CHAPTER I
1.1 STATISTICAL QUALITY CONTROL

Quality control is the engineering and management activity to measure the quality characteristics of the products, to compare it with specification or requirements, and to take appropriate remedial action whenever there is a difference between actual performance and the standard. More specifically, quality control is the regulatory process to measure actual quality performance, to compare it with standards, and to act on the difference. In other word, quality control is the operational techniques and activities which sustain a quality of product or service that will satisfy given needs. The statistical control of quality is the application of principles and techniques in all stages of design, production, maintenance and service, directed towards the economic satisfaction of demand. The Statistical Quality Control techniques are successfully applied to maintain substantial quality, cost and productivity improvements in large industrial concerns for the last seventy years.

In any industrial process, regardless of how well designed or carefully maintained, the quality of manufactured products is always subject to a certain amount of variation. Some stable "system of chance causes" is inherent in a particular scheme of production and inspection. The variation within this stable pattern is inevitable. The
reasons for variation outside this stable pattern may be discovered and corrected. The primary objective of Statistical Quality Control is, thus, the systematic reduction of variability in quality characteristics of the products. To keep the variation under allowable limits, checks are made at two stages. The first stage of check is offered through control chart during production and keeps the manufacturing process stable and capable of operation such that virtually all of the items produced conform to the specification. However, control chart does not resort to screening and elimination of defective items. The second stage of check is done to ensure that lots of manufactured items ready for shipment do not contain excessively large proportion of defectives. This check is done through sampling inspection. Acceptance sampling prescribes a procedure that, if applied to a series of lots, will give a specified risk of accepting lots of given quality. Acceptance sampling, in fact, yield quality assurance. The classical work of Shewhart (1926, 31) on control chart and that of Dodge and Romig (1929, 37) on acceptance sampling plans may be considered as the beginning of Statistical Quality Control methods.

The research development in the field of Statistical Quality Control has mainly confined to the above two directions, namely, development of control charts (as a measure of process control) and that of acceptance sampling plans (as a measure of product control). A good deal of literature developed on classical line of Shewhart and Dodge and Romig has already appeared in the text form in the series

Bissell (1984), Pignatiello (1988), Banerjee and Raim (1988), Tagaros and Lee (1989) on economic control charts; Chiu (1975) and Duncan (1978) on np-chart; and Saniga (1977, 79) on joint \( \bar{x} \) and R-chart are a few of quote. A brief account of researches on control chart techniques have been summarised by Gibra (1975), Montgomery (1980) and Vance and McDonald (1983).

Likewise, on the sampling plan side, attention may be drawn to the contributions of Dodge (1943, 47), Wald and Wolfowitz (1945), Dodge and Torrey (1951), Lieberman (1953), Page (1954), Lieberman and Soloman (1955), Hillier (1964, a, b), Phillips (1969), MIL-STD-1235 A (1974), Shahani (1979), Stephens (1979, 81) on continuous sampling plans; Dodge (1955), Clark (1960), Fred Frishman (1960), Dodge and Stephens (1966), Soundararajan (1978), Soundararajan and Govindaraju (1982, 83), Soundararajan and Raju (1982), Shankar (1988) on chain sampling plans; Gregory and Resnikoff (1955), Savage (1955), Schilling and Dodge (1967, a, b, c; 1969) and Schilling (1967) on mixed variables and attributes sampling plans. Owen (1969) and Wetherill and Chiu (1975) have reviewed the research work done on the sampling plans developed in different periods.

The development of researches in the field of Statistical Quality Control has mainly been made through two approaches: (1) the classical approach based on the pioneering work of Shewhart (1926) and Dodge and Romig (1929, 37), and (2) the economic approach introduced among others by
Since then various modifications and improvements have been made by several workers in the above directions.

Graphical Evaluation and Review Technique (GERT) originally introduced for systems design and analysis has been used successfully for studying a few types of quality control problems. The study of quality control system through GERT methods has been the subject of considerable attention during the last three decades. A brief account of researches in quality control through GERT methods has been summarised by Shankar (1993). An introduction to the GERT methodology has been given in the next section.

1.2 BRIEF REVIEW OF GERT

Graphical Evaluation and Review Technique (GERT) initiated by Pritsker and Happ (1966), Pritsker and Whitehouse (1966) and Whitehouse and Pritsker (1969) is a new graphical procedure which combines the discipline of flowgraph theory, moment generating function and PERT (Program Evaluation and Review Technique) to obtain the solution of stochastic problems. It represents the most general network where both the graphs and functions are probabilistic. A node in a stochastic network consists of an input (receiving, contributive) function and an output (omitting, distributive) function. Three logical relations Exclusive-OR, Inclusive-OR and AND exist for the input side. Two relations deterministic and probabilistic, exist for the
output side depending on whether the activity omitted from the node has probability of realization equal to or less than one. The input and output symbols are combined to give six possible types of nodes as described in Fig. (1.1).

An Exclusive-OR node is one which is realized if one and only activity incident on it is realized. However, such a node is also realized if a feedback activity is realized. An Inclusive-OR node is realized when any activity leading into the node causes the node to be realized. An AND-node is realized if all the activity incident on it are realized. A Deterministic node is one which is realized if all the branches emanating from it are taken i.e. all the branches emanating from this node have a probability parameter equal to one. In the case of probability node, at most one branch emanating from the node is taken if the node is realized. The detailed discussion on the node structure of the GERT has been presented by Whitehouse (1973).

GERT is developed as a technique for the analysis of stochastic networks having the following characteristics:

1. Each network consists of nodes denoting logical operation and transmittances (or directed branches).
2. A branch associated with it has a probability that the activity represented by the network will be performed.
3. Others parameters describe the activities which the branches represent. These parameters may be additive such as time or cost or multiplicative like probability.
or reliability. However, a moment generating function of such additive parameters convert them into multiplicative ones.

(4) A realization of a network is a particular set of transmittances and nodes which describe the network for one experiment.

(5) If parameter associated with a branch is a random variable, then a realization also implies that a fixed parameter has been selected for each branch.

In order to include the probability and the parameter (say, time parameter) which could be treated uniformly throughout the network, the creation of a new function is required. The duration $t$ associated with a branch is characterized by moment generating function (m.g.f.) of the form:

$$
M(\theta) = \begin{cases} 
\int e^{\theta t} f(t) \, dt, & \text{(if } t \text{ is continuous variable)} \\
\sum e^{\theta t} f(t), & \text{(if } t \text{ is discrete variable)} 
\end{cases}
$$

where $f(t)$ denoted the density function associated with $t$ and $\theta$ is any real variable. The probability $\phi$ that the branch is realized is multiplied by the moment generating function to yield a $W$-function such that

$$
W(\theta) = \phi M_t(\theta) \quad (1.2.1)
$$
The W-function is used to obtain information on a relationship which exist between the nodes.

Pritsker and Happ (1966) and Whitehouse (1973) have shown that a GERT network with Exclusive-OR node is equivalent to a flowgraph and W-function of GERT network is equivalent to the transmittance in the flowgraph. Therefore, Mason's (1953) rule used to evaluate transmittances in a flowgraph is also applicable to GERT network. Once, the equivalent W-function is known, the equivalent probability of the network is given by

\[ p = [ W(\theta) ]_{\theta=0} = W(0) \]  \hspace{1cm} (1.2.2)

Furthermore, the equivalent m.g.f. of the network is obtained by

\[ M_t(\theta) = W(\theta) / p \]  \hspace{1cm} (1.2.3)

and finally, mean (say, time) of the realization of the network can be found as

\[ E(t) = [ \frac{d}{d\theta} M_t(\theta) ]_{\theta=0} \]  \hspace{1cm} (1.2.4)

The steps employed in applying GERT to the analysis of a stochastic network may be summarized as follows:

1. Convert a qualitative description of a system or problem
to a model in the network form.

(2) Collect necessary data to describe the branches of the network.

(3) Determine the equivalent function or functions of the network.

(4) Convert the equivalent function into the following two performance measures of the network:
   (a) The probability that a specific node is realized.
   (b) The moment generating function of the parameters associated with an equivalent network.

(5) Make inferences concerning the system under study from the information obtained in step (4).

1.3 REVIEW OF PREVIOUS WORK

Graphical Evaluation and Review Technique originally introduced for systems design and analysis has been found useful in solving quality control problems for several years. Fry (1966), Powell (1967) and Mullen (1968) studied Dodge's continuous sampling plans through GERT approach and proposed some modifications to the plan. Later on, Dodge-Romig plan and Chain sampling inspection plans have been investigated respectively by Kase and Ohta (1977) and Ohta (1978) by means of GERT. Kase and Ohta (1974, 76) studied the dynamics of corrective action schemes for semi-Markovian production processes by GERT approach. A GERT network analysis of a manufacturing process to increase the acceptability of the product without affecting operating sequence has been presented by Mohapatra and Banerjee (1977).
An economic design of control chart has been studied by Ohta (1980). An investigation into the effect of inspection error on the performance characteristics of continuous and single and double sampling plans by means of GERT has been done by Ohta and Kase (1984), Chakraborty and Rathie (1989) and Chakraborty (1990). Shankar (1989) modelled and analyzed the Multiple deferred sampling plans through GERT. Subsequently, Shankar and others in a series of papers investigated various quality control models through GERT approach. To be more specific, the contributions due to Shankar (1988, 89) on Deferred Sampling plan; Singh et al. (1989), Shankar and Mohapatra (1991, 93), Shankar and Joseph (1993) and Shankar (1996) on Repetitive Group Sampling plan; Shankar and Mohapatra (1990) and Shankar and Joseph (1996) on Chain Sampling plan and Shankar (1998) on Corrective Action plan are a few to quote. A brief account of such studies has been presented by Shankar (1991, 93, 97).

1.4 PROBLEMS UNDER INVESTIGATION

From the references quoted in previous sections, it may be observed that the statistical quality control problems have very little been studied so far by means of GERT. This growing field of knowledge needs systematic investigation and development which is the main objective of the present study. The purpose of present investigation is to model and analyze through classical as well as GERT methodology the properties of statistical decision procedures for product and process control. This thesis is divided into
nine chapters. Chapter-I is introductory with brief review of GERT methodology and an outline of literature survey on statistical quality control.

In Chapter II, a new deferred sampling plan designated as Two-Stage Deferred Sampling plan have been proposed using different sample sizes in two deferred stages of the inspection. This plan contains a particular attention to the specification of sample sizes used in different stages of the criterion. The formulae for Operating Characteristic (OC), Average Sample Number (ASN) and Average Cost Functions of the plan have been derived by GERT. Numerical examples have been included to illustrated the mathematical findings. In order to facilitate the operation and construction of the plan, Poisson unity values have also been tabulated.

In Chapter III, a Two-Stage Conditional Repetitive Group Sampling plan have extensively been studied using different sample sizes in the normal and tightened phases of inspection. The OC and ASN functions of the plan have been derived by applying Mason’s rule on the GERT network representation of the inspection system. Poisson unity values have been tabulated to facilitate the construction and selection of the plan. Finally, a comparison of the proposed plan with Two-phase inspection Repetitive Group Sampling plan [ Shankar (1996) ] has been made in terms of ASN and it is found that the proposed plan is better than Two phase-inspection RGS plan in sample size efficiency.
The Chapter IV provides detailed procedures and tables for the construction and selection of Chain-Deferred (ChDF-2) sampling plans due to Shankar and Joseph (1994) using various combinations of entry parameters, like (a) Two points on the OC curve, (b) The indifference quality level $p_0$ and relative slope of the OC curve at $p_0$, (c) Quality level corresponding to inflection point $p^*$ and relative slope of the OC curve at $p^*$ and (d) Average outgoing quality limit and one point on the OC curve. A procedure for conversion from one set of parameters to the other equivalent set has also been illustrated.

In Chapter V, a new continuous sampling plan has been proposed as an extension of classical continuous sampling plan CSP-1 in which one defective is also allowed during detailing (100% inspection) state of the CSP-1 plan. For convenience, the proposed plan is designated as continuous sampling plan CSP-(1,1). An attempt has also been made to model and analyze the dynamic characteristics of CSP-(1,1) plan through GERT approach. Procedures and tables have been provided to find an unique combination of plan parameters $i$ and $f$ to ensure the AOQL requirements, and also to optimize the average amount of inspection, when the process level $p = p_w$ is known, on the line of Shankar and Mohapatra (1994).

Chapter VI, is devoted to the study of Chain-sampling plans. This chapter is divided into two parts. The first part deals with the investigation of Bagchi's (1976)
extented Two-Stage chain sampling plan and the second part with that of Tightened-Normal-Tightened (TNT) plan given by Calvin (1977). Various performance characteristics of the plan have been derived by GERT network representation of the plan. Lastly, tables of Poisson unity values have been developed for the construction and selection of respective plans.

Chapter VII, develops the process control plans for the production processes in lieu of classical control charts. The problem under consideration may be stated as follows: "Given a production process being subject to some deterioration, then the questions arise when to perform which corrective action in order to maintain high quality level of the process. Not having full information of actual state of the process, the decisions have to be based on some partial information obtained for instance by sampling. Possible corrective actions are (minimal) repairs, adjustments, renewals which all can be performed after detection of the state of deterioration or as preventive actions. A policy includes the rules for getting the information, and deciding what to do." Burr (1976; pp. 26) remarked that "Common sense is simply not a reliable guide for avoiding two types of errors: (a) taking action when none is desirable, and (b) missing opportunities for taking appropriate action or learning what makes the process behaves as it does. In order to minimize the chances of making these two types of errors, we need to use the control chart and other statistical tools."
The present chapter is an outcome of the follow-up study based on the above remarks. This study has a specific view point. It claims that the function of sampling inspection can be translated into the measure of process control beyond its original purpose. Furthermore, the suggested plan requires no direct inspection of machines and equipment of the process. The dynamics of the corrective action plan have been modelled and analyzed through GERT approach. The formulae for OC and ASN functions of the plan have been derived and illustrated numerically. Lastly, Poisson unity values have been tabulated to facilitate the operation and construction of the plan.

Ohta (1980) investigated corrective action schemes for semi-Markovian production processes using fraction defective control chart through GERT. They assumed that the process is characterized by two states namely the in-control state and the out-of-control state. In Chapter VIII, it is assumed that process is characterized by three states: the in-control state, marginal-control state and out-of-control state. The GERT analysis of the economic design of fraction defective control chart for three state of the semi-Markovian production processes has been investigated analytically. The major contribution of the analytical approach in this chapter is due to Ohta (1980).

The last Chapter, i.e., Chapter IX is devoted to the study of life testing plans where quality characteristic is given by product's life time. In many life testing
problems, it is customary to truncate the experiment at a pre-assigned time and to note the number of failures. The limitations in most of the life testing plans are two-fold. First, a lot is accepted at time $t_0$ when the number of failures is less than or equal to a given number $C$ even if all the units fail at the early stage of the experiment. Hence, it fails to maintain the minimum life time (quality) of an unit. Second, no attention has been paid to the lots of marginal quality. This chapter is divided into two parts. The first part of this chapter develops life testing plans in the case of three quality classes say good, marginal and bad. The second part deals with the development of Repetitive Group Sampling plan on the line of Sherman (1965). In both the cases life time quality characteristics has been assumed to follows a Gamma failure distribution. Lastly, tables have been provided to determine the smallest sample size $n$ necessary to ensure a certain mean life or the quality of the product.
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Fig. (1.1): NODE STRUCTURES OF GERT NETWORKS