3.0 Introduction

The design of the prototype filter is an important aspect in the design of the filter bank since all the sub channel filters are based on this filter. In the chapter the various errors occurring in the prototype filter of the cosine modulated filter banks (CMFBs) are listed. The cost function that is to be minimized in order to design the optimum prototype filter is defined. Several methods of design for the prototype filter available in literature are compared. As part of this work two methods for the design of the prototype filter have been proposed. These methods are described and compared with existing methods. The comparison is made based on performance, computational cost and ease of design.

3.1 The reconstructed signal

In the analysis filter bank the signal $x(n)$ is split into $M$ subband signals $x_k(n)$ by the $M$ analysis filter $H_k(z)$. Each of these signals is decimated by $N$ to obtain the signals $v_k(n)$. The synthesis filter bank reconstructs the signal $\hat{X}(z)$ from the subband signals $v_M(n)$ and $u_M(n)$ using the synthesis filters $F_M(z)$. The signal $\hat{X}(z)$ is the reconstructed signal.

Fig.3.1.1 An M-Channel QMF Filter Bank

In the analysis filter bank the signal $x(n)$ is split into $M$ subband signals $x_k(n)$ by the $M$ analysis filter $H_k(z)$. Each of these signals is decimated by $N$ to obtain the signals $v_k(n)$. The synthesis filter bank reconstructs the signal $\hat{X}(z)$ from the subband signals $v_M(n)$ and $u_M(n)$ using the synthesis filters $F_M(z)$. The signal $\hat{X}(z)$ is the reconstructed signal.
Each subband signal is given by
\[ X_k(z) = H_k(z) X(z) \] (3.1.1)

The decimated signals are given by
\[ V_k(z) = \frac{1}{N} \sum_{l=0}^{N-1} H_k(z^{1/N} W^l) X(z^{1/N} W^l) \] (3.1.2)
\[ W = W_N = e^{-\frac{j\pi}{N}} \] (3.1.3)

These signals pass through the N-fold expander and through the M synthesis filters where the M subband signals are combined to obtain the reconstructed signal \( x(n) \).

\[ U_k(z) = V_k(z^{1/N}) = \frac{1}{N} \sum_{l=0}^{N-1} H_k(z W^l) X(z W^l) \] (3.1.4)

The reconstructed signal is given by
\[ \hat{X}(z) = \sum_{k=0}^{M-1} F_k(z) U_k(z) = \frac{1}{N} \sum_{l=0}^{N-1} X(z W^l) \sum_{k=0}^{M-1} H_k(z W^l) F(z) \] (3.1.5)
\[ \hat{X}(z) = \sum_{l=0}^{N-1} A_l(z) X(z W^l) \] (3.1.6)

where
\[ A_l(z) = \frac{1}{N} \sum_{k=0}^{M-1} H_k(z W^l) F_k(z) \quad 0 \leq l \leq N-1 \] (3.1.7)

### 3.2 Distortion in Filter Banks

In a filter bank there are three sources of distortion, namely amplitude distortion, phase distortion and aliasing distortion. Filter banks rely on the flatness and the linear phase of the filters as well as of the orthogonality of the modulation functions to ensure proper amplitude and phase reconstruction. To reduce distortion caused by aliasing the filter banks cancel it out between adjacent bands and assume that there is no aliasing between nonadjacent bands. This is possible only if the stopband attenuation of the given filter is
infinite. In practice, it is sufficient if the attenuation is suitably high to ensure that the required output resolution in the reconstructed signal is obtained.

3.3 Errors in the filter bank system

Errors such as aliasing, imaging, amplitude and phase distortion error occur in the filter bank system as a result of which the reconstructed signal could differ considerably from the original signal [49].

3.3.1 Aliasing and Imaging errors

Due to interpolation and decimation, in the frequency domain there occurs imaging and aliasing of the spectrum respectively. It is practically impossible to design the analysis filters to have a zero transition bandwidth and zero stopband gain. The signals $x_k(n)$ are not band limited and hence decimation leads to aliasing as shown in Fig.2.6.4. Decimation of these signals results in aliasing even though the attenuation in the stopband of the filters is sufficiently high. In the M-channel filter bank the responses of the M analysis filters overlap, and each of the subband signals can have substantial energy in the bandwidth exceeding the passband. Errors due to aliasing can be completely eliminated by proper choice of synthesis filters $F_k(z)$. These filters are chosen such that the aliasing terms cancel out.

3.3.2 Amplitude and phase distortion

Unless aliasing is completely cancelled the M-Channel QMF bank will be periodically time varying one. So we should ensure that this is done. Once complete alias cancellation is obtained that is if

$$A_l(z) = 0 \quad \text{for } 1 \leq l \leq M-1$$

then

$$\hat{X}(z) = T(z)X(z) \quad (3.3.1)$$

$T(z)$ is the distortion function or the overall transfer function of the filter bank system.

$$T(z) = A_0(z) = \frac{1}{M} \sum_{k=0}^{M-1} H_k(z)F_k(z) \quad (3.3.2)$$
On cancellation of aliasing the filterbank system becomes a linear time invariant system with a transfer function $T(z)$.

If $|T(e^{j\omega})|$ is a constant then the system is allpass and there will be no distortion in the output. However if that is not the case then there will be amplitude distortion. If $|T(e^{j\omega})|$ has nonlinear phase then there will be phase distortion. If the analysis and the synthesis filters $H_k(z)$ and $F_k(z)$ are such that aliasing is completely cancelled and if $T(z)$ is a pure delay i.e. if $T(z) = cx^{-n_0}$ then the output $\hat{x}(n) = c(n-n_0)$ and the system is free of all kinds of distortion, aliasing, amplitude and phase distortion and the system is said to be perfect reconstruction (PR) filter bank.

In multicarrier transmission a large portion of the delay is a result of modulation and demodulation. Hence it is imperative to minimize the delay of the prototype filter and the choice of the number of coefficients (length of filter) is a tradeoff between delay and filter performance mainly, stopband attenuation.

To get high quality reconstruction in a pseudo QMF bank the low pass prototype $H(\omega)$ must satisfy as much as possible the following two conditions\cite{83}.

$$|H(\omega)|^2 + |H(\omega - \pi/M)|^2 = 1, \quad \text{for} \quad 0 < \omega < \pi/M \quad (3.3.3)$$

and

$$|H(\omega)| = 0 \quad \text{for} \quad \omega > \pi/M \quad (3.3.4)$$

If (3.3.3) is satisfied exactly then all amplitude distortion is eliminated in the combined system, while if (3.3.4) alone is satisfied then there is no aliasing between non adjacent subbands. Since it is not possible to satisfy both (3.3.3) and (3.3.4) simultaneously using finite length filters, it is necessary to design a filter that will approximately satisfy both. Therefore a cost function is derived that will combine both conditions and the prototype filter is designed using methods that will minimize this cost function.

The cost function $\Phi$ that will be minimized during the design process of the prototype filters of the modulated filter bank can be defined as

$$\Phi = \max \{ |H(\omega)|^2 + |H(\omega - \pi/M)|^2 - 1 \}, \quad \text{for} \quad 0 < \omega < \pi/M \quad (3.3.5)$$
Modulated filter banks have the advantage of computational efficiency and high performance filter design. The prototype filter is linear phase and generally of length $2mM$ where $m$ is any integer. They have the advantage of using longer length filters to obtain high stopband attenuation. Later arbitrary length filters came into use in Biorthogonal filter banks.

Systems that used these linear phase filters have to make a hard choice between whether to opt for reduced delay, or high stopband attenuation. In multicarrier transmission a large portion of the delay is a result of modulation and demodulation. Hence it is imperative to minimize the delay of the prototype filter and the choice of the number of coefficients (length of filter) is a tradeoff between delay and filter performance mainly, stopband attenuation.

To obtain reduced latency smaller length filters had to be used but this would result in poor stopband attenuation and consequently in improper subchannelisation. Biorthogonal filter banks, filter banks in which the synthesis filter is no longer the time reversed version of the analysis filter, would help offset this dilemma to some extent because they would allow a more general value of delay $D=2sM-1$. In the case of telecommunication applications it is imperative for the delay to be minimal whereas linear phase filters are useful in image processing applications.

### 3.4 Design of Prototype Filter

In this section various methods of prototype filter design, for the cosine modulated pseudo qmf filter bank suggested by Creusere and Mitra[84], Cruz and Roldan[85], Lin and Vaidyanthan[86], J Kliewer[87], are studied in relation to lengths of filter used, and hence coefficients to be optimized, time for optimization and peak to peak ripple in the passband for an M channel filter bank. Next, two methods [46][47] are proposed for the design of the prototype filter and the performance of the filters designed will be analysed and compared with earlier designs on the basis of filter lengths, total distortion in the passband, and optimization time. The amount of computation involved in terms of Multiplications Per Unit(MPU) and Additions Per Unit(APU) will also be compared.
In the method suggested by Creusere and Mitra in [84] the stopband edge is fixed and the pass band edge is varied so as to minimize the objective function which is defined as the sum of the admissible pass band ripple and the stopband ripple. The prototype was designed using the Parks-McClellan algorithm[88]. This method is designated as Method 1 in this work. The prototype filter design method proposed in this paper was simulated using MATLAB and the program for the same is given in Appendix A. The value of total distortion obtained is very good and compares very well with the value obtained by the method suggested by Lin and Vaidyanathan in [86]. The main drawback is the time taken for the optimization process. The time taken for the optimization of the filter coefficients increases greatly as the number of channels in the filter bank increases. For instance in the case of M=128 the time taken for the optimization process is almost 9 minutes, whereas in [86] the optimization process is completed in a few seconds. In keeping with the requirement of having filter of length equal to 2mM in some cases it was required to slightly alter the filter order from that suggested in the method. This did not alter the responses obtained in a major way. The magnitude response plots Fig. 3.4.1, Fig.3.4.2 and the distortion function Fig.3.4.3 are obtained. The results obtained using this method using M=8,16,32,64 and 128 are as tabulated in Table.3.4.1.

![Magnitude response for M=16](image)

**Fig.3.4.1** Magnitude response of the prototype filter using Method 1.
Fig. 3.4.2 Magnitude responses for the M=16 channel filter bank using Method1

Fig. 3.4.3 Amplitude distortion function for M=16 channel filter bank using Method1
TABLE: 3.4.1 Performance parameters for various M and Lp

<table>
<thead>
<tr>
<th>M</th>
<th>Attenuation</th>
<th>Lp</th>
<th>Time (in secs)</th>
<th>p-p passband ripple</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>100</td>
<td>127</td>
<td>1.288</td>
<td>.01088</td>
</tr>
<tr>
<td>16</td>
<td>100</td>
<td>255</td>
<td>4.388</td>
<td>.01095</td>
</tr>
<tr>
<td>32</td>
<td>100</td>
<td>511</td>
<td>24</td>
<td>.0112</td>
</tr>
<tr>
<td>64</td>
<td>100</td>
<td>1023</td>
<td>120</td>
<td>.01075</td>
</tr>
<tr>
<td>128</td>
<td>100</td>
<td>2047</td>
<td>500</td>
<td>.0051</td>
</tr>
</tbody>
</table>

The method suggested by Cruz Roldan and others in [84], involves the adjustment of the 6db frequency to lie at π/2M. This method has been designated as Method 2 in this work. The design method suggested in this paper has been implemented using MATLAB program given in Appendix B. Plots representing the magnitude response of the prototype filter, the response of the M analysis filters and the total distortion of the M-channel filter bank have been obtained. The prototype filters are obtained for a pseudo– QMF bank and the analysis and the synthesis filters are the cosine modulated versions of the same prototype filter.

The plots shown below in Fig. 3.4.4, Fig.3.4.5 and Fig. 3.4.6 indicate that this design does produce high quality filters. The authors have suggested that the windows that are used to design the filter can be of any type, Blackman, Hamming, Kaiser etc. In [84] the Hamming window has been chosen. It has been observed that that the choice of the window does not affect the nature of the response of the filter bank to any great extent. Plots for values of M = 16, are shown in Fig (3.4.4) and Fig.(3.4.5). Since the width of the transition band reduces as the number of channels is increased the order the filters increases by the same factor. The process of optimizing the filters is by coefficient optimization hence the complexity of this process is directly affected by the order of the filter. However it was observed that the time taken for the optimization of coefficients in each case is not significantly different for M=8,16,32 and 64. This is shown in Table 3.4.2.
Optimisation of filter coefficients requires the traversal of two loops. The method involves the adjustment of the 6db frequency to lie at \( \pi/2M \), and each time the filter needs to be designed. This is one drawback of this method.

**Fig. 3.4.4** Magnitude response of the prototype filter using Method 2.

**Fig. 3.4.5** Magnitude responses for the M=16 channel filter bank using Method2

46
The process of obtaining specific length filters i.e. 2mM though not impossible is a process of trial and error and hence tedious. From Table 3.4.2 it is seen that for M=8, the length of the filter for an attenuation of 100 dB is 171. To meet the specifications on filter length the filter length has to be 176. When the length of the filter changes the objective function is no longer minimum. By trial and error the value of the cut off frequency is slightly adjusted to obtain the same output. In the same fashion the filters for the other values of M are also obtained. This is a limitation of the method if its required to meet our requirement on filter lengths.
In the method suggested by Lin and Vaidyanathan in [85] the initial filter $p(n)$ is designed as an FIR low pass filter using the Kaiser window approach. This method is designated as Method 3 in this work. The design method suggested in this paper has been implemented using MATLAB program given in Appendix C. Next a filter $G(e^{j\omega})$ is designed such that $G(e^{j\omega}) = |p(e^{j\omega})|^2$. The filter $G(e^{j\omega})$ is a Nyquist$(2M)$ filter[50] and $g(2Mn) = (1/2M)\delta(n)$. The objective function is

$$\Phi = \max_{\omega} \ |g(2Mn)| \quad (3.4.1)$$

As in the earlier cases the objective function is a convex function of the cutoff frequency $\omega_c$. The cutoff frequency is varied to obtain the best filter impulse response sequence $p(n)$ for minimum value of the objective function. This shown in Fig.3.4.7. The method suggested is a simple yet provides very good design for the prototype filter. This is the method that has been adopted in the methods based on the IFIR approach that have been proposed. Plots shown in Figs.3.4.8,3.4.9 and 3.4.10 indicate good design. For applications requiring high stopband attenuation the filters will be of high order which increases computation complexity but little increase in design time as it was in the case of method suggested by Creusere and Mitra[83].

Fig. 3.4.7 Plot of objective function vs. cutoff frequency for Method3.
Fig. 3.4.8 Magnitude response of the prototype filter using Method 3.

Fig. 3.4.9 Magnitude responses the M=16 channel filter bank using Method 3.
The method suggested by J. Kliewer [87] was explored as a possible choice of method for the design of the prototype filters for the oversampled CMFBs. This method is designated as Method4 in this work. The design method suggested in this paper has been implemented using MATLAB program given in Appendix D. As opposed to the methods discussed earlier [84][85][86] this method is not based on coefficient optimisation. Here the prototype filter is obtained by first designing an initial filter as a Nyquist Eigen filter [90][91] and then using frequency sampling techniques. The responses obtained by this method are again very good and comparable to the responses obtained using other methods. Fig. 3.4.10

TABLE 3.4.3 Performance parameters for various M and Lp

<table>
<thead>
<tr>
<th>M</th>
<th>Attenuation</th>
<th>Lp</th>
<th>Time (in secs)</th>
<th>p-p passband ripple</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>100</td>
<td>111</td>
<td>.38</td>
<td>.04047dB</td>
</tr>
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<td>16</td>
<td>100</td>
<td>239</td>
<td>.68</td>
<td>.0406dB</td>
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<tr>
<td>32</td>
<td>100</td>
<td>479</td>
<td>1.28</td>
<td>.03430dB</td>
</tr>
<tr>
<td>64</td>
<td>100</td>
<td>959</td>
<td>3.14</td>
<td>.03221dB</td>
</tr>
<tr>
<td>128</td>
<td>100</td>
<td>1919</td>
<td>9.18</td>
<td>.03431dB</td>
</tr>
</tbody>
</table>
shows the magnitude response of the prototype filter designed for an M=8 channel filter bank. The filter is designed for attenuation of 50dB and has a length of 129. As seen from the Fig. 3.4.11 and Fig.3.4.12 the filter has good stopband characteristics. The stopband falloff rate is also good. The total distortion is small and compares well with the values obtained for the earlier methods.

Fig. 3.4.11 Magnitude response of the prototype filter using Method 4

Fig. 3.4.12 Magnitude responses of the M=8 channel filter bank using Method 4
Fig. 3.4.13 Amplitude distortion function for M=8 channel filter bank using Method 4, shown for one interval.

As the number of channels M is increased the method becomes very slow. This is a limitation as compared to the other methods based on the coefficient optimisation approach which are comparatively very fast for the same quality of performance.

### 3.5 Proposed Methods for the Design of the Prototype Filter

On comparing the above methods on the basis of performance, computational complexity and time required for the optimization routine it was concluded that all the methods were comparable on the basis of performance. However based on the ease of optimization and computational time the method suggested by Lin and Vaidyanathan was found to be the best. Keeping in view the fact that the filters were to be designed for an oversampled filter bank, the methods had to be computationally efficient. The need therefore was to simplify the design process further and reduce the time for optimization by reducing the number of filter coefficients to be optimized further, while maintaining good performance. Since the filter bank was to be used in a transmultiplexer system for a highly dispersive channel there it would be designed as a pseudo-QMF filter bank.
Two methods for the design of the prototype filter are proposed. The filters are designed based on the interpolated FIR filter (IFIR) approach[91]. These methods have been proposed on the basis of simplicity of design and efficiency of computation.

3.5.1 Method 1

The Proposed IFIR-Kaiser Method

The first method proposed is the IFIR-Kaiser method[46], which is an extension of the Kaiser window based method that was suggested by Lin and Vaidyanathan in[85]. The method of design using the IFIR filter involves designing the prototype filter as a cascade of two filters one, the model filter and the other the image suppressor filter. These two filters are designed based on the method suggested in[92].

The filter \( H(z) \) is designed using the IFIR approach as follows

\[
\begin{align*}
X(z) & \quad \rightarrow \quad H(z) \quad \rightarrow \quad Y(z) \\
\text{Fig.3.5.1. Analysis Filter for which the IFIR representation is derived} \\
\end{align*}
\]

\[
\begin{align*}
& X(z) \\
& \quad \rightarrow \quad G(z) \\
& \quad \uparrow L \quad \rightarrow \quad G(z') \quad \rightarrow \quad I(z) \quad \rightarrow \quad Y(z) \\
& \text{Fig.3.5.2 Steps in the design of IFIR filter.} \\
\end{align*}
\]

The model filter \( G(z) \) in Fig.3.5.2 is the \( L \) times stretched version of the desired filter \( H(z) \), in Fig.3.5.1. The pass band and the transition band are \( L \) times the pass band and the transition band of the desired filter. The transition band determines the order of the filter. Larger is the transition band desired, smaller will be the order of the filter. Hence if the transition band is increased by a factor of \( L \), the order of the filter is decreased by a factor of \( L \). Hence the order of the model filter is approximately \( 1/L \) times the order of the desired
filter. The model filter is interpolated by an upsampling factor of \( L \). It has been shown in Fig. 2.6.7 that interpolation leads to images of the original spectrum being produced. Hence when the model filter is interpolated by a factor of \( L \) several images of the original spectrum are produced. In order to suppress these images the filter \( G(z^L) \) is cascaded with another filter \( I(z) \) which is called as the image suppressor filter. The image suppressor filter \( I(z) \) has a very large transition band and hence its order is very low. Fig. 3.5.3 shows the various steps in the formulation of the IFIR filter.

(a) Desired narrow band filter \( H(z) \)

(b) Model filter \( G(z) \)

(c) Response of \( G(Z^2) \)

(d) Response of \( I(z) \)
Fig. 3.5.3 (a) shows the desired frequency response of the filter $H(z)$. In (b) is shown the response of the model filter $G(z)$ in which the pass band edge and stop band edge are twice that of the filter $H(z)$ since the interpolation ratio has been assumed to be 2.

The impulse response of the model filter is denoted by $g(n)$. Inserting $L-1$ zeroes between the original samples of $g(n)$ the upsampled sequence $g'(n)$ is obtained.

$$g'(n) = g(n/L) \quad n=0,1,2,3,\ldots$$  (3.5.1)

$$= 0 \quad \text{otherwise.}$$

$$G'(z)=G(z^L).$$  (3.5.2)

For $L=2$ $G'(z)=G(z^2)$.

$G(z^2)$ is the two times upsampled version of $G(z)$ and is shown in (c). The model filter $G(z)$ has a transition band that is two times that of the desired filter $H(z)$. The frequency response of $G(z^L)$ is periodic with period $2\pi/L$, in this case the period is $\pi$. Any of the passbands in the interval 0 to $\pi$ may be selected as the desired one. The image suppressor filter $I(z)$ is used to attenuate the unwanted replicas of the desired passband. This shown in Fig.3.5.3 (d). This filter is convolved with $G(z^2)$ to obtain the desired frequency response $H(z)$ as shown in Fig.3.5.3.(e).

For the design of the filters the gain of the prototype filter is assumed to be 1 in the passband and zero in the stopband with a deviation of $\delta_p$ in the passband and $\delta_s$, in the stopband. The passband of the overall response has ripples that are larger than that of $G(z^L)$ and $I(z)$. To meet the desired specifications the peak passband ripple is taken to be $\delta_p/2$ for the model filter and for the image- suppressor, and the stopband ripple to be a maximum of
\( \delta \). The length of the model filter and the image suppressor required to meet the desired response is calculated using the formula

\[
\text{Filter length} = \frac{(-20 \log \delta ps - 13)}{(14.36 + \Delta f)} + 1 \tag{3.5.3}
\]

\( \Delta f = (\omega_p - \omega_s) / 2\pi \) denotes transition bandwidth.

Fig.3.5.4 Implementation of the IFIR filter

The filter \( H(z) \) has been designed using the above method for \( L=2 \). The filters are designed for number of channels \( M = 8, 16, 32, 64 \) and for a minimum stopband attenuation of 100dB. The design method suggested in this paper has been implemented using MATLAB program given in Appendix E. It is seen that the method performs uniformly well for all \( M \) even though the increasing number of channels means that the transition band becomes narrower and the filter orders are higher. The plot of the objective function vs the cutoff frequency is shown in Fig.3.5.5. The magnitude response of the model filter, the interpolation filter and the interpolated model filter are shown in Fig.3.5.6. Fig.3.5.7 shows the magnitude response of the prototype filter.

<table>
<thead>
<tr>
<th>( M )</th>
<th>Minimum Stopband Attenuation</th>
<th>( L_p-1 ) (order)</th>
<th>( \text{p-p passband ripple in dB} )</th>
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<tr>
<td>8</td>
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<td>57</td>
<td>18</td>
</tr>
<tr>
<td>16</td>
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<tr>
<td>64</td>
<td>100</td>
<td>456</td>
<td>19</td>
</tr>
</tbody>
</table>

56
Fig. 3.5.5 Plot of objective function vs. cutoff frequency for the design of prototype filter using proposed IFIR-Kaiser method.

Fig. 3.5.6 Plot of magnitude response of model filter, interpolation filter and L=2 times stretched model filter.
Fig. 3.5.7 Magnitude response of the prototype filter designed using the proposed IFIR-Kaiser method for M=16

Fig. 3.5.8 Magnitude responses of the M=16 channel filter bank using the proposed IFIR-Kaiser method
The magnitude responses and the distortion function for the proposed method are shown in Fig. 3.5.8 and Fig. 3.5.9.

In order to be able to use the DCT IV transform for cosine modulation of the prototype filter to derive the M channel cosine modulated filter bank, it is required that the length of the filter be $2mM$ as previously mentioned. Hence the filter lengths obtained as per specifications given in the table above have been modified such that the prototype filter has the length $2mM$. Note that in some cases only the model filter length or the interpolation filter length is altered whereas in others both have been modified to satisfy the criteria.

In practice, when the transition width is $L$ times that of a conventional FIR filter the order of the model filter is a little more than $1/L$ times that of the conventional FIR filter, as it is required to meet more stringent passband requirements than the desired filter. For the IFIR filter required to meet the same set of specifications as an equivalent FIR filter, the IFIR filter requires approximately $1/L$ times the number of multipliers and adders as the FIR, neglecting the image-suppressor which is a poor quality filter. Also coefficient sensitiveness and the output round-off noise properties being dependent on the order of the filter these effects will be considerably reduced in the case of the IFIR filter.
In the earlier section the interpolation factor is arbitrarily chosen as 2. However this may not be the best value. The best value of $L$ remains to be determined, namely the value of $L$ that will yield minimum computation without compromising on performance. The number of multipliers required for the implementation of the FIR filter is dependent on the filter order $N$, which is in turn dependent on the transition bandwidth for a chosen values of $\delta_p$ and $\delta_s$. Since the transition width is directly proportional to the stretch factor, $L$, it may be concluded that the order of the model filter and therefore the number of multipliers for the implementation of the IFIR filter will depend on the value of $L$. As derived by Mehrnia and Wilson[89], the value of $L$ that will yield minimum number of multipliers is given by the expression

$$L_{opt} = \frac{2\pi}{(\omega_p+\omega_s + \sqrt{2\pi(\omega_p-\omega_s)})}$$ (3.5.4)

The optimum choice of $L$ for minimum multipliers is the value of $L_{opt}$ rounded off to nearest integer.

However when this method is used for prototype filter design of the $M$ channel cosine modulated filter bank the choice of the stretch factor cannot be based only on the above. This is because the passband and the stopband edges are also dependent on the value of $M$, the number of channels. For this different values of $L$ were chosen for the design of the filters for value of $M = 8, 16, 32, 64$. Hence the value of $L$ based on the number of channels has been used for the prototype filter design.

An interesting fact emerges from the design procedure of the IFIR-based prototype filter. As has been mentioned earlier the transition band of the interpolation filter is very large. This filter has been designed using the Kaiser window. The choice of the cutoff frequency determines the shape of the transition band. In earlier designs using this method[73] it was assumed that since this filter was of poor quality it was not necessary to optimize the interpolation filter. However in the process of trying to improve the quality of the overall prototype by optimizing only the coefficients of the model filter a very interesting observation was made. The shape of the interpolation filter’s transition band affected the total magnitude distortion. This in turn meant that the choice of the cutoff frequency for the interpolation affects the total magnitude distortion obtained. This meant therefore that the optimal value of the cut off frequency for the interpolation filter had to be determined to get better performance of the overall filter The change in the magnitude response characteristics
for changing values of the cut-off frequencies has been shown explicitly for the case of the filter bank with $M=16$. The change in the cut-off frequency of the interpolation filter changes not only the main lobe of the filter magnitude response but also the stopband edge, and the stopband characteristic, namely the fall off rate in the stopband.

![Magnitude response of interpolation filter for $M=16$, $L=4$](image)

**Fig. 3.5.10** Plot of magnitude response of the interpolation filter for different cut-off frequencies

As shown in Fig.3.5.10 with increase in the cut-off frequency the width of the main lobe of the interpolation filter, for a given number of channels $M$ and given length of the filter increases. This could mean that the filter is better able to suppress the images of the model filter that has been interpolated. The fall off rate in the stopband however decreases with an increase in the cut-off frequency. This means that the stopband energy could reduce.

The graph in Fig.3.5.10 above shows the frequency response of interpolation filter for three different values of the cut-off frequency. The fall off rate of the frequency response in the stop band is determined by the cut-off frequency and it is seen that this shape of magnitude response of the interpolation filter affects the frequency response of the overall prototype filter thereby influencing the overall magnitude distortion and the peak to peak pass band ripple. In Fig.3.5.11 the variation in response of the interpolation filter and hence that of the prototype filter for three different values of the cutoff frequency are shown. From
the figures it is obvious that for a given length of the interpolation filter the variation of
cutoff frequency is increasing size of the main lobe and this effects the image suppression,
giving rise to changes in the magnitude response of the prototype filter.

Hence the objective function is to be minimized by varying the cutoff frequency of
the interpolation filter. This is an important finding. It was felt that since the interpolation
filter was an extremely poor quality filter with a very large transition band and hence of very
small order, the shape of its magnitude response would have no bearing on the characteristics
of the overall filter. Earlier designs of the IFIR filter too have concentrated only on the
optimization of the model filter coefficients.

As shown in the Table 3.5.4 the values of peak to peak ripple obtained are comparable to
the values in the case of Lin and Vaidyanathan[85]. Time required for optimization is less as
only the coefficients of the model filter are to be optimized.

Consider the design of the prototype filter \( H(z) \) having length \( L_p \) and a transition
width of \( \Delta f \). Let \( \omega_p^H \) and \( \omega_s^H \) denote the passband and stopband edge of \( H(z) \).

\[
\omega_p^H = \pi / M \quad \text{(3.5.5)}
\]

and \( \omega_s^H = \pi / 2M \quad \text{(3.5.6)} \)

Then the passband and stopband edge of the model filter \( G(z) \) will be \( 2\omega_p^H \) and \( 2\omega_s^H \)
respectively for \( L=2 \).

The filterbank has 32 channels, \( M=32 \). The minimum stopband attenuation of the
prototype filter is specified as 100dB. The transition bandwidth is \( 0.01406\pi \).

The prototype filter is obtained by cascading the two filters \( G(z) \) and \( I(z) \). The image
suppressor filter and the model filter will be designed separately. First the Image suppressor
filter \( I(z) \) is designed as an FIR filter using the Kaiser window for the specifications given.
The procedure is described below.
Any FIR filter of length $L_p$ designed using the window method can be written as follows:

$$p(n) = h_d(n) \cdot w(n) \quad (3.5.7)$$

$h_d(n)$ is the desired impulse response and $w(n)$ is the window function.

$$h_d(n) = \frac{\sin(\omega_c(n-0.5L_p))}{\pi(n-0.5L_p)} \quad (3.5.8)$$

$\omega_c$ is the cutoff frequency of the desired filter.

$w(n)$ is the Kaiser window

$$w(n) = \begin{cases} 
  I_0(\beta) \sqrt{1 - \left(\frac{n-0.5L_p}{0.5L_p}\right)^2} & 0 \leq n \leq L_p \\
  0 & \text{otherwise}
\end{cases} \quad (3.5.9)$$

$I_0(\beta) = 1 + \sum_{k=1}^{\infty} \left(\frac{0.5\beta}{k!}\right)^2$

The value of $\beta$ depends on the desired attenuation in the stop band.

The order of the window function $N$ is given by

$$L_p = \frac{A_s - 7.95}{14.36\Delta\omega/2\pi} + 1 \quad (3.5.10)$$

Since $L_p$ and $\beta$ are dependent on the filter specifications namely attenuation and transition bandwidth $\Delta\omega$ in order to optimize the coefficients and minimize the objective function the only factor that can be varied is the cutoff frequency $\omega_c$. The process has been described in Section 3.4.
Fig. 3.5.11. Plots showing the variation in the magnitude response of the prototype filter
for (a) \( wc=0.35 \) (b) \( wc=0.4 \) (c) \( wc=0.5 \) and \( M=16 \)
The coefficient optimization will be done for the model filter. The prototype filter is designed to have an attenuation greater 100dB in the stopband. For a stretch factor L to be 2 the transition band of this filter will be 2 times the specified value. The steps for the design of the prototype filter are as shown in Fig. 3.5.12

Fig. 3.5.12 Procedure for design of IFIR-Kaiser filter
1. Determine the order of the model filter \( G(z) \) so as to obtain desired attenuation, for the given transition bandwidth, in the prototype filter \( H(z) \).

2. Determine the order of \( I(z) \) for pass band edge \( \pi/2M \) and stop band edge of \( 2\pi/L - \pi/M \).

3. The optimal value of the cutoff frequency of \( I(z) \) is determined.

4. The filter \( I(z) \) is designed using the Kaiser window.

5. The optimal model filter is designed by varying the cutoff frequency as to minimize the cost function as stated in [3].

6. The optimized model filter is now stretched by a factor of \( L \).

7. The stretched model filter is convolved with the image suppressor filter \( I(z) \), to obtain the desired prototype filter \( H(z) \).

Now filters are designed for different values of \( M=8,16,32,64 \). The filter bank is designed as a cosine modulated filter bank. The \( M \) filters of the \( M \)-channel filter bank are cosine modulated versions of the prototype filter. For \( M=32 \) order of the model filter obtained by the above method is 274. That is only 274 coefficients have to be optimized whereas in [86] the filter has an order of 466. Also the total distortion was found to be comparable with that obtained in [86]. As the number of channels is increased the bandwidth decreases and the order of the filters increases. Note that the filter is to be designed for each iteration and hence considerable saving in computation can be obtained when filter order is reduced. The minimum stopband attenuation obtained is about 100dB.

Table 3.5.5 Performance analysis of IFIR- Kaiser filter

<table>
<thead>
<tr>
<th>( M )</th>
<th>Minimum Attenuation in Stopband</th>
<th>( L_p -1 ) (order)</th>
<th>p-p overlapping Ripple in the passband</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>100</td>
<td>65</td>
<td>26</td>
</tr>
<tr>
<td>16</td>
<td>100</td>
<td>130</td>
<td>26</td>
</tr>
<tr>
<td>32</td>
<td>100</td>
<td>274</td>
<td>26</td>
</tr>
<tr>
<td>64</td>
<td>100</td>
<td>502</td>
<td>18</td>
</tr>
<tr>
<td>128</td>
<td>100</td>
<td>1014</td>
<td>18</td>
</tr>
</tbody>
</table>
3.5.2 Method 2

The Proposed Multistage IFIR-Kaiser Method

Consider the analysis filter bank in the figure shown below. For number of subchannels $M$ that is large the analysis filter $H(z)$ will be narrowband and may be designed using a method based on the earlier IFIR approach.

Starting from the earlier IFIR based design where the design of the prototype filter is reduced to the design of two poor quality filters thereby reducing computation a method by which computation could be reduced still further is proposed[47]. The analysis filters in the filter bank shown below are implemented using a multistage approach. The procedure can be repeated for the synthesis filters as well.

![IFSFilter](image)

**Fig. 3.5.13 Analysis Filter Bank for which the multistage IFIR filter representation is to be derived**

The stretch factor $M_1$ is taken to be a factor of the decimation factor $M$. As shown in Fig 3.5.4. the filter $H(z)$ may be represented as a cascade of the image suppressor and the interpolated model filter. Then the decimator structure in each channel is represented as a cascade of the image suppressor filter the downsampling by $M_1$, the model filter $G(z)$ and the downsampling by $M_2$ as shown in Fig. 3.5.14 below.

![IFSFilter](image)

**IFIR implementation of $H(z)$**
The computational complexity involved in the implementation of the filters is dependent on the order of the filter and the order of the Model filter $G(z)$, $N_g$, and the Image Suppressor filter $I(z)$, $N_i$, can be obtained from the equations below.

$$N_g = \frac{20 \log \sqrt{0.5 \delta \delta \frac{1}{2} - 13}}{14.6(\omega_s - \omega_p) / 2\pi M_1}$$  \hspace{1cm} (3.5.11)$$

$$N_i = \frac{20 \log \sqrt{0.5 \delta \delta \frac{1}{2} - 13}}{14.6(2\pi - (\omega_s + \omega_p)M_i) / 2\pi}$$  \hspace{1cm} (3.5.12)$$

The number of multiplications per unit time (MPU) is

$$N_g/M + N_i/M_1$$  \hspace{1cm} (3.5.13)$$

In Table 3.5.6 the direct and multistage designs are compared on the basis of the filter order and the amount of computation involved namely the number of additions and multiplications involved in the implementation of the filter for different values of $M$ and $M_i$. $M$ will remain fixed and $M_i$ can take different values for given $M$.

From Table 3.5.6, the following inference can be drawn. As the value of $M$ is increased by a factor of 2 the order of the conventional filter approximately doubles and the order of the model filter $G(z)$ and of $I(z)$ are significantly less. Also the order of $G(z)$ and $I(z)$ is dependent on the value of the factor $M_i$. As $M_i$ reduces the order of $G(z)$ increases and that of $I(z)$ reduces.
Table 3.5.6 Comparison on the basis of filter orders

<table>
<thead>
<tr>
<th>M</th>
<th>Direct Design</th>
<th>Filter order</th>
<th>Multistage design</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$G(z)$</td>
<td>$I(z)$</td>
</tr>
<tr>
<td>8</td>
<td>$M_1=4$</td>
<td>75</td>
<td>21</td>
<td>17</td>
</tr>
<tr>
<td>16</td>
<td>$M_1=8$</td>
<td>150</td>
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<td>32</td>
</tr>
<tr>
<td></td>
<td>$M_1=4$</td>
<td></td>
<td>41</td>
<td>13</td>
</tr>
<tr>
<td>32</td>
<td>$M_1=16$</td>
<td>298</td>
<td>21</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>$M_1=8$</td>
<td></td>
<td>41</td>
<td>25</td>
</tr>
<tr>
<td>64</td>
<td>$M_1=32$</td>
<td>596</td>
<td>21</td>
<td>129</td>
</tr>
<tr>
<td></td>
<td>$M_1=16$</td>
<td></td>
<td>41</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>$M_1=8$</td>
<td></td>
<td>81</td>
<td>23</td>
</tr>
</tbody>
</table>

Table 3.5.7 Comparison on the basis of MPU

<table>
<thead>
<tr>
<th>M</th>
<th>Direct Design</th>
<th>MPU</th>
<th>Multistage design</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$G(z)$</td>
<td>$I(z)$</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$M_1=4$</td>
<td>4.69</td>
<td>1.31</td>
<td>2.61</td>
</tr>
<tr>
<td>16</td>
<td>$M_1=8$</td>
<td>4.69</td>
<td>.656</td>
<td>2.656</td>
</tr>
<tr>
<td></td>
<td>$M_1=4$</td>
<td></td>
<td>2.56</td>
<td>4.19</td>
</tr>
<tr>
<td>32</td>
<td>$M_1=16$</td>
<td>4.66</td>
<td>.328</td>
<td>2.328</td>
</tr>
<tr>
<td></td>
<td>$M_1=8$</td>
<td></td>
<td>.641</td>
<td>2.2</td>
</tr>
<tr>
<td>64</td>
<td>$M_1=32$</td>
<td>4.66</td>
<td>.164</td>
<td>2.184</td>
</tr>
<tr>
<td></td>
<td>$M_1=16$</td>
<td></td>
<td>.32</td>
<td>1.88</td>
</tr>
<tr>
<td></td>
<td>$M=8$</td>
<td></td>
<td>.633</td>
<td>2.073</td>
</tr>
</tbody>
</table>

Table 3.5.7. Shows the comparison on the basis of MPU. The specifications chosen for the design of the prototype filter are $\delta_1=.02$, and $\delta_2=.001$. The passband and stopband edges have been chosen to be $\pi/2M$ and $\pi/M$ respectively.
Table 3.5.8 Comparison on the basis of APU

<table>
<thead>
<tr>
<th>M</th>
<th>Direct Design</th>
<th>Multistage design Method</th>
<th>APU</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G(z)</td>
<td>I(z)</td>
<td>Total</td>
</tr>
<tr>
<td>8</td>
<td>M1=4</td>
<td>9.38</td>
<td>2.63</td>
</tr>
<tr>
<td></td>
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</tr>
<tr>
<td>16</td>
<td>M1=4</td>
<td>2.56</td>
<td>3.25</td>
</tr>
<tr>
<td>32</td>
<td>M1=16</td>
<td>.656</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>M1=8</td>
<td>1.28</td>
<td>3.13</td>
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<tr>
<td>64</td>
<td>M1=32</td>
<td>.328</td>
<td>4.03</td>
</tr>
<tr>
<td></td>
<td>M1=16</td>
<td>.641</td>
<td>3.13</td>
</tr>
<tr>
<td></td>
<td>M1=8</td>
<td>1.27</td>
<td>2.88</td>
</tr>
</tbody>
</table>

The value of $M_1$, where the cost of implementation in terms of MPU is minimum for a fixed $M$ is determined. Substituting (3.5.11) and (3.5.12) in (3.5.13), differentiating the result with respect to $M_1$ and equating to zero the value of $M_1$ that will obtain minimum MPU for a given value of $M$ is obtained. This is given by

$$M_1(\text{optimal}) = -\frac{2\pi}{M(\omega_s - \omega_p) - (\omega_s + \omega_p)} + \frac{2\pi \sqrt{M(\omega_s^2 - \omega_p^2)}}{M(\omega_s^2 - \omega_p^2) - (\omega_s + \omega_p)^2}$$ (3.5.14)

Since the value of $M_1$, obtained from (3.5.14) may not exactly be a factor of $M$ we must choose an appropriate value that is close to it. On testing the values for different $M_1$ and comparing, the values that have been tabulated in Table 3.5.8 match very closely. So it may be concluded that by factoring $M$ suitably maximum saving in computation cost can be obtained.

From Tables 3.5.7 and 3.5.8 the following may be inferred. The computational cost per unit time of implementation of the filter using direct design methods is almost independent of the number of subchannels. But the cost using multistage designs varies for different $M$. Also for a given $M$ the computational cost is different for different for different values of $M_1$. This means therefore that there must be some value of $M_1$, where cost is minimum, and this value may be used to reduce cost of implementation still further.
In the above the design of decimators of the analysis filter bank using the multistage approach has been described. The interpolators in the case of the synthesis filter bank may also be designed in the same way using the multistage approach.

Comparisons are obtained in the case of a 32 channel filter bank, using the method proposed by Creusere and Mitra in [84] where the order of the filter by direct design is 511 where as for the same method using the multistage implementation the order of $G(z)$ is 69 and $I(z)$ is 73 all specifications of the filters being the same for both methods. $M_1$ was chosen to be 8 which is the factor closest to the optimal value calculated. In the case of [86] the peak to peak amplitude distortion was marginally lower at .01. The plots for the magnitude responses and the amplitude distortion are shown in Fig. 3.5.15, Fig. 3.5.16, Fig. 3.5.16.

In conclusion it may be said that the design of the prototype filter using the IFIR based approach and its further multistage implementation allows the design of filter banks using filters of much less order and it would be highly desirable to explore this method for implementation of filter banks and filter bank based transmultiplexers.

![Plot of magnitude response of model filter and interpolation filter](image)

**Fig. 3.5.15** Plot of magnitude response of model filter and interpolation filter
Fig. 3.5.16 Magnitude responses of the M=32 channel filter bank using the proposed multistage IFIR-Kaiser method

Fig. 3.5.17 Amplitude distortion function for M=8 channel filter bank using the proposed multistage IFIR-Kaiser shown for one interval