CHAPTER 4
REJECTION OF INTERFERENCE OVER RAYLEIGH FADING CHANNELS BY MMSE CRITERION

4.1 Introduction
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4.1 INTRODUCTION

Wireless communication is one of the most active areas of research for technology enhancement in current times. Videos, images, text and data can be more efficiently transmitted now-a-days with the help of latest state of art technologies.

As a result of technological developments in the field/area of wireless communication, the demand for higher transmitted power and bandwidth is increasing. But these two parameters are severely limited in the deployment of modern wireless networks. These two resources are not improving at the rates that can support increasing demands for wireless capacity. So, current effort in recent years is aimed at developing new wireless capacity through the deployment of greater intelligence in wireless networks. To obtain maximal benefit from the existing transmission techniques, these techniques are aimed at addressing the physical properties of wireless channels like interference.

Spread spectrum in the form of Direct-Sequence (DS) is very common signaling scheme in current and emerging wireless services. Interference arises in practical spread spectrum systems due to various reasons. It can be a significant factor in another situation of spread spectrum systems, which is for systems deployed in unregulated bands, such as the industrial, scientific, and medical bands, in which wireless LANs (Local Area Networks), cordless phones, and Bluetooth Piconets operate as spread spectrum systems.

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Similarly, shared access between military Very High Frequency Systems (VHF) with civilian VHF traffic gives rise to Narrowband Interference (NBI). Thus, the issue of NBI in spread spectrum systems is of increasing importance in the development of future advanced wideband telecommunication systems.

Further improvements in NBI suppression can be made by going beyond random modeling at the chip level and taking advantage of the fact that the spreading code of at least one user of interest must be known in order to begin data demodulation. Techniques taking advantage of this fact are termed as code-aided techniques. These works have been based primarily on detectors originally designed for linear multi-user detection. In the context of multi-user detection, linear detectors operate by estimating the data sequence and then quantizing the resulting estimates to get estimates of the data symbols themselves. Two of the principal such multi-user detection techniques used are decorrelator, and the linear MMSE (Minimum Mean Square Error) detector.

The decorrelator completely eliminates Multiple Access Interference (MAI), with the attendant disadvantage of enhancing ambient noise. The Minimum Mean Square Error (MMSE) detector reduces the latter effect by minimizing the Mean Square Error (MSE) between the linear estimate and the transmitted symbols. Also, the linear MMSE detector can be easily adapted than the decorrelator and it results in a lower bit-error rate under most practical circumstances [97]. Advantage of MMSE scheme is that explicit knowledge of interference parameters is not required, since filter parameters can be adapted to achieve the MMSE solution. Also, complexity of these schemes can be adjusted to achieve a given level of performance [98]. When the channel is perfectly estimated, the more complex detectors such as 2S (Two Stage) Detector and DF (Decision Feedback) Detector can achieve the single user bound and are near far resistant. However the simpler schemes Serial Interference Cancellation (SIC) and Parallel Interference Cancellation (PIC) are not suitable for the high bandwidth efficient channels, and should be applied to systems with relatively low cross correlations. In the present work rejection of interference by MMSE criterion is used. It is applied to fading channels and is found to suppress narrowband and multiple access interference.

The proposed work is on analyzing the linear MMSE detector. In the present chapter a simple MMSE technique using a bank of filters is incorporated for rejection of
multiple access interference (MAI). SIR and error probability are used as measuring metrics. It is realized that MAI is not a real limitation as far as processing gain is large enough. The detector is analyzed in the fading channels. For the receiver to adapt to dynamically varying channels, MMSE is made adaptive and is made to work in flat fading channels. The tone signal is used as an interfering signal and thus code aided techniques are used to reject varying number of sinusoids. MMSE is analyzed for various power levels and fading parameters. The results are compared with the conventional methods and are found to outperform. In the present work, a mathematical model of the proposed receiver has been designed and analyzed keeping into consideration the realistic and practical environment.

4.2 MULTIUSER DETECTION

The multi-user detection, upon which the working of linear MMSE detector is based, is explained here. Consider a wireless cellular communication system shown in Fig. 4.1. Let there be K users operating in the same bandwidth at the same time [99]. The signals $s_1[i]$, $s_2[i]$... $s_K[i]$ are the transmitted signals and are destined to receivers 1, 2.....K respectively and $r_1[i]$, $r_2[i]$...... $r_K[i]$ are the received signals. One of the signals is destined to the desired base. The other signals are destined to other bases, and they interfere with the desired signal constituting co-channel interference. The received signal is constituted of K co-channel signals from the desired users and the interferers plus the ambient channel noise. The first user transmitter transmits a sequence $\{b[0], b[1]..., b[M-1]\}$ of M channel symbols over the wireless channel.

![Wireless communication system employing adaptive arrays at the base station.](image)
The symbols can be represented by a column vector $\mathbf{b}$ after sorting them by symbol number first and user number next \([100]\).

$$
\mathbf{b} = \begin{bmatrix}
    b_1[0] \\
    b_2[0] \\
    \vdots \\
    b_k[0] \\
    \vdots \\
    b_1[M-1] \\
    b_2[M-1] \\
    \vdots \\
    b_k[M-1]
\end{bmatrix}
$$

(4.1)

The $n$th element of $\mathbf{b}$ is

$$
[b]_n = b_i[n]
$$

(4.2)

With

$$
k = [n-1]_K
$$

and

$$
i = \left\lfloor \frac{n-1}{K} \right\rfloor, \quad n = 1, 2, \ldots, KM
$$

(4.3)

Where $k$ is the user referred out of $K$ total number of users and $i$ is the symbol referred of the $k$th user. At the receiver, optimal inferences about detected symbol can be made by the observation $r(.)$ conditioned on entire frame of symbols as-

$$
L\left( \frac{r()}{b} \right) = \exp \left\{ \frac{1}{\sigma^2} \left[ 2R\{b''y\} - b''Hb \right] \right\}
$$

(4.4)

Where $L[.]$ denotes likelihood function, $H$ denotes $kM \times kM$ Hermitian cross correlation matrix of composite waveforms associated with symbols in vector $\mathbf{b}$ and $R\{\cdot\}$ denotes the real part of its argument and $\sigma^2$ represents noise variance.
Also, \( y \) is a column vector, whose \( i \)th component is given by
\[
y_M = \sum_{i=0}^{M-1} y_i, \quad i = 0, 1, \ldots, M - 1
\]

\( M \) is the number of symbols in the symbol alphabet.

The composite modulation waveform \( f_{i,k}(t) \) associated with symbol \( b_k[i] \) i.e. \( i \)th bit of \( k \)th user, is now given as-
\[
f_{i,k}(t) = \int_{-\infty}^{\infty} g_k(t,u)w_{i,k}(u)du
\]

\( g_k(t,u) \) being the time invariant impulse response of a linear filter representing the channel between \( k \)th transmitter and receiver and \( w_{i,k}(t) \) is the signaling waveform given as-
\[
w_{i,k}(t) = A_k s_k(t - iT)
\]

Where \( A_k \) is complex amplitude of the \( k \)th user and \( T \) is inverse of single user symbol rate and \( s_k(.) \) is normalized signaling waveform.

The \((n,m)\)th element of Hermitian matrix is cross correlation between \( f_{i,k}(t) \) and \( f_{j,i}(t) \).
\[
H[n,m] = \int f_{i,k}^*(t)f_{j,i}(t)dt
\]

With
\[
l = \lfloor m - 1 \rfloor_k, \quad j = \left\lfloor \frac{m - 1}{K} \right\rfloor
\]

\( m \in \{0, 1, \ldots, M - 1\} \)

The vector \( y \) gives enough statistics about \( b \).

The \( i \)th symbol decision of \( k \)th user is
\[
b_{i,k} = q(My_i)
\]
With
\[ k = [n - 1]_K \]
\[ i = \left[ \frac{n - 1}{K} \right], \]
(4.11)

My is a continuous estimate of the symbol column vector \( \vec{b} \). Here, \( q(\cdot) \) denotes a quantizer mapping the complex numbers to the symbol alphabet.

It implies that the received signal is correlated with composite modulation waveform and the received bits are quantized and hence estimated. N dimensional noise vector \( n \) is Gaussian with zero mean and covariance matrix \( \sigma^2 I_N \), where \( I_N \) represents \( N \times N \) identity matrix and \( \sigma^2 = N_0/2 \). The choice of matrix \( M \) leads to different detections. The choice \( M = (H + \sigma^2 \sum \vec{b} \vec{b}^H)^{-1} \) is known as linear MMSE (Minimum Mean Square Error) multi-user detector. Here, \( \sum \vec{b} \) denotes the covariance matrix of symbol vector \( \vec{b} \).

Thus, continuous-time observations are directly processed to obtain sufficient data and the symbols are then detected by algorithm processing.

### 4.3 LINEAR MMSE DETECTOR

Linear MMSE detector and linear decorrelating detector are two popular forms of linear detectors for multi-user detection. Both suppress multiple access interference (MAI) but linear decorrelator enhances ambient noise whereas MMSE detector minimizes the total effect of MAI and the ambient noise at the detector output.

The block diagram of MMSE receiver is given in Fig.4.2. The received signal is filtered by a chip matched filter and then sampled at the chip rate. Then the sampled bit vector is estimated and quantized. The detector then minimizes the difference between detected bit and the transmitted bit justifying the name Minimum Mean Square Error (MMSE) detector. The front end of the receiver utilizing MMSE detection principle is shown in Fig. 4.3. The receiver works in decision directed mode, where the decision of the detected bit directs the mode to be operated. But when there is sudden change in environment, it switches to blind adaptation mode and stays there until it converges, then it switches back to decision directed mode.
FIG. 4.2 Block Diagram of MMSE Receiver

FIG. 4.3 Block diagram of Receiver Front end
4.3.1 THEORY OF MMSE DETECTORS

The linear MMSE detector minimizes total effect of MAI and ambient noise at the detector output \([99]\). The linear MMSE detector \(w_1 = m_1 \in C^N\) for 1st user is given by solution to the optimization problem given as-

\[
m_i = \arg\min_{w} \mathbb{E} \left\{ \| A_i b_i [t] - w^H r [t] \|_2^2 \right\}
\]

Where \(A_i\) represents the received complex amplitude of 1st user and

\[
\|A\| = \text{diag}(A_1, \ldots, A_K)
\]

This can be proved as-

The linear MMSE detector for user 1 is given as-

\[
m_i = S[R + \sigma^2 |A|^2]^{-1} e_i
\]  

(4.13)

The MMSE detector \(m_i\) lies in column space of \(S\) [i.e. \(m_i \in \text{range}(S)\)], where \(S\) is the set of all users signaling waveforms

Therefore, \(m_i\) can be written as-

\[
m_i = Sx_i \text{ for some } x_i \in C^K
\]

where

\[
x_i = \arg\min_{x} \mathbb{E} \left\{ \| A_i b_i [t] - x^H S_i^H r [t] \|_2^2 \right\}
\]

\[
= \arg\min_{x} x^H \left[ S^H r [t] r^H [t] S \right] x - 2x^H S_i^H R \{ A_i^* E(b_i [t] [t]) \}
\]

\[
= \left( R + \sigma^2 |A|^2 \right)^{-1} e_i
\]  

(4.15)

Hence Equation 4.13 has been obtained.

\(R = S^H S\) i.e. the correlation matrix of user signature sequences, where \(S\) is full column rank(K) matrix.

The output of the linear MMSE detector is given by

\[
z_i [t] = m_i^H r [t] = A_i (m_i^H s_i) [t] + \sum_{k=2}^K A_k (m_k^H s_k) b_k [t] + v_i [t]
\]  

(4.16)
With
\[ v_t[n] = m_i[n] n_t[n] - N_c(0, \sigma^2 \| m_i \|^2) \tag{4.17} \]
where first term in Equation 4.16 is useful signal of desired user, second term is residual MAI and last term ambient Gaussian noise, which is negligible compared to that for linear decorrelating detector. The detector \( m_i \) is correlated with signaling waveform \( s_k \) to obtain the detected bit \( \| m_i \|^2 \) given as-
\[
\begin{align*}
| m_i |^2 &= (R + \sigma^2 | A |^{-2})^{-1} R (R + \sigma^2 | A |^{-2})^{-1} R_k \\
&= (R + \sigma^2 | A |^{-2})^{-1} R (R + \sigma^2 | A |^{-2})^{-1} R_k
\end{align*}
\tag{4.18}
\]
\[
\| m_i \|^2 = (R + \sigma^2 | A |^{-2})^{-1} R (R + \sigma^2 | A |^{-2})^{-1} R_k
\tag{4.19}
\]
This gives the bit detected at the receiver in the presence of MAI and ambient noise.
Thus the theory of linear MMSE detector has been discussed and system model of MMSE receiver is discussed next.

### 4.3.2 SYSTEM MODEL OF MMSE RECEIVER

Code Division Multiple Access (CDMA) implemented with Direct Sequence Spread Spectrum (DS-SS) modulation is the most popular multiple access technology for most of the wireless channels.

A basic model with \( K \) user time invariant synchronous Additive White Gaussian Noise (AWGN) channel employing periodic spreading sequences with coherent BPSK format [101] is considered.

The received signal \( r_k(t) \) is sum of \( K \) simultaneous CDMA transmissions plus additive white Gaussian noise. The received signal due to the \( k \)th user is
\[
r_k(t) = \sqrt{2P_k} \sum_{j=-\infty}^{\infty} b_{i,k} s_k(t - iT - v_k) \cos(\omega_c t + \theta_k), \quad 1 \leq k \leq K \tag{4.20}
\]
Where \( T \) is the bit interval, \( b_{i,k} \in \{1,-1\} \) is the \( i \)th bit of the \( k \)th user, \( P_k, v_k \) and \( \theta_k \) are the power, delay and carrier phase of the \( k \)th user respectively, \( \omega_c \) is the carrier frequency and \( s_k(t) \) is a spreading waveform given by-
\[
s_k(t) = \sum_{p=0}^{K-1} a_{p,k} \delta(t - pT_c) \tag{4.21}
\]

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Where $a^p \in \{-1, 1\}$ is the $p$th element of the spreading sequence for user $k$, $\psi(t)$ is the chip waveform, $N$ is the processing gain, and $T_c = T/N$ is the chip duration.

Thus the received signal is

$$r(t) = \sum_{p=1}^{N} r^p(t) + n(t)$$  \hspace{1cm} (4.22)

Where $n(t)$ is white Gaussian noise with power spectral density $N_0/2$.

Upon demodulation, the $p$th sample $r[p]$ at the output of chip matched filter correlated with the transmitted waveform matched to this transmission is given as-

$$r[p] = \sqrt{2} \int_{V_{\omega,\theta}}^{} r(t) \psi(t) \cos(\omega_c t + \theta) \, dt$$  \hspace{1cm} (4.23)

It is assumed that power and delay of the desired first transmission are 1 and 0. Also, carrier phase $\theta_k=0$ for each $\theta_k$. For $2 \leq k \leq K$ constituting the co channel interferers, the relative delay $\upsilon_k = (\tau_k + \delta_k) T_c$ where $\tau_k$ is an integer between 0 and $N-1$ and $\delta_k = \upsilon_k / T_c - \tau_k$ lies in the interval $\{0, 1\}$.

The transmitted symbol may be estimated from received sample within a single symbol period.

The estimate of $b_{0,k}$ depends on received signal for $t \in [0,T]$ or on vector of received samples $\vec{r} = (r[0],...,r[N-1]),$ where $N$ is the processing gain. During this symbol interval, $k$th interfering signature sequence is modulated by symbol $b_{0,k}$ for $\upsilon_k < t \leq T$, and by symbol $b_{1,k}$ for $0 < t \leq \upsilon_k$.

Therefore, the received signal $r(t)$ is-

$$r(t) = b_{0,k} a_k + \sum_{i=2}^{N} \sqrt{r_i} \left( b_{0,k} a_{0,i} + b_{1,k} a_{-1,i} \right) + n(t)$$  \hspace{1cm} (4.24)

Where $N$ vector $\vec{a}_k$ with $N$ elements is given as-

$$\vec{a}_k = [a_k[0], a_k[1], ..., a_k[N-1]]^T$$  \hspace{1cm} (4.25)

and

$$[\alpha_{0,k}]_p = \Phi_{1,k} a_k [p - \tau_k]_{x_p \geq \tau_k} + \Phi_{2,k} a_k [p - \tau_k - 1]_{x_p < \tau_k}$$  \hspace{1cm} (4.26)

$$[\alpha_{1,k}]_p = \Phi_{1,k} a_k [p - N - \tau_k]_{x_p \geq \tau_k + 1} + \Phi_{2,k} a_k [p + N - \tau_k - 1]_{x_p < \tau_k + 1}$$  \hspace{1cm} (4.27)
Where 0 ≤ p ≤ N − 1 and 2 ≤ k ≤ K where \( \chi_A \) is the indicator function for the set A

\[
\Phi_{1,k} = \int_0^{T_c} \Psi(t)\Psi(t + \delta_k T_c) dt \quad \text{and} \\
\Phi_{2,k} = \int_0^{T_c} \Psi(t)\Psi(t + (1 - \delta_k) T_c) dt
\]

(4.28)

(4.29)

\( \Psi(t) \) is a rectangular pulse of width \( T_c \). Thus, \( \Phi_{1,k} = 1 - \delta_k \) and \( \Phi_{2,k} = \delta_k \) where \( \Phi \) represents the autocorrelation function of the chip waveform \( \Psi(t) \). Also, \( a_{0,k} \) and \( a_{1,k} \) are linearly independent and are modulated by different bits so that the kth asynchronous interferer contributes two interference vectors during a single symbol interval.

### 4.4 SUPPRESSION OF INTERFERENCE

For wireless communications, frequencies are often assigned by federal authorities on a regional basis. These same frequencies can be assigned to another user or service provider in a different region, sufficiently distant that there is no interference. To maximize frequency reuse, reuse distances are made as small as possible. As a consequence, a service is not interference free, but rather must have a certain interference tolerance. A typical example, as shown in Fig. 4.4, is that of a FM receiver that must tolerate a signal to interference ratio (SIR) of 20 dB at its edge of coverage. ISI, crosstalk interference, NBI and MAI are the most relevant types of interferences existing in the MMSE detection system environment.

![FIG 4.4 Example of traditional Frequency Reuse.](image)
The spectrum for Third Generation (3G) systems in some parts of the world is being allocated to bands not yet vacated by existing narrowband services, creating Narrow Band Interference (NBI). Multiple Access Interference (MAI) is the existence of desired user signal with other users of the same system. MAI and NBI are the most relevant types of interferences existing in the MMSE detection system environment. Although spread spectrum systems are naturally resistant to narrowband interference, active methods of NBI suppression can significantly improve the performance of such systems. It improves error-rate performance and leads to increased CDMA cellular system capacity. In the present work, mathematical models for suppression of NBI and residual MAI are developed.

4.4.1 PERFORMANCE OF MMSE DETECTORS

Next, the performance of MMSE detector has been analyzed in this section.

To detect symbol $b_j$, it is assumed that user signature sequences are linearly independent [i.e. the matrix $S = \begin{bmatrix} s_1 & \ldots & s_K \end{bmatrix}$ has full column rank, rank $(S) = K$]. Let $R = S^H S$ be the correlation matrix of the user signature sequences, where $R$ is invertible. Given the received vector

$$\tilde{r} = \sum_{k=1}^{K} b_k \tilde{P}_k + \tilde{n}$$

(4.30)

Where $\tilde{r} \in \tilde{R}^M$ with dimension $M$ as the processing gain $N$, the vector $\tilde{P}_1$ is the desired signal vector, $b_k$ for $2 \leq k \leq L$ are symbols contributed by interferers and $\{P_k\}, k=2,\ldots,L$ is the set of interference vectors. It is assumed that $b_k \in \{-1,1\}$ and the transmitted symbols are independent with zero mean and noise vector $\tilde{n}$ is Gaussian with zero mean and covariance matrix $\Gamma$.

The detectors used here have the form-

$$\hat{b}_j = \text{sgn}(c^T \tilde{r})$$

(4.31)

Where $c \in \mathbb{R}^M$ is so chosen as to minimize the mean squared error

$$MSE = E\left\{ (c^T \tilde{r} - \hat{b}_j)^2 \right\}$$

(4.32)
In addition to Mean Square Error (MSE), there are two other performance measures in the form of Signal to Interference Ratio (SIR) and error probability. The SIR is defined as the ratio of the desired user signal power to the sum of powers due to noise and multiple access interference at the output of filter \( c \) \[102\] and is given as-

\[
SIR = \frac{c^T \bar{P}}{c^T \Gamma_c + \sum_{k=2}^{K} (c^T \bar{P}_k)^2}
\]  

(4.33)

Keeping the condition of \( b_1 = 1 \), because the users transmit binary, equiprobable, antipodal symbols and conditioning further on vector of interference bits \( \tilde{b}_i = (b_2, \ldots, b_i)^T \), the error probability is obtained as

\[
P_e(\tilde{b}_i) = Q \left( \frac{c^T \bar{P} + \sum_{k=2}^{K} b_k (c^T \bar{P}_k)}{(c^T \Gamma_c)^{1/2}} \right)
\]  

(4.34)

Where

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-t^2/2} dt
\]  

(4.35)

The MMSE solution for \( c \) satisfies

\[
\Lambda c = (I - c^T \bar{P}_i) \bar{P}_i
\]  

(4.36)

Where \( \Lambda = \sum_{k=2}^{K} \bar{P}_k \bar{P}_k^T + \Gamma \) \n
(4.37)

All solutions to Equation 4.36 minimize the Mean Square Error (MSE) and also maximize the SIR being denoted by \( SIR_{\text{MAX}} \).

\[
SIR_{\text{MAX}} = \frac{c^T \bar{P}_i}{(I - c^T \bar{P})} = \bar{P}_i A^{-1} \bar{P}_i = \text{MMSE}^{-1} - 1
\]  

(4.38)

Thus the working of MMSE receiver to evaluate its performance has been studied here. The system performance measuring parameters are Mean Square Error (MSE), Signal to Interference Ratio (SIR) and error probability \( (P_e)\).
4.4.2 MMSE RECEIVER AND FADING CHANNELS

The theoretical performance of a MMSE receiver for a general fading channel that may be either frequency-selective or nonselective is evaluated here. MMSE receiver has been analyzed considering Gaussian noise model. Multipath fading can cause significant degradation in performance of an MMSE receiver [103]. A standard model for asynchronous DS-CDMA is assumed and further mathematical model of MMSE receiver has been developed taking into consideration effect of fading.

4.4.2.1 MATHEMATICAL MODEL OF MMSE RECEIVER IN FADING CHANNELS

The received signal is taken to be of the form

\[ r(t) = \sum_{k=1}^{Q_k} r_k(t) + N_w(t) \]  

(4.39)

Where \( r_k(t) \) is the received signal from the \( k \)th user and \( N_w(t) \) is complex white Gaussian noise having the property of expected value given in Equation 4.40 as-

\[ E[N_w(t)N_w^*(s)] = 2N_0 \delta(t - s) \]  

(4.40)

Where \( N_0 \) represents noise power.

Assuming a fading multipath channel, each received signal takes on the form-

\[ r_k(t) = \sum_{n=1}^{Q_k} \sqrt{P_k} y_{k,n}(t) S_{k}(t - \tau_{k,n}) \]  

(4.41)

Where \( Q_k \) is the number of the paths for the \( k \)th signal, and \( P_k, y_{k,n}(t) \) and \( \tau_{k,n} \) are the received power, the complex fading process and the relative delay for the \( n \)th received path of the \( k \)th user’s signal, respectively. The intersymbol interference (ISI) is assumed to be a minimum.

The MMSE receiver [101] takes the signal at complex base band and passes it through a chip matched filter and samples the output of that filter at the chip rate and is synchronous with the reception of the desired user’s first path. \( N \) chip samples are stored for each symbol received and together these chip samples form the received vector \( \hat{r}(m) \), which (for the \( m \)th symbol) is given as-

\[ \hat{r}(m) = (r_{mN}, r_{mN+1}, \ldots, r_{mN+1})^T \]  

(4.42)
The MMSE receiver filters this received vector with a finite impulse response discrete filter characterized by the N-element tap weight vector \( \mathbf{w}(m) \). During each symbol interval, the data decisions are made on the basis of output of this filter \( z(m) \) given as:

\[
z(m) = \mathbf{w}^H (m) \mathbf{r}(m)
\]  

(4.44)

The MMSE receiver is operated in a coherent manner, where decisions \( d_1(m) \) are given as:

\[
d_1(m) = \text{sgn}(\text{Re}[z(m)])
\]  

(4.45)

Where \( d_1(m) \) is the detected symbol.

If the transmitted data bits are differentially encoded, the coherently decoded data is differentially decoded and represented as \( \hat{b}_1(m) \):

\[
\hat{b}_1(m) = \hat{d}_1(m) \hat{d}_1(m-1)
\]  

(4.46)

Where \( b_1(m) \) is the detected bit and \( d_1(m) \) and \( d_1(m-1) \) are coherently decoded data.

To avoid difficulties faced on a fading channel, differential detection on the output of the MMSE filter is used and data decisions are given as:

\[
\hat{d}_1(m) = \text{sgn}(\text{Re}[z(m)z^*(m-1)])
\]  

(4.47)

The tap weights of the MMSE filter are chosen to minimize the mean squared error \( e(m) \).

The Minimum Mean Square Error \( J(m) \) is given as:

\[
J(m) = \mathbb{E}[|e(m)|^2]
\]  

(4.48)

\[
= \mathbb{E}[|d_1(m) - z(m)|^2]
\]  

(4.49)

The tap weight vector which minimizes this mean squared error is given by:

\[
\mathbf{w}(m) = \mathbf{R}^{-1}(m)\bar{p}(m)
\]  

(4.50)

where,

\[
\mathbf{R}(m) = \mathbb{E}[\mathbf{r}(m)\mathbf{r}^*(m)]
\]  

and

\[
\bar{p}(m) = \mathbb{E}[d_1^*(m)\bar{r}(m)]
\]  

(4.51)

(4.52)

i.e. \( \mathbf{R} \) is the sample autocorrelation matrix and \( \bar{p} \) is the steering vector.
4.4.2.2 EFFECT OF FADING PARAMETER (\(\gamma\))

In the present work the factor \(\gamma\) has been introduced for analysis of the effect of fading on the system performance. It is defined as under

\[
\alpha = \gamma \sqrt{\frac{E_b}{N_0}} \tag{4.53}
\]

Where \(\alpha\) is the instantaneous signal to noise ratio of the received signal, \(\gamma\) is the fading parameter, \(E_b\) is the bit energy (carrier power / bit rate) and \(N_0\) is the noise power.

Using the model of received vector, now the received vector after considering effect of fading parameter is modeled as [104]-

\[
\tilde{r}(m) = \sum_{k=1}^{K} \sum_{r=1}^{R} \left( \frac{P_k}{P_1} \right) \gamma_{k,r}(m) \left[ d_k(m - L_{k,r} - 1) \mu_k(r) (NT_c - \mu_k(r)) + d_k(m - L_{k,r}) \mu_k(r) \right] + n(m) \tag{4.54}
\]

Where

\[
L_{k,r} = \left\lfloor \frac{\tau_{k,r}}{T_s} \right\rfloor
\]

\[
\mu_{k,r} = \tau_{k,r} - L_{k,r} T_s
\]

i.e. \(L\) is the integral part of the relative delay and \(\mu_{k,r}\) is the fractional part of delay offered by \(r\)th path of \(k\)th user. \(P_1\) is the total power received from the desired signal. The fading processes do not change over the duration of a symbol.

The received vector has been scaled by \(\sqrt{2P_1T_c}\) and the noise component has a covariance matrix given by \(\sigma^2 I\) where \(I\) is an identity matrix and \(\sigma^2 = \frac{N}{\left( \frac{E_b}{N_0} \right)}\) is the variance of noise.

4.4.3 ADAPTIVE MMSE RECEIVER IN FLAT FADING CHANNELS

The mobile receiver, in the downlink of a CDMA system has knowledge of only its own signature sequence, but not of those of other users. Hence, the problem of blind implementation (adaptive) of linear detector is considered, where there is no requirement of knowing the signature sequence of interfering users.

In the case of a flat fading channel model, the received signal vector \(\tilde{r}(m)\) becomes [104]
\[ \hat{r}(m) = \hat{d}_1(m)\hat{r}_1(m)k^*_m + \hat{r}(m) \]  
\[ (4.55) \]

Where \( \gamma \) is the fading parameter.

The first part is the desired signal. The second term represents noise due to a finite observation interval and has to be neglected.

For slow fading case, the fading process is invariant over the interval of observation. The MMSE detector is then

\[ z(m) = \hat{w}^H \hat{r}(m) \]  
\[ (4.56) \]

Where \( \hat{w} \) is the weight vector and \( \hat{r}(m) \) is the received signal vector.

The MMSE tap weights are given as

\[ \hat{w}(m) = R^{-1}(m)\hat{p}(m) \]  
\[ (4.57) \]

Where \( R \) is the sample autocorrelation matrix and \( \hat{p} \) is the steering vector, which is formed according to

\[ \hat{p} = \frac{1}{M} \sum_{m=1}^{M} h^*_m(m)z^*(m-1)\hat{r}(m) \]  
\[ (4.58) \]

Thus the MMSE tap weights are given as

\[ \hat{w}_k = R^{-1}z^*(\mu_{k,\lambda}) \]  
\[ (4.59) \]

**4.4.3.1 MATHEMATICAL MODEL OF ADAPTIVE MMSE RECEIVER**

The working of an adaptive MMSE receiver is analyzed using decomposition of linear detector.

Hence, in the work carried out in the thesis, the mathematical model of MMSE receiver on the above assumption has been proposed to find out \( P_e \) of the signal.

The operation of adaptive receiver is based on the decomposition of the linear detector as

\[ \bar{c} = s + x \]  
\[ (4.60) \]

Where \( s \) is the signature sequence of the user and the \( x \) is an adaptive component.
This analysis results in better estimation of the detected symbol and hence results in lesser probability of error $P_e$, which has been analyzed as under-

The analysis has been carried out keeping into consideration the following condition-

$$s^T x = 0 \quad (4.61)$$

The detector $c$ can be found by the method of Lagrange multipliers.

Taking fading into consideration

Let

$$L(c) = MSE - 2\eta(s^T c - 1) \quad (4.62)$$

where $\eta$ is the Lagrange's multiplier.

By taking differential of $L$ and setting it to zero as-

$$\Delta L = 0 \quad (4.63)$$

**4.4.3.2 EFFECT OF FADING PARAMETER ($\gamma$)**

The effect of fading is taken into consideration according to the formula-

$$\alpha = \gamma \frac{E_b}{N_0}$$

Where $\alpha$ is the signal to noise ratio of the received signal, $\gamma$ is the fading parameter, $E_b$ is the bit energy (carrier power /bit rate) and $N_0$ is the noise power.

Now keeping fading into consideration the model developed for detector $c$ is given as-

$$c = (P_s + \gamma)R^{-1}s \quad (4.64)$$

The tap weights are decomposed as-

$$w_k = c_k^*(\mu_{t,k}) + x_k \quad (4.65)$$

$$c_k = (P_s + \gamma)R^{-1}s_k \quad (4.66)$$

This leads to the solution

$$w_k = R^{-1}(P_s + \gamma)R^{-1}s_k \left[ (P_s + \gamma)R^{-1}s_k \right]^T R^{-1}(P_s + \gamma)R^{-1}s_k \right] c_k^* c_k (\mu_{t,k}) \quad (4.67)$$

The performance of this system can be evaluated by parameter probability of error $P_e$ given as [105]-

$$P_{e^k} = \frac{1}{2 + 2(P_s + \gamma)R^{-1}s_k^T R^{-1}(P_s + \gamma)R^{-1}s_k} \quad (4.68)$$
The relationship between $P_{e/k}$ and $K$ is based on

$$P_{e/k} = \frac{1}{2^4-1} \sum_{k=1}^{K-1} \frac{Q\left(\frac{A_i w_i^T s_i + \sum_{i=2}^{K} A_i b_i w_i^T s_i}{\|w_i\|\sigma}\right)}$$  \hspace{1cm} (4.69)

The performance of adaptive MMSE receiver has further been analyzed by way of various graphs and tables in terms of relationship between probability of error $P_e$ and number of users using equations given above.

### 4.5 INTERFERENCE SUPPRESSION USING CODE-AIDED TECHNIQUES

When improvements in NBI suppression are made by going beyond random modeling at the chip level and making use of the fact that spreading code of at least one user must be known for data demodulation, these techniques are called code aided techniques [106]. The interfering signal has been taken as a sinusoidal signal (tone).

Progress in the area of NBI suppression for spread systems until late 1980s were frequency-domain techniques and predictive or interpolative techniques based on linear predictors or interpolators. Now days there have come up various techniques that improve upon the performance of predictive and interpolative methods. Performance of the systems based on code-aided techniques can be further improved by making use of the structure of the useful data signal and of the NBI wherever possible.

Existing active NBI suppression techniques can be grouped into three basic types: frequency-domain techniques, predictive techniques and code-aided techniques. Frequency-domain techniques operate by transforming the received signal into the frequency domain, masking frequency bands in which NBI is dominant and then passing the signal off for subsequent despreading and demodulation as shown in Fig. 4.5 [99].
Predictive systems operate in the time domain. They make use of the difference in bandwidths of spread spectrum signal and the NBI without making use of any knowledge of the specific structure data signal. The discrepancy in predictability of narrowband signals form an accurate replica of the NBI that can be subtracted from the received signal to suppress the NBI. The received signal $r(t)$ consists of wideband components $\{S(t)+N(t)\}$, where $S(t)$ is useful data signal, NBI is wideband ambient noise and the narrowband interference component $I(t)$. If a linear prediction of $\{r(t)\}$ is generated, the values predicted will consist of a prediction of $\{I(t)\}$. Such a prediction forms a replica of NBI, which can then be suppressed from the received signal. A residual signal $r(t) - \hat{r}(t)$ is formed, where $\hat{r}(t)$ is a prediction of $r(t)$ from past observations. The effect of subtraction is to significantly reduce the narrowband component of $\{r(t)\}$. The prediction residual is then passed on for despreading and demodulation as shown in Fig.4.6. The code aided techniques make use of structure of useful data signal and hence provide improved performance.
4.5.1 MATHEMATICAL MODEL OF CODE AIDED RECEIVER

The mathematical model of system rejecting NBI and MAI is developed here. The code of the weight vector is broken into signature sequence and additional component into which effect of fading is considered in form of fading parameter \[107\].

The analysis of code aided receiver is carried out using the received signal for receiver already discussed in section 4.3.2.

The received samples estimate a given transmitted symbol within a single symbol period. Therefore, estimates of \( b_c \) depends on received signal for \( t \in [0, T] \).

The symbol \( \hat{b}_1 \) can be detected given the received vector-

\[
\hat{r} = \sum_{i=1}^{L} b_i \hat{P}_i + \tilde{n}
\]  

(4.70)

Where \( \hat{r} \in \mathbb{R}^M \), \( \hat{P}_i \) is desired vector; \( b_i, 2 \leq k \leq L \) are symbols contributed by interferers; \( \hat{P}_i \) is set of interference vectors.

The bit \( b_1 \) is detected according to the following rule:

\[
\hat{b}_1 = \text{sgn}(c^T r)
\]

(4.71)

Where \( c \in \mathbb{R}^M \) is chosen to minimize mean square error

\[
MSE = E \left\{ (c^T r - b_1)^2 \right\}
\]

(4.72)

\[
= (c^T P_1 - 1)^2 + \sum_{k=2}^{L} (c^T P_k)^2 + c^T \Gamma_c
\]

(4.73)

Now, the effect of fading is taken into consideration.

Assuming a fading multipath channel, received signal can be modeled as

\[
r(t) = \sum_{k=1}^{K} r_k(t) + I(t) + N(t)
\]

(4.74)

where

\[
r_k(t) = \sqrt{Q_k} \gamma_k(t) s_k(t - \tau_{k,r})
\]

(4.75)

\( Q_k \) is the number of paths for \( k \)th signal, \( \gamma_k(t) \) is complex fading process and \( \tau_{k,r} \) is relative delay for \( r \)th received path of \( k \)th user.
4.5.2 EFFECT OF FADING PARAMETER (γ)

The effect of fading is taken into consideration according to the formula

\[ \alpha = \gamma \sqrt{\frac{E_b}{N_0}} \]  

(4.76)

Where \( \alpha \) is the signal to noise ratio of the received signal, \( \gamma \) is the fading parameter, \( E_b \) is the bit energy (carrier power /bit rate) and \( N_0 \) is the noise power.

The received signal vector can be represented as

\[ \tilde{r} = h\sqrt{P_s} s + i + n \]  

(4.77)

The MMSE detector has the form [108]-

\[ \hat{b} = \text{sgn}(c^T r) \]  

(4.78)

Mean Square Error after taking effect of fading into consideration is given as-

\[ \text{MSE} = E\left( c^T r - b\gamma \sqrt{P_s} \right)^2 \]  

(4.79)

MMSE receiver forms-

\[ z(m) = w^H(m) r(m) \]  

(4.80)

The fading parameter is incorporated into analysis for calculation of weight as follows-

Using canonical representation of MMSE detector

\[ \tilde{w} = s + x \]  

(4.81)

Where all the symbols used have the same meaning as used earlier.

Taking effect of fading into account as-

\[ = \gamma s(m) + x \]  

(4.82)

The weight vector is given as-

\[ \tilde{w}'' = s''(m)\gamma + \tilde{x} \]  

(4.83)

This is subject to constraint \( w^T s = ||s||^2 = 1 \)  

(4.84)

\( \tilde{w} \) can be found by method of Lagrange’s multipliers.

\[ L(w) = \text{MSE} - 2\eta (s^T w - 1) \]  

(4.85)

where \( \eta \) is the Lagrange’s multiplier.
The system equations are analyzed on the above lines to get the expression for MSE as under-

Taking differential of $L$ w.r.t $w$ and setting it to zero as-

$$\nabla_w L = 0$$  \hspace{1cm} (4.86)

$w$ is obtained as-

$$w = (P_s + \eta) R^{-1}s$$  \hspace{1cm} (4.87)

Under the constraint

$$s^T w = 1$$  \hspace{1cm} (4.88)

The weight vector is given as-

$$w = \frac{1}{s^T R^{-1}s} R^{-1}s$$  \hspace{1cm} (4.89)

Next the mean square error is obtained

$$MSE = w^T \left( R_i + \sigma_i^2 I + \gamma^2 P_s s_s^T \right) w - 2\gamma^2 P_s w^T s + \gamma^2 P_s$$ \hspace{1cm} (4.90)

Taking gradient of MSE w.r.t. $w$ and setting it to zero.

$$\left( R_i + \sigma_i^2 I + \gamma^2 P_s s_s^T \right) w - \gamma^2 P_s s = 0$$ \hspace{1cm} (4.91)

The parameter for measurement of system performance in MMSE receiver is defined as-

$$SIR = \frac{E[w^T r]}{\text{var } w^T r}$$ \hspace{1cm} (4.92)

$$E[w^T r] = \beta y \sqrt{P_s}$$ \hspace{1cm} (4.93)

$$\text{var } w^T r = E[w^T r - \beta y \sqrt{P_s}]$$ \hspace{1cm} (4.94)
The parameter for measurement of system performance for the proposed work is SIR which is hence defined as

$$\text{SIR} = \frac{\gamma^2 P_s}{\sigma_n^2} \left[ \frac{1}{\left( R_s + \sigma_n^2 I \right)^{-1}} \right] s$$

(4.95)

4.5.3 SUPPRESSION OF TONE (INTERFERENCE) SIGNALS

Considering a sinusoidal interfering signal of the form

$$i(k) = \sum_{i=1}^{m} \sqrt{P_i} e^{j(2\pi f_i t + \Phi_i)}$$

(4.97)

Where $P_i$ and $f_i$ are the power and normalized frequency of the $i^{th}$ sinusoid and $\Phi_i$ are random phases distributed over $(0,2\pi)$.

The expected value of SIR with respect to $s$ is

$$E\{\text{SIR}_i\} = \left[ 1 - \frac{P_i / \sigma_n^2}{1 + N \left( P_i / \sigma_n^2 \right)} \right] \frac{P_s}{\sigma_n^2}$$

(4.98)

$$= \left( 1 - \frac{1}{N} \right) \frac{\gamma^2 P_s}{\sigma_n^2} \quad \text{as} \quad P_i \to \infty$$

(4.99)

It is found that expression for $P_i$ vanishes and hence the energy of the strong interferer is suppressed to a large extent by the MMSE detector.

For $m=2$

$$E(\text{SIR}_2) = \left( 1 - \frac{2}{N} \right) \frac{\gamma^2 P_s}{\sigma_n^2}$$

(4.100)

It is concluded that MMSE receiver suppresses the tone interference signals, one and two in number respectively. As the fading severity is reduced ($\gamma$ is higher), interference is rejected in a better fashion. As the processing gain ($N$) increases, it is observed from Equation 4.100 that the SIR decreases and the interfering energy almost suppresses.
4.6 RESULTS AND DISCUSSION

The results of the equations analyzed previously are represented in the form of graphs. For adaptive case Mean Square Error (MSE) is evaluated and plotted as a function of signal power for varying values of fading parameter $\gamma$ ranging from 0 to 1 (practically used values in the present scenario), probability of error and number of users. For code aided case, SIR is analyzed for one and two number of interfering tone signals. Also, graph between SIR and interfering power is drawn.

Figure 4.7 depicts the behavior of MMSE receiver in the presence of fading. The same is represented by drawing graph between MSE and signal power. Fig. 4.8 is the representation of relation between probability of error ($P_e$) and capacity of the system in terms of the number of users for the adaptive MMSE receiver. The graph also compares the performance of the already existing cases and adaptive MMSE.

The next set of graphs is for code aided MMSE receiver for different number of interfering tones (one and two in number respectively). It shows two set of graphs (i.e. Fig. 4.9, 4.12 and Fig. 4.10, 4.13) between SIR and $P_s$ (signal power) and SIR and $\gamma$ (fading parameter). Fig. 4.11 shows the comparison of suppression of interference tone signal by conventional MMSE receiver and the code aided MMSE receiver. Also, comparison of performance by different techniques is done in Fig. 4.14.
4.6.1 RESULTS FOR PERFORMANCE OF LINEAR MMSE DETECTOR

FIG. 4.7 Plot between Mean Square Error (MSE) and signal power ($P_s$) for fading parameter ($\gamma$) values of 0.2, 0.6 and 0.8.

<table>
<thead>
<tr>
<th>Signal Power $P_s$ (dBm)</th>
<th>Mean Square Error (MSE) $\gamma$=0.2</th>
<th>Mean Square Error (MSE) $\gamma$=0.6</th>
<th>Mean Square Error (MSE) $\gamma$=0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6400</td>
<td>0.1600</td>
<td>0.0400</td>
</tr>
<tr>
<td>2</td>
<td>0.5143</td>
<td>0.0229</td>
<td>0.0173</td>
</tr>
<tr>
<td>3</td>
<td>0.4272</td>
<td>0.0015</td>
<td>0.1487</td>
</tr>
<tr>
<td>4</td>
<td>0.3600</td>
<td>0.0400</td>
<td>0.3600</td>
</tr>
<tr>
<td>5</td>
<td>0.3056</td>
<td>0.1167</td>
<td>0.6223</td>
</tr>
<tr>
<td>6</td>
<td>0.2602</td>
<td>0.2206</td>
<td>0.9208</td>
</tr>
<tr>
<td>7</td>
<td>0.2217</td>
<td>0.3451</td>
<td>1.2468</td>
</tr>
<tr>
<td>8</td>
<td>0.1886</td>
<td>0.4859</td>
<td>1.5945</td>
</tr>
<tr>
<td>9</td>
<td>0.1600</td>
<td>0.6400</td>
<td>1.9600</td>
</tr>
<tr>
<td>10</td>
<td>0.1351</td>
<td>0.8053</td>
<td>2.3404</td>
</tr>
</tbody>
</table>

TABLE 4.1 Mean square error versus signal power for $\gamma=0.2$, 0.4 and 0.6.
The curves in Fig. 4.7 have been drawn in accordance with Equation 4.79. Table 4.1 shows the variation of MSE with signal power ($P_s$) for three different values of $\gamma$ (gamma) = 0.2, 0.6 and 0.8. It is observed from Fig. 4.7 that in all the three cases MSE varies differently with signal power $P_s$. Further, MSE decreases asymptotically with $P_s$ for $\gamma$ (gamma) = 0.2. This implies that the difference between the estimated and transmitted bit will decrease with increasing signal power as the effect of noise also decreases during the transmission of the signal if the signal power is less. It is observed that the error decreases with decrease in fading effect. The fading severity has reduced from higher values of $\gamma = 0.6, 0.8$ to a lower value of $\gamma = 0.2$. It is mentioned that the value of fading parameter is higher in magnitude ($\gamma = 0.8$), when the fading severity is lesser in effect. The MSE is decreased with decrease in fading effect because the lesser the constructive and destructive cancellation of the multipath signals, lesser is the mean square error. Hence, the bits are detected with more accuracy. This is so because if the effect of fading is lessened to negligible extent, it becomes a linear detector with fading not coming in picture.
4.6.2 RESULTS FOR ADAPTIVE MMSE RECEIVER

FIG. 4.8 Graph between Probability of error and the number of users

<table>
<thead>
<tr>
<th>Number of Users (K)</th>
<th>Probability of Error (Pe)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Matched Filter</td>
</tr>
<tr>
<td>5</td>
<td>.080</td>
</tr>
<tr>
<td>10</td>
<td>.100</td>
</tr>
<tr>
<td>15</td>
<td>.109</td>
</tr>
<tr>
<td>20</td>
<td>.117</td>
</tr>
<tr>
<td>25</td>
<td>.120</td>
</tr>
<tr>
<td>30</td>
<td>.126</td>
</tr>
</tbody>
</table>

TABLE 4.2 Probability of error versus number of users for different detectors
The above figure plots between the probability of error and the permissible number of users for adaptive MMSE applied to fading, and the comparison is done for matched filter and MMSE (slow), which already exist. It is observed that performance of proposed adaptive MMSE receiver shows significant improvement as compared to conventional matched filter receiver. Also, the performance is improved as compared to conventional MMSE receiver. And it is observed that probability of error reduces from 0.080 to 0.004 for 5 users, from 0.109 to 0.010 for 15 users and from 0.120 to for 0.017 for 25 users. Though improvement is lesser for higher number of users for the proposed receiver, but still it is preferred because it can adapt itself to changing channel conditions in a better manner.
4.6.3 RESULTS FOR CODE AIDED MMSE RECEIVER

FIG. 4.9 Graph between Signal to interference ratio (SIR) and the signal power (Ps) for fading parameter (γ) values of 0.4, 0.6 and 0.8 for code aided technique (m=1).

<table>
<thead>
<tr>
<th>Signal Power Ps (pWatts)</th>
<th>Signal to Interference Ratio (SIR) (×1.0e+006)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>γ = 0.4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>0.1548</td>
</tr>
<tr>
<td>200</td>
<td>0.3097</td>
</tr>
<tr>
<td>300</td>
<td>0.4645</td>
</tr>
<tr>
<td>400</td>
<td>0.6194</td>
</tr>
<tr>
<td>500</td>
<td>0.7742</td>
</tr>
</tbody>
</table>

TABLE 4.3 SIR versus Ps for code-aided MMSE receiver with γ=0.4, 0.6 and 0.8 (m=1).
The plot shown above is between the signal to interference ratio (SIR) and the signal power \( P_s \) for fading parameter of 0.4, 0.6 and 0.8 in accordance with Eq. (4.99). \( N=31 \) and \( \sigma_n^2=0.1 \) The results obtained are also shown in the Table 4.3. It implies that when MMSE is operated with code aided technique, the Signal to interference ratio (SIR) increases with increase of signal power. The interfering sinusoid is suppressed almost completely. The suppression is linear. For a step increase of 100 in the signal power, the SIR almost doubles as shown in Table 4.3. It also increases with the fading parameter \( \gamma \). As the effect of fading is reduced i.e. for increased value of fading parameter, SIR can be observed to increase. The SIR gets doubled as the signal power increases from 200 to 400pW. But, it degrades with increase in fading severity (reduction in the value of fading parameter \( \gamma \)). SIR decreases from 0.6194 to 0.1548 as fading parameter decreases from \( \gamma = 0.8 \) to \( \gamma = 0.4 \). This is so because if the cancellation of signal is less, the estimated SIR at the detector is higher. It is mentioned again that as the fading severity increases, the corresponding value of the fading parameter is taken to be lesser in magnitude.
FIG. 4.10 Graphs between Signal to interference ratio (SIR) and fading parameter \( \gamma \) for signal power \( P_s = 100, 200, 300, 400 \) and 500pW \((m=1)\)

<table>
<thead>
<tr>
<th>Fading Parameter ( (\gamma) )</th>
<th>Signal to Interference Ratio (SIR) ( (\times 1.0e+006) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.006 0.0194 0.0290 0.0387 0.0484</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0387 0.0777 0.1161 0.1548 0.1935</td>
</tr>
<tr>
<td>0.3</td>
<td>0.0871 0.1742 0.2613 0.3484 0.4355</td>
</tr>
<tr>
<td>0.4</td>
<td>0.1548 0.3097 0.4645 0.6194 0.7742</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2419 0.4839 0.7258 0.9677 1.2097</td>
</tr>
<tr>
<td>0.6</td>
<td>0.3484 0.6968 1.0452 1.3935 1.7419</td>
</tr>
<tr>
<td>0.7</td>
<td>0.4742 0.9484 1.4226 1.8968 2.3710</td>
</tr>
<tr>
<td>0.8</td>
<td>0.6194 1.2387 1.8581 2.4774 3.0968</td>
</tr>
<tr>
<td>0.9</td>
<td>0.7839 1.5677 2.3516 3.1355 3.9194</td>
</tr>
<tr>
<td>1.0</td>
<td>0.9677 1.9355 2.9032 3.8710 4.8387</td>
</tr>
</tbody>
</table>

TABLE 4.4 SIR versus Gamma for \( P_s = 100, 200, 300, 400 \) and 500 pW.
Fig. 4.10 analyzes Signal to Interference ratio as a function of parameter quantifying fading severity. It is observed that performance improves with increase of fading parameter (gamma) from 0 to 1 i.e. with fall of fading severity. It shows a nonlinear increase in SIR with gamma, the fading parameter. As the fading parameter increases, SIR also increases. Fading being the constructive and destructive addition of multipath signals, shows an increase of SIR with lesser fading severity. The curves are in the linear region for moderate values of fading parameter $\gamma$ (between 0.3 and 0.8). The results are found to be in good agreement with the results for linear MMSE detector. Moreover, MSE shows practical results and the receiver state is stable for value of $\gamma$ in this range. Hence, it is recommended to use moderate fading severity. The SIR is further analyzed for signal powers of 100, 200, 300, 400 and 500(pW). For higher values of signal power, the SIR is higher for the same value of gamma. SIR follows similar increasing trend with fading parameter for different signal power. However, nature of increase is more curvilinear in nature for higher values of signal power. Fig. 4.10 is showing variation of the MMSE detector in the presence of both MAI and NBI. The MAIs are synchronous with the desired SS user, with random signature sequences and are of the same power.
FIG 4.11 Graph between SIR (dB) and the tone power (pW) for the two cases of conventional MMSE and code aided MMSE in fading.

Series 1, 3, 5 (Red)  One set of three tone interference frequencies suppressed by MMSE.
Series 2, 4, 6 (Green) Frequencies suppressed by the code aided MMSE detector.

The above graph is drawn between Signal to Interference ratio (SIR) in dB and interference signal power $P_I$ in pW for two different sets of three tone frequencies. The frequency can be randomly chosen. It is observed that the proposed receiver works irrespective of interference signal power because SIR is nearly constant with interference signal power. The proposed code aided MMSE receiver gives a better SIR for the same set of three frequencies.
4.6.3.1 RESULTS FOR CODE AIDED MMSE RECEIVER FOR TWO NUMBER OF INTERFERING TONE SIGNALS \((m=2)\)

FIG. 4.12 Graph between SIR and signal power \((P_s)\) for fading parameter \((\gamma) = 0.4, 0.6\) and 0.8 \((m=2)\).

<table>
<thead>
<tr>
<th>Signal Power (P_s) (pWatts)</th>
<th>Signal to Interference Ratio (SIR) ((\times1.0e+006))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\gamma = 0.4)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>0.0374</td>
</tr>
<tr>
<td>200</td>
<td>0.0748</td>
</tr>
<tr>
<td>300</td>
<td>0.1123</td>
</tr>
<tr>
<td>400</td>
<td>0.1497</td>
</tr>
<tr>
<td>500</td>
<td>0.1871</td>
</tr>
</tbody>
</table>

TABLE 4.5 SIR versus \(P_s\) for code-aided MMSE receiver with \(\gamma=0.4, 0.6\) and 0.8 for \(m=2\)
The above plots are between SIR and Signal power $P_s$ for $m=2$ drawn in accordance with Equation 4.100, where $N=31$ and $\sigma_n^2=0.1$. It is seen that the interference is suppressed as in the previous case ($m=1$) shown in Fig. 4.9 but in a slight worse fashion. The results obtained are also shown in the Table 4.5. This is so because as the number of complex sinusoids increases, the interference power also increases. For the increase in fading severity, the relative SIR decreases as observed from the above graphs. Hence, this method of suppressing the interference in fading channels outperforms other methods. It is assumed that the receiver knows the signature sequences of all users and has perfect synchronization and channel estimation for each user and perfect power control for slow fading and path loss. The channel for each user has independent identically distributed frequency flat Rayleigh fading with normalized Doppler frequency. The graph shows linear increase of SIR with power.
FIG. 4.13 Signal to interference ratio (SIR) and fading parameter $\gamma$ for various values of signal power for two number of interfering signals

<table>
<thead>
<tr>
<th>Fading Parameter ($\gamma$)</th>
<th>Signal to Interference Ratio (SIR) ($\times 1.0e+006$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_s=100\text{pW}$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0094</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0374</td>
</tr>
<tr>
<td>0.3</td>
<td>0.0842</td>
</tr>
<tr>
<td>0.4</td>
<td>0.1497</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2339</td>
</tr>
<tr>
<td>0.6</td>
<td>0.3368</td>
</tr>
<tr>
<td>0.7</td>
<td>0.4584</td>
</tr>
<tr>
<td>0.8</td>
<td>0.5987</td>
</tr>
<tr>
<td>0.9</td>
<td>0.7577</td>
</tr>
<tr>
<td>1.0</td>
<td>0.9355</td>
</tr>
</tbody>
</table>

TABLE 4.6 SIR versus Gamma for $P_s=100, 200, 300, 400$ and $500$ pW ($m=2$)
Fig. 4.13 depicts the graph between SIR and fading parameter for varying values of power $P_s$ i.e. 100, 200, 300, 400 and 500 pW as also shown in Table 4.6. It shows a nonlinear increase of SIR with gamma. The increase is almost two times with twofold increase in gamma. The rate of increase in SIR is more for higher value of signal power $P_s$. The value of SIR increases almost threefold on doubling the fading parameter for $P_s=200$ (pW). These values are for narrowband interference signal having two numbers of complex sinusoids. As the value of signal power increases the rate of increase becomes higher for the same value of fading parameter gamma. The higher value of fading parameter means lesser fading severity and hence higher SIR. After comparing the graphs for $m=1$ and $m=2$, it is observed that the rejection of interference slightly worsens as the value of $m$ increases.
4.6.3.2 COMPARATIVE PERFORMANCE OF VARIOUS INTERFERENCE SUPPRESSION TECHNIQUES

FIG 4.14 Comparison of performance by different interference suppression techniques in terms of SIR (dB) vs. narrowband interference power (dB).

<table>
<thead>
<tr>
<th>Series</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Matched filter</td>
</tr>
<tr>
<td>2</td>
<td>MMSE detector</td>
</tr>
<tr>
<td>3</td>
<td>No interference</td>
</tr>
<tr>
<td>4</td>
<td>Code aided MMSE</td>
</tr>
</tbody>
</table>

The above graph is between SIR (dB) and the narrowband interference power (dB) for the various Narrowband interference rejection techniques i.e. matched filter, MMSE detector and the proposed code aided MMSE technique. The noise power is held constant at $\sigma_n^2 = -20$ dB relative to the Spread Spectrum (SS) signal after despreading. The spreading signature sequence is a length-31 m sequence. It is observed that the results are better than the conventional methods of matched filter and linear MMSE detector i.e. SIR is higher for the code aided MMSE receiver (for the same values of interference power) than matched filter and conventional MMSE detector. When code aided MMSE is applied, there is improvement in rejection of interference up to the extent of 10%.

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