (from 1600-50,000 bits/sec) tied to the NASDAQ Central Processing Centre in Trumbull through control units and data concentrators [53].

The ARPA network has probably generated more interest in the field of computer networking than any other network in the world. The ARPA network provides store-and-forward communications between a set of computers distributed across the United States. The message handling at each centre in the
network is performed by a special purpose interface message processor (IMP). The centres are connected through the IMPs by fully duplex high-speed communication lines. Messages are broken up into sets of packets each with header information. Each packet independently makes its way through the network to its destination. A goal of the system is to achieve an average delay of 0.2 sec. for single packet. The network is required to undertake the variation of information flow without degradation of its performance. For reliability, the network is so designed that the network will fail only if at least two of its lines fail. Control is provided which can alter the routing and the traffic flow on the basis of the present status of the system.

TELENET is another example of the commercial data carrier which is operational for quite sometime. SITA (Societe Internationale de Telecommunication Aeronautiques) is another computer communication system which is operative as an airline reservation system.

Normally all types of CCNs are having different philosophy of operation and their design and development depends upon:

(i) The design of the IMPs, concentrators and multiplexers to be used.

(ii) The topological modelling and design, to specify the capacity and location of each communication link within the circuit.
(iii) The design of the protocol to meet the user's requirements.

In this chapter, an attempt is made to evolve some heuristic algorithms to optimize the system in regard to the problem mentioned at (ii) above.

There are some inevitable difficulties which are encountered during the topological optimization of the computer communication network. Some of these are listed below:

(a) **Network Choice**

In general, there are:

$$\frac{[n(n-1)/2]}{[\{\binom{n-1}{2} - m\}]m!]$$

ways of arranging m links amongst n nodes. Finding an exact optimal topological layout of links for a sufficiently large system is impossible even with a powerful computing system.

(b) **Discrete Elements**

The components used in the CCN are available only in discrete sizes. Thus line speeds will have discrete values such as 2, 2.4, 3.6, ..., 50, 240 K bits/sec. This means the integer optimization problem must be solved, for which no theoretical methods are feasible for the large practical problems except for centralized tree design.
(c) **Non-Linearities**

Time delay functions, component cost, structure and reliability functions are all non-linear. Typical life cycle cost functions are neither 'Concave' nor 'Convex' and thus no analytical method is easily applicable to determine the optimal solution. A typical cost versus reliability plot is shown in Fig. 4.1.

The object of the network optimization is to provide a 'network design' which meets all design constraints and has the lowest possible cost. One of the fundamental considerations in the design of a CCN is the reliability and availability of the communication paths between specified pairs of nodes. These characteristics strongly depend upon the topological layout of the communication links in addition to the reliability and availability of the individual computer system and communication facilities. It is assumed that communication is desired from a specified node, called source, to another specified node, called sink.

Having the knowledge of $n$ geographically located nodes, $m$ interconnecting links and cost and reliability of all connecting links, an optimal network topology is to be evolved, which gives a maximum terminal reliability.

Simple and efficient heuristic methods for the solution of the above problem are discussed here. The additional and significant advantage of designing the system by the proposed methods will be that one has not to redesign
Fig 41 COST VERSUS RELIABILITY
the whole system, if the permissible cost is increased at a later stage by the budgetary provisions. The existing system can be updated depending upon the additional resources available for the installation of more links.

The following assumptions are made in addition to that of section 1.6:

(i) The location of the various computers or concentrators and the possible locations of the connecting links are known.

(ii) Cost of establishing each possible connecting link is known.

(iii) Reliability of every possible link is also known.

(iv) Cost of interconnection of a link to an already existing node is negligible.

4.1 METHOD

The method proceeds with the determination of all the minimal paths from the source to the sink (terminal) node, assuming all possible links in position. Since a path is defined as an open walk from the source node to the sink node [81], the cost of this path is equal to the sum of all the link costs contained in this path. The reliability of a path is the product of the reliabilities of the links contained in this path. We choose that
particular path for which the ratio of path-reliability to path-cost is maximum. Depending upon the balance of the cost available, branches are now sequentially added to the network as explained below:

For all presently available link positions, a ratio of increase in the s-t reliability to the increase in the cost is worked out, if any particular link is to be added to the network. A link for which this ratio is maximum, is added, subject to the cost constraint. The augmented network, now becomes the starting point and the whole procedure is repeated for the remaining possible links. This is continued as long as cost constraint is not violated. The method is illustrated by considering typical examples.

4.2 ALGORITHM

An algorithm based upon the method described in section 4.1 is outlined by following steps:

Step 1: Assuming all possible links in position, determine all the minimal s-t paths using any of the existing methods such as [81,84,85,89,90,91].

Step 2: Generate the path-cost matrix, $P_c$ and path-reliability matrix $P_r$, such that,

$$P_c(i,j) = \begin{cases} 
c^*_j; & \text{the cost of the } j^{th} \text{ branch if this branch exists in } i^{th} \text{ path} \\
0; & \text{otherwise} 
\end{cases}$$
and

\[ P_r(I,J) = \begin{cases} \Pr(I,J) & \text{the reliability of } j^{th} \text{ branch if this branch exists in } i^{th} \text{ path} \\ 1 & \text{otherwise} \end{cases} \]

**Step 3:** Determine the cost of each path and generate a column matrix, defined by,

\[ F(I) = \sum_{J=1}^{m} P_c(I,J) ; \quad I = 1,2,\ldots,H. \]

**Step 4:** Determine the path-reliabilities and form a matrix \( T \),

\[ T(I) = \prod_{J=1}^{m} P_r(I,J) ; \quad I = 1,2,\ldots,H. \]

**Step 5:** Determine the ratio of path-reliability to that of path-cost and generate a column matrix \( D \), where

\[ D(I) = \frac{T(I)}{F(I)} ; \quad I = 1,2,\ldots,H. \]

**Step 6:** Choose \( k \) such that \( D(k) \geq D(I) \neq k \). Determine \( F(k) \) and \( T(k) \). The balance cost available is \([CS - F(k)]\).

If \([CS - F(k)]\) is negative, let \( D(k) = 0 \) and repeat this step to find another value of \( k \).

If \([CS - F(k)]\) is zero, this \( k^{th} \) path is the optimal solution.
If \([CS - F(k)]\) is positive, go to the next step.

**Step 7:** Remove the branches already used from further consideration and arrange the remaining branches in order of their ascending costs. Further remove any branches whose addition is not possible because balance cost available is less than the cost of the link under consideration. If all the branches are removed, stop; otherwise go to next step.

**Step 8:** For each of the remaining branches, compute

\[\Delta R(I) \text{: increment in the } s-t \text{ reliability if } I^{th} \text{ branch is added to the network,}\]

and

\[\Delta C(I) \text{: cost of } I^{th} \text{ branch, and hence}\]

\[\Delta D(I) = \frac{\Delta R(I)}{\Delta C(I)}\]

**Step 9:** Choose \(k\) such that \(\Delta D(k) \geq \Delta D(I)\) for all \(I\) under consideration in the above step. Augment the network with branch \(k\) and go back to step 7. If, however, \(\Delta D(I) = 0\) for all \(I\) under consideration in the above step, add the first branch in the network and go back to step 7.

This algorithm is computerized making use of FORTRAN IV and the program listing is shown in the Appendix A-II. The program was run on a DEC-20 system. The flow chart of the program is presented in Appendix A-III.
4.3 TERMINAL RELIABILITY OPTIMIZATION BASED ON PATH ENUMERATION

Another method for designing an optimal computer communication network having the same constraints as mentioned in section 4.1 and section 4.2, is discussed in this section. It proceeds with the determination of all s-t paths of the possible network topology. We choose the $k^{th}$ path which has the maximum reliability to cost ratio. The other paths are now arranged in the ascending order of their costs. All these paths are added sequentially and on every iteration, the increase in s-t reliability and the cost increase is determined. All those links of that path are placed in position, which yield the maximum ratio of the increase in reliability to the cost of that path. This process is continued till the total cost available is exhausted.

Example 4.1: An ARPANET has five nodes fixed in position and it is possible to have at most 7 links connected in the topology shown in Fig. 4.2 and has the following data:

<table>
<thead>
<tr>
<th>Branch</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Reliability</td>
<td>0.9</td>
<td>0.6</td>
<td>0.7</td>
<td>0.9</td>
<td>0.9</td>
<td>0.6</td>
<td>0.7</td>
</tr>
</tbody>
</table>

The total permissible cost $CS = 20$ units.
The solution takes the following steps:

**Step 1:** The minimal s-t paths are:

- b, f; a, c, g; a, c, f; a, d, e, f; b, e, g; b, c, d, g; a, c, e, g;

**Step 2:**

\[
P_c = \begin{bmatrix}
0 & 3 & 0 & 0 & 0 & 4 & 0 \\
5 & 0 & 0 & 4 & 0 & 0 & 2 \\
5 & 0 & 2 & 0 & 0 & 4 & 0 \\
5 & 0 & 0 & 4 & 6 & 4 & 0 \\
0 & 3 & 0 & 0 & 6 & 0 & 2 \\
0 & 3 & 2 & 4 & 0 & 0 & 2 \\
5 & 0 & 2 & 0 & 6 & 0 & 2
\end{bmatrix}
\]

\[
P_r = \begin{bmatrix}
1 & 0.6 & 1 & 1 & 1 & 0.6 & 1 \\
0.9 & 1 & 1 & 0.9 & 1 & 1 & 0.7 \\
0.9 & 1 & 0.7 & 1 & 1 & 0.6 & 1 \\
0.9 & 1 & 1 & 0.9 & 0.9 & 0.6 & 1 \\
1 & 0.6 & 1 & 1 & 0.9 & 1 & 0.7 \\
1 & 0.6 & 0.7 & 0.9 & 1 & 1 & 0.7 \\
0.9 & 1 & 0.7 & 1 & 0.9 & 1 & 0.7
\end{bmatrix}
\]
Steps 3, 4 and 5:

\[
\begin{bmatrix}
7 & 0.3600 \\
11 & 0.5670 \\
11 & 0.3780 \\
15 & 0.3969 \\
\end{bmatrix}
\]

\[
F(I) = 19 ; \ T(I) = 0.4374 ; \ D(I) = 0.02302
\]

\[
\begin{bmatrix}
0.05143 \\
0.05155 \\
0.03436 \\
0.02646 \\
\end{bmatrix}
\]

Step 6: Max (D) = 0.05155 for I = 2,

\[
F(2) = 11
\]

and

\[
T(2) = 0.5670
\]

thus branches a,d and g are connected in the network, as shown in Fig. 4.3.

As \([CS - F(2)]\) i.e., \([20 - 11]\) > 0 ; go to next step.

Step 7: The branches for further consideration at this stage, after properly arranging are: \{c,b,f,e\}.

Step 8: Now AR(I) = 0 for all I, as inserting any of the links c,b,f or e does not yield any additional path as shown in Figs. 4.4, 4.5, 4.6 and 4.7.

Step 9: Add the branch c with cost 2 units as shown in Fig. 4.4.
**Step 7:** The branches still under consideration are: \{b, f, e\}.

**Step 8:**

(i) Inserting the branch b in Fig. 4.4, augmented network of Fig. 4.8 will have s-t paths a, d, g; and b, c, d, g; for which, after disjointing, the reliability expression is:

\[ R = P_a P_d P_g + q_a P_b P_c P_d P_g \]

Therefore,

\[ \Delta R(1) = q_a P_b P_c P_d P_g = 0.02646 \]

hence

\[ \Delta C(1) = 3 \]

\[ \Delta D(1) = 0.00882 \]

(ii) Inserting the branch f, the augmented network of Fig. 4.9 will have paths a, d, g and a, c, f; for which after disjointing give the reliability expression as:

\[ R = P_a P_d P_g + P_a P_c Q_d P_f + P_a P_c P_d P_f Q_g \]

Therefore,

\[ \Delta R(2) = P_a P_c Q_d P_f + P_a P_c P_d P_f Q_g = 0.13986 \]

hence

\[ \Delta C(2) = 4 \]

\[ \Delta D(2) = 0.034965 \]

(iii) Now insert branch 'e' to form the augmented network of Fig. 4.10, giving paths a, c, e, g and a, d, e. The reliability expression will become:

\[ R = P_a P_d P_g + P_a P_c Q_d P_e P_g \]
Therefore,

\[ \Delta R(3) = p_a p_c q_d p_e 0_g = 0.03969 \]

\[ \Delta C(3) = 6 \]

hence

\[ \Delta D(3) = 0.006615. \]

**Step 9:** As \( \Delta D(2) > \Delta D(1) \) and \( \Delta D(2) > \Delta D(3) \), add branch \( f \) (as shown in Fig. 4.9) also to the network and go back to step 7.

**Step 7:** Noticing that the balance cost available now is \([20 - 11 - 2 - 3] = 4\), the set of branches under consideration is \{b\} only.

**Step 8:** Insert the branch \( b \) in Fig. 4.9 to form Fig. 4.11.
Determine the s-t paths (b,f; a,d,g; a,c,f; b,c,d,g) and s-t reliability.

**Step 7:** As all the branches are now removed from further consideration, the desired optimal solution is obtained as shown in Fig. 4.11.

Therefore, the cost of this designed system is 20 units and the reliability is 0.7894.

**Example 4.2:** A computer communication network containing 6 nodes and 8 possible links has the topological layout as shown in Fig. 4.12 and has the following data:
<table>
<thead>
<tr>
<th>Branch</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Reliability</td>
<td>0.9</td>
<td>0.7</td>
<td>0.8</td>
<td>0.6</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Maximum permissible cost $CS = 20$ units.

**Step 1:** The minimal $s$-$t$ paths are: $a,d,g$; $b,e,h$; $a,c,e,h$; $b,c,d,g$; $a,d,f,h$; $b,e,f,g$; $a,c,e,f,g$; $b,c,d,f,h$.

**Step 2:**

$P_c = \begin{bmatrix}
2 & 0 & 0 & 6 & 0 & 0 & 4 & 0 \\
0 & 5 & 0 & 0 & 4 & 0 & 0 & 3 \\
2 & 0 & 3 & 0 & 4 & 0 & 0 & 3 \\
0 & 5 & 3 & 6 & 0 & 0 & 4 & 0 \\
2 & 0 & 0 & 6 & 0 & 3 & 0 & 3 \\
0 & 5 & 0 & 0 & 4 & 3 & 4 & 0 \\
2 & 0 & 3 & 0 & 4 & 3 & 4 & 0 \\
0 & 5 & 3 & 6 & 0 & 3 & 0 & 3
\end{bmatrix}$

$P_r = \begin{bmatrix}
0.9 & 1 & 1 & 0.6 & 1 & 1 & 0.7 & 1 \\
1 & 0.7 & 1 & 1 & 0.9 & 1 & 1 & 0.8 \\
0.9 & 1 & 0.8 & 1 & 0.9 & 1 & 1 & 0.8 \\
1 & 0.7 & 0.8 & 0.6 & 1 & 1 & 0.7 & 1 \\
0.9 & 1 & 1 & 0.6 & 1 & 0.8 & 1 & 0.8 \\
1 & 0.7 & 1 & 1 & 0.9 & 0.8 & 0.7 & 1 \\
0.9 & 1 & 0.8 & 1 & 0.9 & 0.8 & 0.7 & 1 \\
1 & 0.7 & 0.8 & 0.6 & 1 & 0.8 & 1 & 0.8
\end{bmatrix}$
Steps 3, 4 and 5:

\[
\begin{bmatrix}
12 & 0.1780 & 0.0315 \\
12 & 0.3040 \\
12 & 0.5184 & 0.0432 \\
18 & 0.2152 & 0.0136 \\
14 & 0.5629 & 0.0226 \\
16 & 0.0247 \\
20 & 0.0107
\end{bmatrix}
\]

\[
F(I) = 10^3, T(I) = 10^3, D(I) = 10^3
\]

Step 6: Max (D) = 0.0432 for I = 3,

\[
F(3) = 12 \\
T(3) = 0.5184
\]

Thus branches a, c, e and h are connected in the network as shown in Fig. 4.13.

Step 7: The branches for further consideration at this stage, after properly arranging are: \{f, g, b, d\}.

Step 8:

(i) Insert the branch f only in Fig. 4.13 to obtain Fig. 4.14, it yields no additional path.

Therefore, \(\Delta R(1) = 0\).

(ii) Insert the branch g only in Fig. 4.13 to form Fig. 4.15, it also yields no additional path.

Therefore, \(\Delta R(2) = 0\).
(iii) Insert the branch $b$ only to obtain the network of Fig. 4.16. The s-t paths of Fig. 4.16 are $a, c, e, h$ and $b, e, h$; which on disjointing give the reliability expression as:

$$R = P_a P_c P_e P_h + q_a P_b P_e P_h + P_a P_b P_c P_e P_h$$

Therefore,

$$\Delta R(3) = q_a P_b P_e P_h + P_a P_b q_c P_e P_h = 0.1411$$

$$\Delta C(3) = 5$$

hence

$$\Delta D(3) = 0.0262.$$  

(iv) Insert the link $d$ in position to form Fig. 4.17, which does not result in any additional path. Therefore, $\Delta R(4) = 0$. 

**Step 9:** Add the branch $b$ having cost of 5 units as shown in Fig. 4.16, and go to step 7. 

**Step 7:** The cost now available is 3 units. The branch under consideration is only $\{f\}$, since addition of branches $g$ or $d$ will violate the permissible cost constraint. 

**Step 8:** Inserting branch $f$ in the augmented network of Fig. 4.16 gives a network of Fig. 4.18, which again does not give any additional path and hence $\Delta R(5) = 0$. 

**Step 7:** As all the branches are now removed from further consideration, no more improvement is possible. Augmented network of Fig. 4.16 is thus the desired solution having cost...
of 17 units which is less than the maximum permissible cost of 20 units and reliability of 0.6595.

4.4 OPTIMAL SOLUTION INCORPORATING AN ADDITIONAL CONSTRAINT

In this section, we have considered a slightly different but equally realistic optimization problem. It is desired to maximize the reliability between two specified centres of the computers communication system having fixed geographical location of the various computers under the limitation that the total cost of establishing the various communication links at their predetermined locations is not exceeded beyond the required cost. The link costs and reliabilities are of course known. In addition, it is now made sure that the two specified computer centres can communicate with any of the computers located at their preassigned geographical positions.

This heuristic method is efficient and general and proceeds on the basis of the path enumeration between the two specified centres and makes use of the same assumptions as are prescribed for the method 4.1.

The method starts with the generation of only those possible success paths between the two specified nodes i.e., source and sink node, in which all the nodes are encountered once. These paths will constitute of (n-1) links, where n is the total number of nodes contained in the communication
network. Since any path is defined as an open walk from the source to the sink node, the cost of any path will be the arithmetic sum of all its constituent link-costs, while the reliability of any path is the product of the link-reliabilities of the links contained in it. Out of these possible paths, we choose that path for which reliability to cost ratio is maximum. Now depending upon the balance of cost (out of the permissible cost) available, branches are added to the network sequentially as follows.

Out of the available branches, a ratio of the increase in the s-t reliability to the increase in the cost is worked out after adding the particular branch or link into the network. The link, the addition of which yields the maximum reliability to cost ratio is retained and connected permanently into the network subject to the permissible cost constraint. The augmented network so created, thus forms the starting point and the whole procedure is repeated for the remaining possible links. This is carried out as long as permitted by the maximum allowable cost.

The algorithm for this method follows similar steps as mentioned in section 4.2.

**Example 4.3:** A modified graph of an ARPA network shown in Fig. 4.19 with six nodes and nine possible links having link cost-reliability data as follows:
Maximum permissible cost = 25 units.

Step 1: All the possible minimal s-t paths are: a, b, c; d, e, f; a, g, e, f; a, h, f; a, b, i, f; a, h, i, c; a, g, e, i, c; d, e, i, c; d, e, i, c; d, g, b, c; d, g, b, i, f; d, e, h, b, c; d, g, h, i, c; d, g, b, i, f; d, e, h, b, c; d, g, h, i, c;

Paths with cardinality (n-1) = 5 are: a, g, e, i, c; d, g, b, i, f; d, e, h, b, c; d, g, h, i, c.

Step 2:

$$\begin{bmatrix}
2 & 0 & 2 & 0 & 3 & 0 & 2 & 0 & 5 \\
0 & 3 & 0 & 4 & 0 & 5 & 2 & 0 & 5 \\
0 & 3 & 2 & 4 & 3 & 0 & 0 & 4 & 0 \\
0 & 0 & 2 & 4 & 0 & 0 & 2 & 4 & 5
\end{bmatrix}$$

$$PC = \begin{bmatrix}
0.8 & 1 & 0.8 & 1 & 0.8 & 1 & 0.8 & 1 & 0.7 \\
1 & 0.9 & 1 & 0.7 & 1 & 0.7 & 0.8 & 1 & 0.7 \\
1 & 0.9 & 0.8 & 0.7 & 0.8 & 1 & 1 & 0.8 & 1 \\
1 & 1 & 0.8 & 0.7 & 1 & 1 & 0.8 & 0.8 & 0.7
\end{bmatrix}$$
Steps 3, 4 and 5:

\[ F = \begin{bmatrix} 14 \\ 19 \\ 16 \\ 17 \end{bmatrix}, \quad T = \begin{bmatrix} 0.28672 \\ 0.24696 \\ 0.22256 \\ 0.20088 \end{bmatrix}, \quad D = \begin{bmatrix} 0.02048 \\ 0.01299 \\ 0.02016 \\ 0.01475 \end{bmatrix} \]

Step 6: \( \text{Max}(D) = 0.02048 \) for \( I = 1 \),

\[ F(1) = 14 \]

thus the links a, c, e, g, i are established in their position as shown in Fig. 4.20.

Since \([CS - F(1)] > 0\); go to next step.

Step 7: The links for further consideration after proper arrangement are : \( \{b, d, h, f\} \). Inserting the links b, d, h and f one by one in Fig. 4.20 to form Figs. 4.21, 4.22, 4.23, 4.24, respectively, we do not get any additional path having cardinality (n-1).

Step 9: Insert the lowest cost link b as shown in Fig. 4.21 and go to step 7.

Balance cost = \([25 - 14 - 3] = 8\).

Step 7: The links for further consideration are : \( \{d, h, f\} \).

Step 8: Inserting the links d, h and f sequentially in Fig. 4.21, to form Figs. 4.25, 4.26 and 4.27 respectively. No additional paths of cardinality (n-1) are obtained.
Step 9: Install the lowest cost link d as shown in Fig. 4.25 and go to step 7.

Step 7: Since the balance cost = $[25 - 21] = 4$, the link for further consideration is \{h\} only.

Step 8: Insert the link h in position in Fig. 4.25 to form Fig. 4.28. The paths of cardinality (n-1) are: a, g, e, i, c; d, g, h, i, c; d, e, h, b, c;

The reliability expression can be determined by disjointing these paths,

$$R = p_a p_c p_e p_i + q_a p_c p_d p_g p_i + p_a p_c p_q e p_g p_h p_i$$

$$+ q_a p_b p_c p_d p_g p_q h$$

$$+ p_a p_b p_c p_d p_e p_g p_h q_i$$

Substituting the values of the link-reliabilities,

$$R = 0.28672 + 0.28272 = 0.56944$$

Therefore, $\Delta R = 0.28272$ and $\Delta C = 5$

As there is no further branch to be considered, Fig. 4.28 is an optimal solution with maximum s-t reliability subject to the required total cost constraint of 25 units.

Total reliability = 0.56944 and Total cost consumed = 25 units.