CHAPTER 4

NETWORK THEORY OF VOCAL TRANSMISSION AND INTER-RELATIONSHIP WITH WOMB DEVELOPMENT

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CHAPTER 4

NETWORK THEORY OF VOCAL TRANSMISSION AND
INTER-RELATIONSHIP WITH WOMB DEVELOPMENT

Speech production system can be represented as in Fig. 4.1.

Fig. 4.1 Speech production system

The mathematical treatment of the speech production process involves the following successive operations [1,2]. The first one is the mapping of the vocal cavities in terms of an area function describing the cross-sectional area perpendicular to the air stream from the glottis to the radiating surface of the lips. Secondly, this area function has to be approximated by a sufficiently small number of successive parts, each of a constant cross-sectional area. The transmission property of this system are next calculated and added to the assumed frequency characteristics of the source [3,4].

4.1 ACOUSTICAL PARAMETERS OF A SPEECH SYSTEM:

The network representation of the vocal tract is based on the
analogous acoustic impedance. The advantage of adopting the analogous impedance lies in the continuity of both volume velocity and pressure along the resonator network [5,6]. Acoustical impedance ($Z_0$) is the complex quotient of the alternating pressure applied to the system by the resulting volume current. The unit is the acoustical ohm. Acoustical resistance is the real part of the acoustical impedance. This is the part responsible for the dissipation of energy. The unit is the acoustical ohm. Interference in an acoustical system is that coefficient which when multiplied by $2\pi$ times the frequency, gives the positive imaginary part of the acoustical impedance. The unit is the gram/centimeter. Acoustical capacitance in an acoustical system is that coefficient which, when multiplied by $2\pi$ times the frequency, gives the reciprocal negative imaginary part of the acoustical impedance.

An acoustical system is a system adopted for the transmission of sound consisting of one or all of the following acoustical element: acoustical resistance, inertance and acoustical capacitance. An acoustical resistance, reactance or impedance is said to have a magnitude of one acoustical ohm when a pressure of one dyne per square centimeter produces a volume current of one cubic centimeter per second [7].

4.1.1 INERTANCE:

Acoustical inertial energy is associated with inertance in the acoustical system. Acoustical energy increases as the volume current decreases. It remains constant when the volume current of the inertance is constant. Inertance is the acoustical element that opposes a change in volume current. Inertances in gram/centimeter is defined as

$$p = M \frac{dv}{dt} \quad ..(4.1)$$

Where

$M$ — Inertance in grams per centimeter

(64)
\[ \frac{dv}{dt} \] — Rate of change of volume current in cubic centimeter per second per second, and

\( p \) — Driving pressure, in dynes per square centimeter

Equation 4.1 states that the driving pressure applied to an inertance is proportional to the inertance and the rate of change of volume current.

Inertance may be expressed as

\[ M = \frac{m}{s^2} \] ..(4.2)

Where \( m \) — Mass, in grams

\( s \) — Cross-sectional area in square centimeter, over which the driving pressure acts to drive the mass.

The inertance of circular tube is

\[ M = \frac{\rho l}{\pi R^2} = \frac{\rho l}{s} \] ..(4.3)

Where \( R \) — Radius of the tube, in centimeter

\( l \) — Effective length of the tube

\( \rho \) — Density of the medium in the tube in gram per cubic centimeter.

4.1.2 ACOUSTICAL CAPACITANCE:

Acoustical potential energy is associated with the compression of a fluid or gas. Acoustical energy increases as the gas is compressed. It decreases as the gas is allowed to expand. It is constant, and is stored when the gas remains immovably compressed [8].

Acoustical capacitance is an acoustic element which opposes a change in the applied pressure. The pressure, in dynes per square centimeter, in terms of the condensation is

\[ (65) \]
\[ p = c^2 \rho S \]  
\[ \text{Where } c \text{ — Velocity, in centimeter per second} \]
\[ \rho \text{ — Density in gram per cubic centimeter} \]
\[ S \text{ — Condensation} \]

The condensation in a volume \( V \) due to a change in volume from \( V \) to \( V' \) is
\[ S = \frac{(V - V')}{V} \]  
\[ \text{..(4.5)} \]

The change in volume \((V - V')\), in cubic centimeter is equal to the volume displacement, in cubic centimeter
\[ V - V' = X \]  
\[ \text{..(4.6)} \]

Where \( X \) = volume displacement, in cubic centimeter

From equations 4.4, 4.5, and 4.6, the pressure is
\[ p = \frac{\rho c^2}{V} X \]  
\[ \text{..(4.7)} \]

Acoustical capacitance \( C_A \) is defined as
\[ p = \frac{X}{C_A} \]  
\[ \text{..(4.8)} \]

Where \( p \) — Sound pressure, in dynes/square centimeter
\( X \) — Volume displacement, in cubic centimeter

Equation 4.8 states volume displacement in an acoustical capacitance is proportional to the pressure and the acoustical capacitance.

\[ C_A = \frac{V}{\rho c^2} \]  
\[ \text{..(4.9)} \]

4.1.3 REPRESENTATION OF ELECTRICAL, MECHANICAL AND ACOUSTICAL ELEMENTS:

The electrical elements: electrical resistance, inductance and electrical capacitance are represented by the conventional symbols.
Mechanical rectilinear resistance is represented by sliding friction which causes dissipation. Acoustical resistance is represented by narrow slits which cause dissipation due to viscosity when fluid is forced through the slits. These elements are analogous to electrical resistance in the electrical system.

Inertia in the mechanical rectilinear system is represented by a mass. Inertance in the acoustical system is represented as the fluid contained in a tube in which all the particles move with the same phase when actuated by a force, due to pressure. These elements are analogous to inductance in the electrical system [8].

Compliance in the mechanical rectilinear system is represented as a spring. Acoustical capacitance in the acoustical system is represented as a volume which acts as a stiffness or spring element. These elements are analogous to electrical capacitance in the electrical system.

4.2 ACOUSTICAL MODEL OF VOCAL TRACT:

The voice mechanism (Fig. 2.6 and Fig. 2.8) consists of three
parts: the lungs and associated muscles for maintaining a flow of air, the larynx for converting the steady air flow into a periodic modulation, and the vocal cavities of the pharynx, mouth and nose which vary the harmonic content of the output of the larynx. The vocal cord does not receive excitation at the frequency of vibration. The source of power is the steady air stream. The elements of a simplified larynx are shown in Fig. 4.3.

Fig. 4.3: Sectional view of a vibrating system approximating the larynx.

In the electrical analogy and acoustical network: $p$ and $X =$ the d-c air pressure and d-c volume current supplied by the lungs. $M_2$ and $r_{A2} =$ the inertance and acoustical resistance of the aperture formed by vocal cords. $M_1$, $C_{A1}$ and $r_{A1} =$ the inertance, acoustical capacitance and acoustical resistance of the vocal cords. $Z_{AV} =$ the input acoustical impedance to the vocal cords. $r_{AG} =$ the acoustical resistance of the generator. $\mu p_g =$ the pressure of the equivalent a-c generator. $Z_{Al} =$ the acoustical impedance of the trachea and lungs. $M =$ the mutual coupling between branch 1 and branch 2.
The frequency of the vibration is governed by all the elements of the vibrating system, that is, the acoustical capacitance \( C_{A1} \), acoustical resistance \( r_{A1} \) of the vocal cord, inertance \( M_2 \) and acoustical resistance \( r_{A2} \) of the aperture, and the load acoustical impedance \( Z_{AV} \) due to vocal cavities [8].

A schematic view and the acoustical impedance \( Z_{AV} \) due to the vocal cavities can be seen in Fig. 4.4. This shows that the nature of the input acoustical impedance \( Z_{AV} \) to the acoustical cavities is extremely complex. The inertance \( M_1, M_2, M_3 \) and the acoustical capacitance \( C_{A1}, C_{A2} \) can be varied by changing the size of the apertures and the volume of cavities.

![Schematic sectional view and the acoustical network of the vocal cavities.](image)

**Fig. 4.4**: Schematic sectional view and the acoustical network of the vocal cavities.

The acoustical circuit: \( Z_{AV} = \) the input acoustical impedance to vocal cavities. \( C_{A1} = \) the acoustical capacitance of the laryngeal pharynx. \( M_1 = \) the inertance of the narrow passage determined by the epiglottis. \( C_{M2} = \) the acoustical capacitance of the mouth cavity. \( M_2 = \) the inertance of the passage connecting the mouth and the nasal cavities. \( M_3, M_4, r_{A1}, r_{A2} = \) the inertances and acoustical resistances of the mouth and nose openings and the air load upon these openings.

A saw-tooth wave contains the fundamental and all the harmonics. Therefore, the generator, \( \mu P \), should produce the fundamental frequency and all the harmonics of the fundamental frequency. When the
vocal cords are set into vibration as out-lined above, the output of the larynx consists of a steady stream with superimposed impulses (Fig. 4.5).

![Wave shapes of the output of the vocal cords and the mouth and nose for the vowel sound.](image)

**Fig. 4.5 :** Wave shapes of the output of the vocal cords and the mouth and nose for the vowel sound.

The pulsating air stream passes through the air cavities of the head. The harmonic content of the output is modified due to discrimination introduced by the acoustical network (Fig. 4.4) which shows the wave shape of the sound output of the mouth and nose corresponding to the wave shape of the output of the vocal cords. When the shape of vocal cavities is altered the acoustical elements of the acoustical network (Fig. 4.4) are altered which in turn alters the output harmonic content. These changes together with a change in the fundamental frequency of the vocal cord make it possible to produce an infinite number of different sounds. The tongue plays the major role in altering the shape of the vocal cavities. The mouth opening, tongue and epiglottis are the principle elements which are altered [8].

### 4.3 DEVELOPMENT OF TWO PORT ELECTRICAL NETWORK OF VOCAL TRACT:

It allows us to define mathematical quantities (with the dimension of a frequency or a time) that are characteristic of a certain configuration or geometric changes of the vocal tract [9].

When there is sound in the air, the air pressure $P$ at a certain location in space shows small variations denoted by $p$ around its aver-
The variation $p$ is called the sound pressure. This sound pressure $p$ is mathematically indicated as follows:

$$p = p(x,y,z,t)$$

Where $x$, $y$ and $z$ are three dimensions in space and $t$ represents time. Also the density $\rho$ defined as the mass of the air per unit of volume, displays a variation $S$ around its average value so:

$$\rho = \rho_0 + S$$

The variation $S$ is called condensation. For small variations met in practice, $p$ is proportional to $S$ in the following way:

$$p = c^2 S$$

Where $c$ is the well-known velocity with which sound travels in free space.

The vibrating air particles move to and fro around their positions of equilibrium. In doing so, they cover very small distances and develop very low velocities. The so-called particle velocity '$v$' is defined as the distance an air particle travels during a very short time interval, divided by that interval. Particle velocity pertains to the transport of mass whereas the velocity of propagation describes the transport of energy which can take place at much higher velocity.

In order to facilitate the calculation the vocal tract which is virtually boomerang-shaped, is bent in such a way that its axis becomes a straight line which at the same time serves as $x$-axis. Moreover, the streamlines are supposed to be essentially parallel to the $x$-axis so that only the velocity-component is taken into account. The next simplification is to suppose that the particle velocity '$u$' is the same throughout a cross-section area perpendicular to the axis. The same holds good for the sound pressure '$p$'. This means that $u$ and $p$ are functions of $x$ and $t$:

Sound pressure:

$$p = p(x,t)$$
Particle velocity
\[ u = u(x,t) \] ..(4.15)

By straightening the vocal tract and idealizing its streamlines we have straight jacketed our problem into a one-dimensional problem.

An advantage of the one-dimensional approach is that the so called volume velocity \( U \) can be defined in the following way:
\[ U = su \] ..(4.16)

Where \( s \) is the area of the cross-section of the tube over which the particle velocity \( u \) is supposed to be constant with respect to \( x \) [3,9].

The vocal tract as an acoustical device can be represented as in Fig. 4.6.

![Diagram of vocal tract as an acoustic device]

**Fig. 4.6 : The vocal tract as an acoustic device**

Where \( P_0 \) — Sound pressure at throat end  
\( U_0 \) — Volume velocity at throat end  
\( s_0 \) — Area of cross-section of vocal tract at throat end  
\( P_1 \) — Sound pressure at mouth end  
\( U_1 \) — Volume velocity at mouth end  
\( s_1 \) — Area of cross-section of vocal tract at mouth end  
\( X \) — Distance from throat end  
\( l \) — Distance between throat end and mouth end  
\( P_x \) — Sound pressure at a distance \( x \) from throat end
$U_x$ — Volume velocity at a distance $x$ from throat end

$s_x$ — Area of cross-section of vocal tract at a distance $x$ from throat end

From now onwards our calculations will aim at the determination of sound pressure $P$ and volume velocity $U$ in the vocal tract especially at the beginning and the end.

The vocal tract can be seen in the same way with the electric four terminal networks as shown in Fig. 4.7.

![Equivalent Electric Four Terminal Network of the Vocal Tract](image)

Fig. 4.7: Equivalent electric four terminal network of the vocal tract

For sinusoidal oscillations at the input we have

$$P_0 = AP_1 + BU_1 \quad \text{(4.17)}$$
$$U_0 = CP_1 + DU_1 \quad \text{(4.18)}$$

Where $A, B, C, D$ are generalized circuit parameters of the network.

For better analysis throat can be further represented as having twin tube resonator. One resonator having equivalent circuit two port parameters on $A, B, C,$ and $D,$ which in turn depends on $p, X, M_1, M_2, r_{A1}, r_{A2}, r_{A0}, C_{A1}, Z_{AV}, Z_{AL}$ and $\mu p_0$ (Fig. 4.3). The output of this resonator is fed to another resonator, having equivalent circuit two port parameters as $A_2, B_2, C_2,$ and $D_2,$ which in turn depends on $Z_{AV}, M_1, M_2, M_3, M_4, C_{A1}, C_{A2}, C_{A3}, r_{A1},$ and $r_{A2},$ (Fig. 4.4) [8,10].
Thus the throat as a twin tube resonator can be represented by electrical network as in (Fig. 4.8).

\[
P_0 = A_1 P_1 + B_1 U_1
\]

\[
U_0 = C_1 P_1 + D_1 U_1
\]

Where

\[
A_3 = A_1 A_2 + B_1 C_2
\]

\[
B_3 = A_1 B_2 + B_1 D_2
\]

\[
C_3 = A_2 C_1 + C_2 D_1
\]

\[
D_3 = D_1 D_2 + C_1 B_2
\]

4.4 ELECTRICAL AND MECHANICAL MODEL OF WOMB:

Womb, say at 5th month of pregnancy (Fig. 3.13), can be represented analogously as the spring-mass dash-pot (damper) system (Fig. 4.9).
Where $F(t)$ — Diaphragmatic muscular force (muscular force of abdomen, muscular force of uterus) under phonation

$\eta$ — Viscous friction coefficient of amniotic fluid

$K$ — Spring restoring force of umbilical cord

$y(t)$ — Displacement of mass of foetus body in the amniotic cavity.

$m$ — mass of foetus body.

Thus the dynamic mechanical circuit of womb under phonation can be represented as in Fig. 4.10.

![Mechanical circuit diagram of womb](image)

**Fig. 4.10 : Mechanical circuit diagram of womb**

**Table 4.1**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Mechanical translational systems</th>
<th>Electrical system</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Force $F$</td>
<td>Current $i$</td>
</tr>
<tr>
<td>2.</td>
<td>Mass $m$</td>
<td>Capacitance $C$</td>
</tr>
<tr>
<td>3.</td>
<td>Viscous friction coefficient $\eta$</td>
<td>Reciprocal of Resistance $(1/R)$</td>
</tr>
<tr>
<td>4.</td>
<td>Spring constant $K$</td>
<td>Reciprocal of inductance $(1/L)$</td>
</tr>
<tr>
<td>5.</td>
<td>Displacement $y$</td>
<td>Magnetic flux linkage $\phi$</td>
</tr>
</tbody>
</table>

Following the conventional force (torque)—current analogy [11,12].

We have electrical analogy of mechanical system represented as in Fig. 4.11.
Fig. 4.11: Electrical circuit diagram of womb

Where
L — Inductance of womb cavity analogous to 1/K
R — Resistance of womb cavity analogous to 1/\eta
C — Capacitance of womb cavity analogous to \( m \)
i — Current analogous to force \( F \)

4.4.1 MATHEMATICAL FORMULATION OF WOMB:

Now we aim at setting up of equations for the mechanical system (Fig. 4.10), on application of force \( F(t) \) to the mass \( m \). The resulting displacement of the mass being \( y(x,y,z,t) \). From the mechanical circuit diagram (Fig. 4.10), it is noted that the forces developed in the system are as follows:

- Inertial force — \( m \frac{d^2y}{dt^2} \)
- Viscous friction force — \( \eta \frac{dy}{dt} \)
- Spring restoring force — \( Ky \)

According to the nodal analysis, the sum of the forces mentioned above is equal to the applied force \( F(t) \). Thus the equation for the system relating displacement \( y \) and the applied force \( F(t) \) is given below:

\[
m \frac{d^2y}{dt^2} + \eta \frac{dy}{dt} + Ky = F(t) \quad \ldots (4.22)
\]

Assume all initial conditions to be zero before phonation. Taking Laplace transform on both sides of equation 4.22
We have
\[ ms^2Y(s) + \eta s Y(s) + KY(s) = F(s) \] ..(4.23)

If \( Y(s) \) is specified as output and \( F(s) \) as input then the closed loop transfer function of womb is given by the relation
\[
\frac{Y(s)}{F(s)} = \frac{1}{ms^2 + \eta s + K} \] ..(4.24)

Further the characteristic equation of above second order system can be obtained by equating denominator of equation 4.24 to zero, we have
\[ ms^2 + \eta s + K = 0 \] ..(4.25)

or
\[ s^2 + \frac{\eta}{m} s + \frac{K}{m} = 0 \] ..(4.26)

with the conventional way of solution, we have
\[ \omega_n \text{ (natural frequency)} = \sqrt{\frac{K}{m}} \] ..(4.27)
\[ \zeta \text{ (damping factor)} = \frac{\eta}{2\sqrt{Km}} \] ..(4.28)

and \( \omega_d \text{ (damped frequency)} = \omega_n \sqrt{1 - \zeta^2} \) ..(4.29)
\[ = \frac{1}{2m} \sqrt{4Km - \eta^2} \] ..(4.30)

Neglecting \( \eta^2 \) in comparison to \( 4Km \) the equation 4.30 can be approximated as
\[ \omega_d = \frac{\sqrt{K}}{\sqrt{m}} \] ..(4.31)

4.5 PREGNANT WOMAN VOCAL TRACT ELECTRICAL NETWORK:

The throat has been treated as a sinusoidal electromotric force \( e \) in series with an internal impedance \( Z_0 \) and the mouth opening is considered as a load-impedance \( Z_1 \), as visualized in Fig. 4.12.
Fig. 4.12: The throat considered as a source of sinusoidal vibrations

Evidently

\[ P_0 = e - U_0 Z_0 \]  \hspace{1cm} (4.32)

and

\[ P_1 = U_1 Z_1 \]  \hspace{1cm} (4.33)

Where

- \( P_0 \) and \( P_1 \) have usual meanings.
- \( e \) — Source with internal impedance
- \( Z_0 \) — Internal impedance of the throat and thoracic cavity
  (which includes lungs, diaphragm and abdominal cavity)
- \( Z_1 \) — Load impedance at the mouth opening

Combining equations 4.19, 4.20, 4.32, and 4.33 and solving for \( U_1 \) and \( U_0 \), we have

\[ U_1 = \frac{e}{B_1 + D_1Z_0 + A_1Z_1 + C_1Z_0Z_1} \]  \hspace{1cm} (4.34)

\[ U_0 = \frac{e}{Z_0 + \frac{A_1Z_1 + B_3}{C_1Z_1 + D_3}} \]  \hspace{1cm} (4.35)

From equation 4.34 it is clear that \( U_1 \) varies with frequency (both in amplitude and phase). The graph depicting \(|U_1|\) (amplitude of \( U_1 \)) as a function of \( \omega \), is known as the frequency response curve, which is represented in Fig. 4.13.
The frequency locations of the peaks of the frequency response curve are called the resonance frequencies and defined as the formant frequencies. $U_i$ is measured in terms of amplitude [15].

It is well known that the diaphragm separates the upper part of the body from its lower parts, normally having an internal impedance as $Z_0$ (non pregnant condition). As the pregnancy advances, the mass and volume of foetus increases which in turn increases the internal impedance $Z_0$. So when the lady is pregnant let the internal impedance $Z_0$ change to $Z'_0$.

From equations 4.34- and 4.35, accordingly, we have $U'_i$, $U'_o$ as new values of volume velocity at mouth end and throat end respectively. We have

$$U'_i = \frac{e}{B_3 + D_3 Z'_o + A_3 Z_i + C_3 Z'_o Z_i}$$  \hspace{1cm} (4.36)

$$U'_o = \frac{e}{Z'_o + \frac{A_3 Z_i + B_3}{C_3 Z_i + D_3}}$$  \hspace{1cm} (4.37)

### 4.5.1 INTER-RELATIONSHIP OF WOMB WITH VOCAL TRACT:

Thus it is obvious that there is an inter-relationship between womb and vocal tract, representing the relationship of them at the vocal tract output. The womb behavioural changes are reflected at the diaphragmatic level causing change in $Z_0$. Since $Z_0$ is an integral part of vocal tract behavioural characteristic output, it will modify the output signal parameters according to womb development as represented by equation 4.36 and 4.37.
4.6 REFERENCES :


CHAPTER 5

STATEMENT OF PROBLEM AND MEASUREMENT

5.1 Statement of Problem

5.2 Measurements
   5.2.1 Frequency measurement
   5.2.2 Amplitude measurement
   5.2.3 Time duration measurement
   5.2.4 Energy measurement

5.3 References