CHAPTER 6

ITERATIVE DECODING TECHNIQUES

6.1 INTRODUCTION

Major obstacles to realizable digital communications are ISI and AWGN as encountered in transmission over dispersive channels. Thus equalization of ISI at the receiver side is necessary [21, 54, 64]. Linear and Decision-feedback Equalization (DFE) are the two common strategies to cope with the ISI [4, 21, 62, 64, 80]. However, depending on the channel characteristics, their performance may be poor [21, 80]. Optimum performance in terms of error event probability is achieved by Maximum-Likelihood Sequence Estimation which can be implemented using the Viterbi algorithm [31]. However, the complexity of the VA increases exponentially with the channel memory length, which makes the implementation difficult or even impossible for long channel impulse responses (CIRs). For complexity reduction, while retaining high performance, several sub-optimum approaches have been discussed in the literature [27, 28, 29, 30, 37, 76].

A popular method is the direct truncation of the channel response [37]. In this scheme residual ISI terms are not taken into account by the detector and cause severe error propagation. Another approach is to use a Linear Equalizer or Decision feedback equalizer to estimate the input sequence and use these estimates to cancel the tail of the ISI in the received sequence prior to passing it to the Viterbi algorithm. However pre-filtering still results in significant error propagation. Delayed Decision-Feedback Sequence Estimation (DDFSE) is another method to reduce the number of states. As in the Viterbi algorithm at each step, the states describe all possible values taken on by a finite number of previous inputs. In the DDFSE each state provides only partial information on the actual state of the channel. The required residual information is provided by an estimate associated with each state of trellis. In principle, this information can be extracted from the path leading to each state. The channel state estimate and the trellis state are used in computing the branch metric. The complexity of DDFSE is determined by the number of trellis states, which is an exponential variation.
Eyuboglu and Qureshi independently developed a reduced-state algorithm similar to DDFSE. In addition to introduce feedback into the structure of the path metric computations, they proposed to reduce complexity further using ideas of set-partitioning [29,30]. The resulting algorithm, called RSSE, is applicable to finite memory channels with large input alphabets.

In all these reduced state algorithms the decisions which are being used to cancel out the residual ISI are derived from the path history and therefore are not the true estimates of the past symbols. Consequently, this leads to the possibility of error propagation. In general pre-filter is necessary [54,77] for both DFE and RSSE schemes which transforms the CIR into its minimum-phase. In many cases, the real time calculation of this filter poses serious problem e.g., in mobile communications [77]. Even with optimized pre-filter a very large number of states might be necessary for decision feedback sequence estimation in order to obtain high performance.

In contrast to conventional DFE Iterative decision algorithms applying soft decision feedback are shown to perform better [77]. In this approach, the rule for generating soft decisions is adopted continuously to the current state of the algorithm. In most cases standard DFE methods are over performed. In each Iteration of the equalization algorithm soft decision feedback is performed sequentially some definite number of times. After soft cancellation of ISI, a soft decision for symbol is calculated and used for ISI cancellation in the next time step increment of the current iteration.

Generating tentative decisions in an iterative approach is being used in concatenated coding techniques, known as the most powerful channel coding techniques in the recent decade. These are developed based on concatenation of encoders i.e., a combination of smaller encoders at the transmitter [41]. Optimal decoding of concatenated codes on most channels would be in most cases too complex to be practical. So suboptimal decoders have been proposed in the literature [7,8], and applied in practical systems. The basic idea is to utilize decoder of constituent encoders and decode noisy channel data either on one sequence of steps or repetitively [4, 5, 8].

In [2] classes of Block-Iterative nonlinear equalizers are developed for ISI channels. These Equalizers use a multipass algorithm to successively cancel ISI from a block of received data and generates symbol decisions whose reliability increases monotonically with each iteration. The strategy cancels both precursor and postcursor ISI.

Following the approaches given in the literature [2,29,30,37,64,77,80], we propose two new sub-optimum decoding strategies, namely, Iterative Decoding and
Extended-refined Iterative Decoding for M-QAM TCM schemes transmission on bandlimited ISI channels. The strategies we propose comprise an Iterative-algorithm as an integral part of the modified soft output Viterbi decoder, minimize the error propagation that occurs in the conventional reduced state sequence estimation receivers [29, 30].

6.2 ITERATIVE-DECODING

The Fig. 6.1 shows the baseband communication system considered for the study. It comprises the TCM encoder/modulator followed by a Matched Filter in the feed-forward path, cascaded with the Iterative Decoder. The transmission channel is assumed to be linear, causal and time-invariant with a finite duration impulse response \( g(t) \) and ISI length \( L \). The TCM encoder/modulator transforms the input information sequence into a stream of complex data symbol sequence \( a[n] \). The data symbols are assumed to be uncorrelated complex random variables whose mean and variance are given by

\[
E[a[n]] = 0
\]

and

\[
E[a[n] a^*[n]] = \sigma^2 \delta_{nk}
\]  

---(6.1)

The channel output is the convolution of the transmitted symbol sequence \( a[n] \) with the discrete-time impulse response of the channel \( g[n] \).

The received symbols are perturbed by the AWGN and the noisy received symbols are given by

\[
r[n] = g[n] * a[n] + w[n] \]

---(6.2)

or

\[
r[n] = \sum_k g[k] a[n-k] + w[n] \]

---(6.3)

![Fig. 6.1 Discrete-time model of data transmission system with Iterative Decoder](image)
where \( w[n] \) represents AWGN samples characterized as a zero mean, complex valued circularly symmetric, stationary white Gaussian noise sequence with variance \( N_0 \) that is independent of \( a[n] \).

The received noisy data sequence \( r[n] \) is first processed by the linear filter in the feed-forward path \( b[n] \), produces the sequence

\[
\tilde{r}[n] = \sum_k b[k] r[n-k]
\]

or

\[
\tilde{r}[n] = g[n] * b[n] * a[n] + b[n] * w[n]
\]

The filter \( d'[n] \) is an adaptive filter provided in the feedback path of the Iterative decoder. The Iterative-decoder input is given by \( \tilde{a}'[n] \), where

\[
\tilde{a}'[n] = \tilde{r}[n] - \hat{z}'[n]
\]

where, \( \hat{z}'[n] \) is an approximately constructed estimate of ISI during each pass of the Iterative-algorithm, given by

\[
\hat{z}'[n] = d'[n] * \hat{a}^{-1}[n]
\]

the term \( \hat{z}'[n] \) is intended to be some kind of ISI estimate, and it is assumed that\([2,54]\)

\[
\hat{z}'[n] = \sum_k d'[k] \hat{a}^{-1}[n-k]
\]

\[
d'[0] = \frac{1}{2\pi} \int_{-\pi}^\pi D'(\omega) d\omega = 0
\]

as \( \hat{a}^0[n] \) is not required for the first iteration.

The ISI eliminated received symbol \( \tilde{a}'[n] \) reaches the modified Viterbi decoder and the tentative decision generated by the Iterative-decoder is the Likelihood sequence estimation.

During the first iteration of the decoder \( i=1 \), the receiver processes the ISI eliminated input \( \tilde{a}'[n] \) and generates the tentative decision \( \hat{a}'[n] \). The Viterbi algorithm traces the reduced-state combined ISI-code trellis for the Likelihood-Estimation of the symbol sequence. During remaining iterations of the Iterative decoder, the Iterative algorithm error spectrum and the tentative decision generated in order to refine the residual ISI. At the end of \( K^{th} \) iteration more refined received data is generated and the path metrics are optimal, accordingly, more reliable tentative decision is generated. Iterative-refining minimizes precursor and postcursor ISI.
The Iterative-algorithm resets for the next received symbol detection and the total number of iterations the algorithm takes depends on the convergence criteria implemented. In the proposed algorithm the number of iterations is fixed to \( K \), and refining is disabled before the \( i \)-th iteration if the ISI-eliminated received symbol becomes reliable for Likelihood-sequence estimation.

As discussed above, Iterative-decoding strategy generates more reliable tentative decisions and are used in Extended-Refined Decoding of the noisy received symbol.

### 6.3 EXTENDED-REFINED ITERATIVE DECODING

The proposed Extended-Refined Iterative Decoding strategy incorporates Extended-refining of the noisy received symbol sequence at the receiver. It comprises a feed-forward filter which process the noisy received symbol in the first iteration, the soft output VA for likelihood sequence estimation and Extended-Refining algorithm in the feedback path. From the current tentative decision of the VA along with the path history maintained for an Extended interval, ISI cancellation factor is computed.

Extended-refined decoding eliminates the residual ISI due to symbols after the truncated channel memory length \( L \). Extension over just one symbol interval has shown significant performance improvement of Extended-Refining over the conventional PDFD with an additional storage requirement for path history. Extended-refining iterative decoding strategy performs noise cancellation as per the following equation:

\[
\tilde{a}^E[n] = r[n] - \tilde{z}^E[n]
\]

where \( \tilde{a}^E[n] \) is the finely refined symbol input for the VA and \( \tilde{z}^E[n] \) is the ISI estimate details are given in appendix.

### 6.4 RESULTS AND DISCUSSIONS:

In this section we present the error performance evaluation of the two Iterative-decoding strategies, namely, Iterative-Decoding and Extended-Refined Iterative Decoding proposed for sub-optimum decoding of Trellis-coded noisy received symbols transmitted on bandlimited ISI channels in the presence of AWGN. The error performance is evaluated through simulation and is compared with the error performance characteristics of RSSE structures. Various communication channels considered for the data transmission are listed in the Table 6.1.
The baseband communication system is simulated as explained in section 3.4. To simulate the Iterative decoder, RSSE receiver is simulated along with an Iterative-algorithm as an integral part of it. For the implementation of Iterative Decoding algorithm, the parameters discussed in section 6.2 are considered. Iterative-decoder filter coefficients provided in the feedback path of the VA are chosen randomly in the range 0.0001-0.00001 and the number Iterations are limited to 10. In the minimization of the mean square process of the Iterative decoding algorithms the LMS algorithm [54] is considered as the reference.

The Fig. 6.2-6.4 depicts the error performance characteristic of Iterative-decoding strategy designed for 4-state M-QAM TCM schemes. Assumed channel coefficients are as given in the Table 6.1 and the simulation is run for $10^5-10^6$ symbols. In the legend shown in Fig. 6.2-6.4 various labels mean the following: (a) 'Iterative-Decoding' represents error event performance characteristic of the proposed Iterative-Decoding strategy, (b) 'Coded-ISI-free' represents the error event performance characteristics of various TCM schemes transmitted on AWGN channel, (c) 'PDFD' represents the error event performance of the PDFD strategy in the presence of ISI, and is considered as the conventional PDFD (d) 'Extended-Refined' is to represent the error-event performance of Extended-Refined Iterative Decoding strategy, and (e) 'Reference' is to represent the error performance characteristic of the PDFD receiver designed for uncoded QAM scheme. Simulations are run for $10^5-10^6$ symbols transmission. For the convergence of the proposed Iterative algorithms a threshold set over the error spectrum generated in the Maximum Likelihood Sequence Estimation. Necessary weighting factors for the Iterative-algorithms are chosen randomly range 0.00001-0.0001.

The Fig. 6.2(a) shows the error performance of the Iterative-Decoding algorithm designed for 4-state 64-QAM TCM scheme transmission over the communication channel CH12* given in Table 6.1. It is observed that Iterative-Decoding performs better over conventional PDFD with a gain improvement of about 0.4 dB at an error rate of $10^{-4}$. The performance degradation over coded ISI free performance is noted as 1.5 dB . The Fig. 6.2(b) depicts the error performance of Iterative-Decoding strategy designed for 4-state 64-QAM TCM scheme for transmission over the communication channel CH13*. It depicts that the performance improvement of Iterative Decoding strategy is about 0.4 dB over conventional PDFD at an error rate of $10^{-4}$ and the degradation in the performance over coded ISI free performance is 1.0 dB .
The Fig 6.2(c) depicts the error performance of Iterative-Decoding strategy for 4-state 64-QAM TCM scheme for the communicational channel CH12. The impulse response coefficients are fixed complex numbers. It is noted from the characteristics that the Iterative-Decoding strategy performs better over conventional PDFD by 0.8 dB at an error rate of $10^{-5}$ and the performance degradation observed over coded ISI free performance is 1.8 dB. The Fig. 6.2(d) shows the error performance of Iterative-Decoder obtained by transmitting 4-state 64-QAM symbols over bandlimited ISI channel whose impulse response coefficients are the complex numbers generated randomly, which is given as channel CHI2 in the Table 6.1. It is observed that the Iterative-Decoding strategy provided error performance improvement of about 1.0 dB at an error rate of $10^{-5}$ over conventional PDFD.

The Fig. 6.3(a) shows the error performance of 4-state 64-QAM TCM scheme transmission over bandlimited ISI channel CHI12 evaluated for the Extended-Refined Iterative-Decoding strategy. It is noted that Iterative-Decoding provides better performance over conventional PDFD by about 1.4 dB at an error rate of $10^{-5}$. In the Fig. 6.3(b) the error performance of the receiver with the Iterative-Decoding strategy for 4-state 16-QAM TCM scheme for the channel CH12 with real coefficients is given. It is noted from the Fig. 6.3(b) that Iterative-Decoding provides better performance over conventional PDFD by about 0.8 dB at an error rate of $10^{-4}$. From the error performance characteristics obtained for the channel with complex randomly varying channel coefficients, depicted in the Fig. 6.3(c) it is observed that Extended-Refined Iterative Strategy performance is better over conventional PDFD receiver performance by 1.0 dB at an error rate of $10^{-4}$. The degradation in performance is about 1.2 dB over optimum ISI free performance. In the Fig. 6.3(a) Error performance of Iterative-Decoding strategy for 4-state 16-QAM TCM scheme transmission over bandlimited ISI channel CHI2 is shown. From the characteristics it is noted that Iterative-Decoding performs better with improvement in performance of 0.4 dB over conventional PDFD at an error rate of $10^{-4}$. The degradation in error performance noted over optimum coded ISI free performance is 0.6 dB.

The Fig. 6.4 shows the error performance characteristic of Extended-Refined Iterative decoding strategies for 4-state M-QAM schemes for transmission over bandlimited ISI channels. The Fig. 6.3(b) depicts the error performance for 4-state 64-QAM TCM scheme transmission over the communication channel CH12. It is noted from that the proposed Extended-Refined Iterative decoding perform better over
conventional PDFD by about 0.8 dB at an error rate of $10^{-5}$. The degradation in the error performance noted over optimum ISI free performance is 1.4 dB.

The Fig. 6.3(c) shows the error performance characteristics of the proposed Extended-Refined Iterative decoding strategy for 4-state 64-QAM TCM scheme transmission on ISI channel CH13. It is noted that the improvement achieved is about 0.6 dB over the Iterative-decoding and the improvement is about 0.8 dB over conventional PDFD at an error rate $10^{-4}$. The Fig. 6.3(d) depicts the error performance of Extended-Refined Iterative Decoding for 4-state 64-QAM TCM scheme for transmission over the communication channel CH12. The improvement in the error performance is about 0.5 dB over conventional PDFD performance, and the performance degradation over optimum MLSE performance is 1.4 dB.

In the Fig. 6.4(a) we presented the error performance of Extended-Refined Iterative Decoding for 4-state 64-QAM TCM scheme for transmission over the communication channel CH12 with complex coefficient impulse response generated randomly. Improved performance of Extended-Refined Iterative decoding is observed over conventional PDFD by 1.5 dB at an error rate of $10^{-4}$. The degradation in the error performance over optimum MLSE performance obtained for ISI free condition is 0.6 dB. In the Fig. 6.4(b) 4-state 64-QAM TCM scheme error performance evaluation is given.

Performance improvement noted is about 0.6 dB for the Extended-Refined Iterative Decoding strategy over conventional PDFD. And the performance degradation noted over optimum ISI free performance is 0.3 dB.

Comparison of Iterative-Decoding and Extended-Refined Iterative Decoding strategies error performance is given in Fig. 6.4 (c)-(d) and in Fig. 6.5(a)-(c). Comparison it is noted that Extended-Refined Iterative Decoding performs better over Iterative-Decoding strategy with a gain improvement in the range 0.4 dB to 0.8 dB at an error rate of $10^{-4}$.

From the error performance characteristics depicted in Fig. 6.2-6.5 it is noted that the error performance improvement of Iterative-decoding and Extended-refined Iterative Decoding strategies is better for M-QAM TCM schemes with larger signal constellation size M. The error performance improvement is better for communication channels where ISI is severe. It is observed that the proposed Extended-Refined Iterative Decoding strategy error performance improvement is about 0.8-1.2 dB over conventional PDFD.
### Table 6.1

**Equivalent Discrete Time Impulse Response of different ISI Channels**

<table>
<thead>
<tr>
<th>ISI Length</th>
<th>Channel Label</th>
<th>Channel Coefficients</th>
<th>Nature of impulse response</th>
</tr>
</thead>
<tbody>
<tr>
<td>L=1</td>
<td>CH12*</td>
<td>g0: 0.7746, g1: 0.6324</td>
<td>Real coefficients</td>
</tr>
<tr>
<td></td>
<td>CH12**</td>
<td>g0: 0.7746, g1: 0.6324</td>
<td>Complex fixed coefficients</td>
</tr>
<tr>
<td></td>
<td>CH12***</td>
<td>g0: 0.7746, g1: 0.6324</td>
<td>Complex random variables</td>
</tr>
<tr>
<td></td>
<td>CH13*</td>
<td>g0: 0.8367, g1: 0.5477</td>
<td>Real coefficients</td>
</tr>
</tbody>
</table>
Fig. 6.2 Error performance characteristics of Iterative-Decoding receiver for 4-State 64-QAM TCM schemes transmission over various ISI channels

(a) for 4-State 64-QAM TCM, CH12*

(b) for 4-State 64-QAM TCM, CH13*

(c) for 4-State 64-QAM TCM, CH12**

(d) for 4-State 64-QAM scheme, CH12***

channel coefficients:
real: \( g_0 = 0.7746, g_1 = 0.6325 \)

channel coefficients:
real: \( g_0 = 0.8367, g_1 = 0.5477 \)

channel coefficients:
fixed complex numbers
\( g_0 = 0.7745, g_1 = 0.6345 \)

channel coefficients:
complex, random variables
\( g_0 = 0.7746, g_1 = 0.6325 \)
Fig. 6.3 Error performance comparison of Iterative-Decoding and Extended-Refined Iterative Decoding strategies for 4-state 16-QAM and 64-QAM TCM schemes
Fig. 6.4 Error performance comparison of Iterative-Decoding and Extended-Refined Iterative Decoding strategies for 4-state 16-QAM and 64-QAM TCM schemes
Fig. 6.5 Error performance comparison of Iterative-Decoding and Extended-Refined Iterative Decoding strategies for 4-state 16-QAM and 64-QAM TCM schemes
The proposed Iterative-Decoding strategy performs better over conventional PDFD by 0.5-0.8 dB at an error rate of $10^{-4}$. For 4-state 64-QAM TCM schemes the extended refined Iterative Decoding scheme performs better by 0.8-1.2 dB and Iterative Decoding strategy performance improvement is noted as about 0.5-1 dB at an error rate of $10^{-4}$.

Though Iterative-algorithm requires additional computations with lesser complexity increment significant error performance improvement about 1.5 dB is observed. The study shows that the proposed techniques can be extended to severe ISI environments. Further research is required in this field.