APPENDIX - II

EFFECT OF RECEIVER FILTER ON THE FAİING COMPONENT OF SIGNAL.

The stochastic component of the received signal $x_p(t)$ resulting from the fading channels is given by

$$x_p(t) = x_{rs}(t) \cos (v_o t + \gamma) - x_{rs} \sin (v_o t + \theta) \quad (II-1)$$

When this signal component is passed through the receiver filter, with impulse response as $h_p(t)$ as shown in Fig. (4.5).

Since the receiver filter is assumed to be linear, if the signal input consists of number of components, then the output will be the superposition of the outputs, each one corresponding to one of the input components. Thus for the fading component as input, let $w(t)$ be the corresponding output. Hence the variance $\sigma^2_w$ is given by

$$\sigma^2_w = \mathbb{E}[w^2(t)] \quad (II-2)$$

Assuming $w(t) = \text{Re}[\mathcal{W}(t) \exp \{j(v_o t + \theta)\}]$

$$= \frac{1}{2} v(t) \cdot \exp \{j(v_o t + \theta)\} \cdot \frac{1}{2} w(t)$$

$$\exp \{-jw (v_o t + \gamma)\} \quad (II-5)$$
and input signal $x(t)$ is

$$x_r(t) = \Re \left[ X(t) - S(t) \right]$$

(II-4)

where $W(t)$ is a complex random process, $X(t)$ is a complex Gaussian process with zero mean and $\sigma_w^2$ as variance and $S(t)$ is the complex signal. Thus $X(t)$ and $S(t)$ are of the form

$$X(t) = \left[ x_r(t) + j x_i(t) \right]$$

and

$$S(t) = \exp \left\{ j (\omega_0 t + \phi) \right\}$$

$$= \cos (\omega_0 t + \phi) + j \sin (\omega_0 t + \phi)$$

From eq. (II-2) and (II-3)

$$\sigma_w^2 = \mathbb{E} \left[ \frac{1}{2} \left\{ W(t) \exp \left\{ j (\omega_0 t + \phi) \right\} + W^*(t) \exp \left\{ -j (\omega_0 t + \phi) \right\} \right\}^2 \right]$$

$$= \frac{1}{4} \mathbb{E} \left[ W^2(t) \right] \exp \left\{ j (\omega_0 t + \phi) \right\} + 1 \mathbb{E} \left[ W(t) W^*(t) \right]$$

$$\cdot \frac{1}{4} \mathbb{E} \left[ W^2(t) \right] \exp \left\{ -j (\omega_0 t + \phi) \right\}$$

(II-5)

According to the given models of communication system, it can be written as

$$W(t) = \int x(\tau) \cdot S(\tau) \cdot h_r(t-\tau) \, d\tau$$
Hence
\[
\sigma^2(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x(t), x(t')) \rho(x(t'), x(t'')) h_x(t - t') h_x(t'' - t') dt' dt''
\]  
\text{Since } v(t) \text{ is a complex Gaussian process,}
\[
\text{\[\rho(x(t), x(t')) = \mathbb{E}[x(t) \cdot x(t')] = 0 \]}
\]  
\text{is independent Gaussian processes with auto correlation function } \rho(t_1, t_2) = \rho_{xx}(t_1, t_2)
\]
\text{Hence}
\[
\sigma^2(t) = 0
\]
\text{Similarly}
\[
\text{\[\mathbb{E}[\rho^2(t)] = \left\{\mathbb{E}[\rho^4(t)]\right\}^{1/2} = 0 \]}
\]
\text{Now consider}
\[
\rho(v(t_1), v(t_2)) = \mathbb{E}[x_{re}(t_1) x_{re}(t_2)] + \mathbb{E}[x_{im}(t_1) x_{im}(t_2)] + \mathbb{E}[x_{re}(t_1) x_{im}(t_2)] + \mathbb{E}[x_{im}(t_1) x_{re}(t_2)]
\]
That is, \( \psi[x(T_1)x(T_2)] = 2\psi(x(T_0)x(T_0)) \)

\[ = 2 \alpha_0^2 (T_1 - T_2) \quad (II-11) \]

Thus from the Eqn (II-10) it can be written as

\[ \psi[\dot{v}(t), \ddot{v}(t)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(T_1 - T_2) \cdot \delta(T_1) \cdot \dot{h}_2(t - T_1) \cdot \dot{h}_2(t - T_2) \]

\[ = 2 \alpha_0^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(T_1 - T_2) \cdot \dot{v}(T_1) \cdot \dot{v}(T_2) \cdot \dot{v}(t - T_1) \cdot \dot{v}(t - T_2) \]

\[ = 2 \alpha_p^2 u(t) \quad (II-12) \]

where \( u(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(T_1 - T_2) \cdot \dot{v}(T_1) \cdot \dot{v}(T_2) \cdot \dot{v}(t - T_1) \cdot \dot{v}(t - T_2) \)

\[ \alpha_p^2 = \frac{1}{2} \psi[\dot{v}(t), \ddot{v}(t)] = \sigma_n^2 \mu(t) \]

where \( t \) is the sampling time.

Now consider

\[ u(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(T_1 - T_2) \cdot \dot{v}(T_1) \cdot \dot{v}(T_2) \cdot \dot{v}(t - T_1) \cdot \dot{v}(t - T_2) \cdot \dot{v}(t) \cdot \dot{v}(t) dt_1 dt_2 \]

\[ (II-13) \]
Since
\[ \gamma(t) = \exp[j(\omega_0 t + \eta)] \quad \text{and} \quad s^*(t) = \exp[-j(\omega_0 t + \eta)] \]

\[ s(t) s^*(t) = 1 \] (II-14)

Also assuming that the receiver filter is having real impulse response, it can be written as

\[ h_2(t) = h_2^*(t) \] (II-15)

Substituting (II-14) and (II-15) in (11-15)

\[ u(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (r_1 - r_2) h_2(t-r_2) h_2^*(t-r_2) \, dr_1 \, dr_2 \]
\[ = \int_{-\infty}^{\infty} h_2^2(t-r_1) \, dr_1 \]

Thus, the output of the filter corresponding to the fading component of the signal at the input has the variance

\[ \sigma^2 = \sigma_0^2 \mu(t) \]

where \( t_d \) is the sampling time, and

\[ u(t) = \int_{-\infty}^{\infty} h_2^2(t-r) \, dr \]

with the assumption that the receiver filter is having real impulse response \( h_2(t) \).