Chapter 4

Cascade Reliability in Different Types of Failure Models

4.1 Introduction

In many situations a component may fail in several ways as electrical failure, mechanical failure and the like. Each type of failure may be attributed to different stresses which in turn can be represented by different random variables. Let us assume that a component may fail in $m$ different ways for which $m$ different stresses are responsible. Now if $X$ is a random variable representing the strength of a component then the reliability of the component is given by

$$R = P[X \geq Y_1, X \geq Y_2, \ldots, X \geq Y_m]$$

(4.1.1)

An $n$-cascade system where components may fail in different ways is considered in this chapter. The following three models are considered:

Model- I: Here we have assumed that in an $n$-cascade system an active component faces $m$ different stresses (which are responsible for different types of failures) and it fails if the strength of the active component is less than any one of the stresses on it. After the failure of the first component, the second component faces $m$ stresses which are $k$ times the corresponding previous stresses and so on.

Model-II: For an $n$-cascade system we assume that for the working of an active component it is essential that the $m$ stresses on the component lie in an
interval \((a_j, b_j), j=1,2,\ldots, m\). The component fails even if one of stresses on the component falls outside the specified limits. The two limits \(a_j\) and \(b_j\) are assumed to be constants. All the components are assumed to be identical.

**Model- III:** Here also an \(n\)-cascade system is considered. This model is similar to the model II except that the components are not identical. Here the limits of the different stresses, though constants are assumed to be different for different components, say \((a_{ij}, b_{ij})\), for the \(i\) th component, \(j=1,2,\ldots, m\).

For model II and model III strength of the components do not come into the picture directly. But the constants \((a_j, b_j)\) and \((a_{ij}, b_{ij})\) must have been fixed on the basis of the strength of the components.

Sriwastav and Dutta (1986) considered the case of an \(n\)-standby system with different types of failure in S-S model. For multicomponent systems Hilton and Fergen (1960) have considered structural reliability problem where failure modes of the components are independent. A similar problem is considered by Moser and Kinser (1967). Heller and Donat (1967) have evaluated the reliability of ‘multiple-load-path’ structure, in which the system with \(m\) components initially, may work even with \((m-1)\) failed components. The applied stress to the system is redistributed among the surviving components, at every failure. The system fails when all the components fail. They have assumed a statistical dependence among different types of failure modes.

Here we have considered an \(n\)-cascade system. We have not come across any study where cascade model is considered for such a models. The main aim of this chapter is to obtain the system reliability \(R_n\) under this three failure models.

This chapter is organized as follows: In Section-4.2 the general expressions for all the three models are developed. In Section-4.3 the reliability expressions of an \(n\)-cascade system is obtained for all the models when the stress-strength of the components follow particular
distributions. In Sub-Section 4.3.1 to 4.3.2 the expressions of \( R_n, \ n < 4 \) is obtained under the three models when stress-strength distributions are either exponential or Rayleigh. Some graphs are also plotted for some values in Section 4.4. In all the cases, numerical results for particular values of relevant parameters are tabulated in Table 4.1, Table 4.2, Table 4.3, Table 4.4, Table 4.5 and Table 4.6 (cf. Appendix) and some results and discussions are given in Section-4.5.

4.2 Development of the Mathematical Models

Let us consider an \( n \)-cascade system working under the impact of \( m \) different stresses. Here we shall obtain the reliability of this system under the three different models, one-by-one.

**Model I:** Let \( X_1, X_2, \ldots, X_n \) be the strengths of \( n \)-components and each component faces \( m \) stresses simultaneously. The stresses on the first component are \( Y_1, Y_2, \ldots, Y_m \). After the failure of the first component the second component faces \( m \) stresses \( kY_1, kY_2, \ldots, kY_m \), the third component faces \( m \) stresses \( k^2Y_1, k^2Y_2, \ldots, k^2Y_m \) and so on. The stresses on the \( t \)th component are \( k^{i-1}Y_1, k^{i-1}Y_2, \ldots, k^{i-1}Y_m \). Let the attenuation factor \( k \) be a constant quantity. It is assumed that \( X_i, \ i=1,2,\ldots,n \) and \( Y_j, \ j=1,2,\ldots,m \) are independent random variables. Then the probability that the \( i \)th component works is given by

\[
R'(i) = P[X_i \geq k^{i-1}Y_1, X_i \geq k^{i-1}Y_2, \ldots, X_i \geq k^{i-1}Y_m] \ i = 1,2,\ldots,n
\]

Then, the reliability of the system is given by

\[
R_n = R'(1) + [1 - R'(1)]R'(2) + [1 - R'(1)][1 - R'(2)]R'(3) + \ldots + [1 - R'(1)]

[1 - R'(2)] \ldots [1 - R'(n-1)]R'(n)

= R(1) + R(2) + \ldots + R(n), \ \text{say}
\]
where \( R(i), \ i=1,2,\ldots,n \) is the marginal reliability due to the \( i \) th component, given by

\[
R(i) = [1 - R'(1)] [1 - R'(2)] \ldots [1 - R'(i-1)] R'(i)
\]  (4.2.4)

Let \( f_i(x) \) and \( g_j(y) \) be the p.d.f. of \( X_i \) and \( Y_j, \ i=1,2,\ldots,n, \ j=1,2,\ldots,m \) respectively.
Since, the stress and strength are independent, we have

\[
R'(i) = \prod_{j=1}^{m} \int_{-\infty}^{\infty} F_i(k^{i-1} y_j) g_j(y) \, dy_j \quad i=1,2,\ldots,n
\]  (4.2.5)

where \( F_i(x) = \int_{-\infty}^{x} f_i(x) \, dx \) and \( F_i(x) = 1 - F_i(x) \)

Substituting \( i =1,2,3 \) in (4.2.5) and using (4.2.4) we get,

\[
R(1) = \prod_{j=1}^{m} \int_{-\infty}^{\infty} F_1(y_j) g_j(y) \, dy_j
= R'(1)
\]  (4.2.6)

\[
R(2) = [1 - R'(1)] R'(2)
= R'(2) - R'(1) R'(2)
= \prod_{j=1}^{m} \int_{-\infty}^{\infty} F_2(k y_j) g_j(y) \, dy_j - \prod_{j=1}^{m} \int_{-\infty}^{\infty} F_1(y_j) F_2(k y_j) g_j(y) \, dy_j
\]  (4.2.7)

\[
R(3) = [1 - R'(1)] [1 - R'(2)] R'(3)
= R'(3) - R'(2) R'(3) - R'(1) R'(3) + R'(1) R'(2) R'(3)
= \prod_{j=1}^{m} \int_{-\infty}^{\infty} F_3(k^2 y_j) g_j(y) \, dy_j - \prod_{j=1}^{m} \int_{-\infty}^{\infty} F_1(y_j) F_3(k^2 y_j) g_j(y) \, dy_j
- \prod_{j=1}^{m} \int_{-\infty}^{\infty} F_2(k y_j) F_3(k^2 y_j) g_j(y) \, dy_j + \prod_{j=1}^{m} \int_{-\infty}^{\infty} F_1(y_j) F_2(k y_j) F_3(k^2 y_j) g_j(y) \, dy_j
\]  (4.2.8)

Substituting \( R(1), R(2), R(3), \ldots, R(n) \) in (4.2.3) we get \( R_n \).
Model II: We have an \( n \)-cascade system. As in model I let \( Y_1, Y_2, \ldots, Y_m \) be the stresses on the first component. We assume that for the working of a component the \( j \)th stress must lie in a specified interval, say \((a_j, b_j)\), \( j = 1, 2, \ldots, m \). After the failure of the first component the second component faces \( m \) stresses \( kY_1, kY_2, \ldots, kY_m \), the 3rd component faces \( m \) stresses \( k^2Y_1, k^2Y_2, \ldots, k^2Y_m \), and so on. Here we assume that the components are identical i.e., the limits \((a_j, b_j)\) is same for all the components. Here also the reliability \( R \) of the system is given by (4.2.2) and (4.2.3) but now,

\[
R'(i) = P\left( a_i < k^{i-1} Y_1 < b_i, (a_2 < k^{i-1} Y_2 < b_2), \ldots, (a_m < k^{i-1} Y_m < b_m) \right) \\
= P\left( a_i < k^{i-1} Y_1 < b_1 \right) P\left( a_2 < k^{i-1} Y_2 < b_2 \right) \cdots P\left( a_m < k^{i-1} Y_m < b_m \right)
\]  

(4.2.9)

\( i = 1, 2, \ldots, n \), since the stresses on the components are independent random variables.

Let \( g_j(y) \) be the p.d.f. of \( Y_j \), then

\[
R'(i) = \prod_{j=1}^{m} \int_{a_j}^{b_j} g_j(y) \, dy_j \quad i = 1, 2, \ldots, n
\]

(4.2.10)

Now from (4.2.4) and (4.2.10), we get

\[
R(1) = R'(1) \\
= \prod_{j=1}^{m} \int_{a_j}^{b_j} g_j(y) \, dy_j
\]

(4.2.11)

\[
R(2) = R'(2) - R'(1)R'(2) \\
= \prod_{j=1}^{m} \int_{a_j/k}^{b_j/k} g_j(y) \, dy_j - \left( \prod_{j=1}^{m} \int_{a_j}^{b_j} g_j(y) \, dy_j \right) \left( \prod_{j=1}^{m} \int_{a_j/k}^{b_j/k} g_j(y) \, dy_j \right)
\]

(4.2.12)
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\[ R(3) = R'(3) - R'(2)R'(3) - R'(1)R'(3) + R'(1)R'(2)R'(3) \]
\[ = \prod_{j=1}^{m} \int_{a_j}^{b_j/k^2} g_j(y) \, dy_{j} - \left( \prod_{j=1}^{m} \int_{a_j}^{b_j/k^2} g_j(y) \, dy_{j} \right) \left( \prod_{j=1}^{m} \int_{a_j}^{b_j/k^2} g_j(y) \, dy_{j} \right) - \left( \prod_{j=1}^{m} \int_{a_j}^{b_j/k^2} g_j(y) \, dy_{j} \right) + \left( \prod_{j=1}^{m} \int_{a_j}^{b_j/k^2} g_j(y) \, dy_{j} \right) \]

(4.2.13)

Substituting \( R(1), R(2), R(3), \ldots, R(n) \) in (4.2.3) we can obtain \( R_n \).

Model III: This model is similar to model II except the components are not identical. Let \( k^{i-1}Y_1, k^{i-1}Y_2, \ldots, k^{i-1}Y_m \) be the stresses on the \( i \)th active component and let \( (a_{ij}, b_{ij}) \); \( (i=1,2,\ldots,n; \ j=1,2,\ldots,m) \) be the required limits of the \( m \) stresses for the working of the \( i \)th active component where \( (a_{ij}, b_{ij}) \) corresponds to the \( j \)th stress on the \( i \)th component.

Reliability of the system under this model is given by (4.2.2) and (4.2.3)

\[ R'(i) = P\left[ (a_{i1} < k^{i-1}Y_1 < b_{i1}), (a_{i2} < k^{i-1}Y_2 < b_{i2}), \ldots, (a_{im} < k^{i-1}Y_m < b_{im}) \right] \]
\[ = P\left( \frac{a_{i1}}{k^{i-1}} < Y_1 < \frac{b_{i1}}{k^{i-1}} \right) \cdot P\left( \frac{a_{i2}}{k^{i-1}} < Y_2 < \frac{b_{i2}}{k^{i-1}} \right) \cdots P\left( \frac{a_{im}}{k^{i-1}} < Y_m < \frac{b_{im}}{k^{i-1}} \right) \]

(4.2.14)

since \( Y_1, Y_2, \ldots, Y_m \) are independent.

Let \( g_j(y) \) be the p.d.f. of \( Y_j \), then

\[ R'(i) = \prod_{j=1}^{m} \int_{a_{ij}/k^{i-1}}^{b_{ij}/k^{i-1}} g_j(y) \, dy_{j}, \quad i=1,2,\ldots,n \]

(4.2.15)

Substituting the values of \( R'(i), \ i=1,2,\ldots,n \) from (4.2.15) in (4.2.2) we get the system reliability \( R_n \).
4.3 Stress-Strength follows Specific Distributions

In this section we have considered stress-strength, which are either exponential or Rayleigh distributions and they may be with different parameters. In the following Sub-Sections we have obtained the reliability of a 3-cascade system under the three models as discussed in section 4.2.

4.3.1 Exponential Stress-Strength Distribution

Model I: Let \( f_i(x) \) and \( g_j(y) \) be the exponential densities with means \( 1/\theta_i \) and \( 1/\alpha_j \), \( i = 1, 2, \ldots, n \); \( j = 1, 2, \ldots, m \) respectively. Then from (4.2.5) we get,

\[
R'(i) = \prod_{j=1}^{m} \int_{0}^{\infty} e^{-k^{-1}y} \alpha_j e^{-\alpha_j y} dy_j
\]

\[
= \prod_{j=1}^{m} \frac{\alpha_j}{\alpha_j + k^{-1} \theta_i}; \quad i = 1, 2, \ldots, n
\]

(4.3.1)

Now from (4.3.1) using (4.2.4) we get the marginal reliabilities \( R(1), R(2), R(3) \) as follows.

\[
R(1) = R'(1)
\]

\[
= \prod_{j=1}^{m} \frac{\alpha_j}{\alpha_j + \theta_i}
\]

(4.3.2)

\[
R(2) = R'(2) - R'(1)R'(2)
\]

\[
= \prod_{j=1}^{m} \frac{\alpha_j}{\alpha_j + k \theta_2} - \prod_{j=1}^{m} \frac{\alpha_j}{\alpha_j + \theta_1 + k \theta_2}
\]

(4.3.3)

\[
R(3) = R'(3) - R'(2)R'(3) - R'(1)R'(3) + R'(1)R'(2)R'(3)
\]

\[
= \prod_{j=1}^{m} \frac{\alpha_j}{\alpha_j + k^2 \theta_3} - \prod_{j=1}^{m} \frac{\alpha_j}{\alpha_j + \theta_1 + k^2 \theta_3} - \prod_{j=1}^{m} \frac{\alpha_j}{\alpha_j + k \theta_2 + k^2 \theta_3} + \prod_{j=1}^{m} \frac{\alpha_j}{\alpha_j + \theta_1 + k \theta_2 + k^2 \theta_3}
\]

(4.3.4)

Substituting the values of \( R(1), R(2), R(3) \) in (4.2.3) we can obtain \( R_3 \), the reliability of a 3 cascade system.
Model II: Let \( g_j(y) \) be the exponential densities with mean \( 1/\alpha_j \), \( j = 1, 2, \ldots, m \) respectively.

Then from (4.2.10) we get,

\[
R'(i) = \prod_{j=1}^{m} \left( e^{\frac{\alpha_{i j}}{k^{j-1}}} - e^{\frac{\alpha_{p j}}{k^{j-1}}} \right); \quad i = 1, 2, \ldots, n
\]  \hspace{1cm} (4.3.5)

Now from (4.3.5) using (4.2.4) we get the marginal reliabilities \( R(1) , R(2) , R(3) \) as follows:

\[
R(1) = R'(1) = \prod_{j=1}^{m} \left( e^{-\alpha_{i j}} - e^{-\alpha_{p j}} \right)
\] \hspace{1cm} (4.3.6)

\[
R(2) = \prod_{j=1}^{m} \left( e^{\frac{\alpha_{i j}}{k} - \frac{\alpha_{p j}}{k}} \right) - \prod_{j=1}^{m} \left( e^{-\alpha_{i j}} - e^{-\alpha_{p j}} \right) \prod_{j=1}^{m} \left( e^{\frac{\alpha_{i j}}{k} - \frac{\alpha_{p j}}{k}} \right)
\] \hspace{1cm} (4.3.7)

\[
R(3) = \prod_{j=1}^{m} \left( e^{\frac{\alpha_{i j}}{k^2} - \frac{\alpha_{p j}}{k^2}} \right) - \prod_{j=1}^{m} \left( e^{-\alpha_{i j}} - e^{-\alpha_{p j}} \right) \prod_{j=1}^{m} \left( e^{\frac{\alpha_{i j}}{k^2} - \frac{\alpha_{p j}}{k^2}} \right) - \prod_{j=1}^{m} \left( e^{\frac{\alpha_{i j}}{k} - \frac{\alpha_{p j}}{k}} \right) \prod_{j=1}^{m} \left( e^{\frac{\alpha_{i j}}{k^2} - \frac{\alpha_{p j}}{k^2}} \right)
\] \hspace{1cm} (4.3.8)

Substituting the values of \( R(1) , R(2) , R(3) \) from (4.3.6) to (4.3.8) in (4.2.3) we get \( R_3 \).

Model III: Let \( g_j(y) \) be the exponential densities with means \( 1/\alpha_j \), \( j = 1, 2, \ldots, m \) respectively.

Then from (4.2.14) we get,

\[
R'(i) = \prod_{j=1}^{m} \left( e^{\frac{\alpha_{i j}}{k^{j-1}}} - e^{\frac{\alpha_{p j}}{k^{j-1}}} \right); \quad i = 1, 2, \ldots, n
\] \hspace{1cm} (4.3.9)
Now from (4.3.9) and (4.2.4) we get the different marginal reliabilities as follows:

\[
R(1) = \prod_{j=1}^{m} \left( e^{-\alpha_{1j} / \sigma_i^2} - e^{-\alpha_{1j} / \beta_j^2} \right) 
\]

(4.3.10)

\[
R(2) = \prod_{j=1}^{m} \left( e^{-\alpha_{2j} / \sigma_i^2} - e^{-\alpha_{2j} / \beta_j^2} \right) - \prod_{j=1}^{m} \left( e^{-\alpha_{1j} / \sigma_i^2} - e^{-\alpha_{1j} / \beta_j^2} \right) \prod_{j=1}^{m} \left( e^{-\alpha_{2j} / \sigma_i^2} - e^{-\alpha_{2j} / \beta_j^2} \right) 
\]

(4.3.11)

\[
R(3) = \prod_{j=1}^{m} \left( e^{-\alpha_{3j} / \sigma_i^2} - e^{-\alpha_{3j} / \beta_j^2} \right) + \prod_{j=1}^{m} \left( e^{-\alpha_{1j} / \sigma_i^2} - e^{-\alpha_{1j} / \beta_j^2} \right) \prod_{j=1}^{m} \left( e^{-\alpha_{2j} / \sigma_i^2} - e^{-\alpha_{2j} / \beta_j^2} \right) - \prod_{j=1}^{m} \left( e^{-\alpha_{3j} / \sigma_i^2} - e^{-\alpha_{3j} / \beta_j^2} \right) 
\]

(4.3.12)

Substituting the values of \( R(1), R(2), R(3) \) in (4.2.3) we get \( R_1 \).

### 4.3.2 Rayleigh Stress-Strength Distribution

**Model I:** Let \( f_i(x) \) and \( g_j(y) \) be the Rayleigh densities with parameters \( \sigma_i^2 \) and \( \beta_j^2 \), \( i = 1, 2, \ldots, n \); \( j = 1, 2, \ldots, m \) respectively. Then from (4.2.5) we get,

\[
R'(i) = \prod_{j=1}^{m} \frac{\sigma_i^2}{\sigma_i^2 + k^{2i-2} \beta_j^2}; \quad i = 1, 2, \ldots, n 
\]

(4.3.13)

Now from (4.3.13) using (4.2.4) we get the marginal reliabilities \( R(1), R(2), R(3) \) as follows.

\[
R(1) = R'(1) = \prod_{j=1}^{m} \frac{\sigma_1^2}{\sigma_1^2 + \beta_j^2} 
\]

(4.3.14)
\[ R(2) = R'(2) - R'(1)R'(2) \]

\[ = \prod_{j=1}^{m} \frac{\sigma_2^2}{\sigma_2^2 + k^2 \beta_j^2} \prod_{j=1}^{m} \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 \sigma_2^2 + \sigma_2^2 \beta_j^2 + k^2 \sigma_1^2 \beta_j^2} \]  

\[ R(3) = R'(3) - R'(2)R'(3) - R'(1)R'(3) + R'(1)R'(2)R'(3) \]

\[ = \prod_{j=1}^{m} \frac{\sigma_3^2}{\sigma_3^2 + k^4 \beta_j^2} \prod_{j=1}^{m} \frac{\sigma_2^2 \sigma_3^2}{\sigma_2^2 \sigma_3^2 + \sigma_3^2 \beta_j^2 + k^2 \sigma_2^2 \beta_j^2 + k^2 \sigma_1^2 \beta_j^2} \]

\[ \quad + \prod_{j=1}^{m} \frac{\sigma_1^2 \sigma_2^2 \sigma_3^2}{\sigma_1^2 \sigma_2^2 \sigma_3^2 + \sigma_2^2 \sigma_3^2 \beta_j^2 + k^2 \sigma_2^2 \beta_j^2 + k^2 \sigma_1^2 \beta_j^2} \]  

(4.3.16)

Substituting the values of \( R(1), R(2), R(3) \) in (4.2.3) we can obtain \( R_3 \), the reliability of a 3 cascade system.

Model II: Let \( g_j(y) \) be the Rayleigh densities with parameters \( \beta_j^2, \ j=1,2,\ldots,m \) respectively.

Then from (4.2.10) we get,

\[ R'(i) = \prod_{j=1}^{m} \left( e^{-\frac{a_j}{\delta_j}} - e^{-\frac{b_j}{\delta_j}} \right); \quad i = 1,2,\ldots,n \]  

(4.3.17)

Now from (4.3.17) using (4.2.4) we get the marginal reliabilities \( R(1), R(2), R(3) \) as follows:

\[ R(1) = R'(1) \]

\[ = \prod_{j=1}^{m} \left( e^{-a_j} - e^{-b_j} \right) \]  

(4.3.18)

\[ R(2) = \prod_{j=1}^{m} \left( e^{-\frac{a_j}{\delta}} - e^{-\frac{b_j}{\delta}} \right) - \prod_{j=1}^{m} \left( e^{-a_j} - e^{-b_j} \right) \prod_{j=1}^{m} \left( e^{-\frac{a_j}{\delta}} - e^{-\frac{b_j}{\delta}} \right) \]  

(4.3.19)
\[ R(3) = \prod_{j=1}^{m} \left( e^{-\frac{a_j}{k^2}} - e^{-\frac{b_j}{k^2}} \right) - \prod_{j=1}^{m} \left( e^{-a_j} - e^{-b_j} \right) \prod_{j=1}^{m} \left( e^{-\frac{a_j}{k^2}} - e^{-\frac{b_j}{k^2}} \right) - \prod_{j=1}^{m} \left( e^{-\frac{a_j}{k}} - e^{-\frac{b_j}{k}} \right) \]  

(4.3.20)

Substituting the values of \( R(1) \), \( R(2) \), \( R(3) \) from (4.3.18) to (4.3.20) in (4.2.3) we get \( R_1 \).

**Model III:** Let \( g_j(y) \) be the Rayleigh densities with parameters \( \beta_j^2, \ j = 1, 2, \ldots, m \) respectively.

Then from (4.2.14) we get,

\[ R'(i) = \prod_{j=1}^{m} \left( e^{-\frac{a_j}{k^2}} - e^{-\frac{b_j}{k^2}} \right) ; \ i = 1, 2, \ldots, n \]  

(4.3.21)

Now from (4.3.21) and (4.2.4) we get the different marginal reliabilities as follows:

\[ R(1) = \prod_{j=1}^{m} \left( e^{-a_j} - e^{-b_j} \right) \]  

(4.3.22)

\[ R(2) = \prod_{j=1}^{m} \left( e^{-\frac{a_j}{k}} - e^{-\frac{b_j}{k}} \right) - \prod_{j=1}^{m} \left( e^{-a_j} - e^{-b_j} \right) \prod_{j=1}^{m} \left( e^{-\frac{a_j}{k}} - e^{-\frac{b_j}{k}} \right) \]  

(4.3.23)

\[ R(3) = \prod_{j=1}^{m} \left( e^{-\frac{a_j}{k^2}} - e^{-\frac{b_j}{k^2}} \right) - \prod_{j=1}^{m} \left( e^{-a_j} - e^{-b_j} \right) \prod_{j=1}^{m} \left( e^{-\frac{a_j}{k^2}} - e^{-\frac{b_j}{k^2}} \right) - \prod_{j=1}^{m} \left( e^{-\frac{a_j}{k}} - e^{-\frac{b_j}{k}} \right) \]  

(4.3.24)

Substituting the values of \( R(1) \), \( R(2) \), \( R(3) \) in (4.2.3) we get \( R_1 \).
4.4 Graphical Representations

Some graphs are plotted in Fig. 4.1(a) - 4.1(b), Fig. 4.2(a) - 4.2(b), Fig. 4.3(a) - 4.3(b), Fig. 4.4(a) - 4.4(b), Fig. 4.5(a) - 4.5(b) and Fig. 4.6(a) - 4.6(b) by taking different parameters along the horizontal axis and the corresponding reliabilities along the vertical axis. Fig. 4.1(a) - 4.1(b) represents the curves for $\theta_2 = 1, \theta_3 = 2$ and it is seen that reliabilities decrease steadily with increasing $k$. Fig. 4.2(a)-4.2(b) represents the values for $a_1 = 0.1, a_2 = 0.2, a_3 = 0.3 = 1, b_1 = b_2 = b_3 = 4, \alpha_1 = \alpha_2 = \alpha_3 = 0.2$ respectively. One can read the values of $R(1), R(2), R(3)$ and $R_3$ for intermediate values of $k$, from these graphs of exponential distribution. In Fig. 4.3(a) - 4.3(b), some graphs of reliabilities against $k$ are plotted for some fix values of $a_{11} = a_{12} = a_{13} = a_{21} = a_{22} = a_{23} = 0.1, b_{11}, b_{12}, b_{13}, b_{21}, b_{22}, b_{23}, b_{31}, b_{32}, b_{33}$ and $\alpha_1 = \alpha_2 = \alpha_3 = 0.2$. It is to be observed that, reliability steadily decreases with increasing $k$. Again Fig. 4.4(a) - 4.4(b) represents the curves for $\sigma_2 = 1, \sigma_3 = 2$ and it is seen that reliability decreases steadily with increasing $k$. Fig. 4.5(a) - 4.5(b) represents the different values of $a_1, a_2, b_1, b_2$ respectively. One can read the values of $R(1), R(2), R(3)$ and $R_3$ for intermediate values of $k$, from these graphs of Rayleigh distribution. Similarly Fig. 4.6(a) - 4.6(b) it is seen that some graphs of reliabilities against $k$ are plotted for some fix values of $a_{11}, a_{12}, a_{21}, a_{22}, a_{31}, a_{32}$ and $b_{11}, b_{12}, b_{21}, b_{22}, b_{31}, b_{32}$. It is to be observed that, $R(2), R(3)$ and $R_3$ are steadily increases with increasing $k$ but the graph of $R(1)$ seems to be a straight line since it is independent of $k$. 
Fig. 4.1(a) Exponential Stress-Strength for model I: Graph for $R_1, R_3$ for fixed values of $\theta_1$

Fig. 4.1(b) Exponential Stress-Strength for model I: Graph for $R_1, R_3$ for fixed values of $\theta_1$

Fig. 4.2(a) Exponential Stress-Strength for model II: Graph for $R(1), R(2), R(3), R_3$

Fig. 4.2(b) Exponential Stress-Strength for model II: Graph for $R(1), R(2), R(3), R_3$
**Fig. 4.3(a)** Exponential Stress-Strength for model III: Graph for $R_1, R_3$.

**Fig. 4.3(b)** Exponential Stress-Strength for model III: Graph for $R_1, R_2$.

**Fig. 4.4(a)** Rayleigh Stress-Strength for model I: Graph for $R_3$ for fixed values of $\sigma_1$.

**Fig. 4.4(b)** Rayleigh Stress-Strength for model I: Graph for $R_3$ for fixed values of $\sigma_1$. 
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Fig. 4.5(a) Rayleigh Stress-Strength for model II:
Graph for $R(1), R(2), R(3), R_3$

Fig. 4.5(b) Rayleigh Stress-Strength for model II:
Graph for $R(1), R(2), R(3), R_3$

Fig. 4.6(a) Rayleigh Stress-Strength for model III:
Graph for $R(1), R(2), R(3), R_3$

Fig. 4.6 (b) Rayleigh Stress-Strength for model III:
Graph for $R(1), R(2), R(3), R_3$
4.5 Results and Discussions

From the Table 4.1 we observe that when $k$ increases, $\alpha$ and $\theta$ remaining the same, $R(2)$ decreases rapidly whereas there is slight decreases in $R(3)$ which results in decrease in $R_3$. For example, when $k=0.1$, $\alpha_1=1$, $\alpha_2=1.0$ and $\theta_1=0.5, \theta_2=1.0, \theta_3=2.0$ we find $R(2)=0.4358$, $R(3)=0.1122$ and system reliability $R_3=0.9925$ but $k=0.5$, for same set of parameter values $R(2)=0.1944$, $R(3)=0.1044$ and $R_3=0.7433$. When $\theta_1$ increase $R(1)$ increases. Similar conclusion may be drawn for increase in $\theta_2$ and $\theta_3$. But when $\theta_1$ and $\theta_2$ increases we find increase in the values of $R(i)$; $i=1,2,3$ and consequently $R_3$.

From the Table 4.2, we see that $R(i)$, $i=2,3$ increase when $k$ increases. Also change in values of $\alpha_i$, $i=1,2,3$ effects all $R(i)$. When any $\alpha_i$ increases (i.e., mean stress decreases), keeping the limits fixed $R(i)$ increases but there is decrease in $R(2)$ and $R(3)$. For instance when $a_1=0.1, b_1=4.0$, $a_2=0.2, b_2=4.0$, $a_3=0.3, b_3=4.0$, $\alpha_1=0.2, \alpha_2=0.2, \alpha_3=0.2$, then $R(1)=0.1337$, $R(2)=0.3248$, $R(3)=0.2891$ and keeping all other parameters fixed if $\alpha_1=0.1, \alpha_2=1.0, \alpha_3=1.0$, we get $R(1)=0.5217$, $R(2)=0.1466$, $R(3)=0.0309$. When any of the upper limits decreases, $R(i)$, $i=2,3$ will decrease.

From the Table 4.3, we notice that all the limits $(a_{ij}, b_{ij})$, $i,j=1,2,3$ and stress parameter $\alpha_i$, $i=1,2,3$ remaining the same $R_3$ increases when $k \leq 0.5$ and $R_3$ decrease when $k > 0.5$. In general we see that when $\alpha_i$ ($i=1,2,3$) increases, all $R(i)$ will increase. For instance when $\alpha_1=\alpha_2=\alpha_3=0.2$, $a_{ij}=0.1$, $i,j=1,2,3$; $b_{11}=b_{21}=b_{31}=2.0$ and $b_{12}=b_{13}=b_{22}=b_{23}=b_{32}=b_{33}=3$, $R(1)=0.5767$, $R(2)=0.5026$, $R(3)=0.0011$. Whereas when $\alpha_1=\alpha_2=\alpha_3=0.5$, other values remaining constant, $R(1)=0.3093$, $R(2)=0.1541$, $R(3)=0.0001$. Also it is observed that keeping all other parameters fixed $R(i)$ can be increased, by increasing $b_{ij}$ values.
From the Table 4.4, we observe that when $k$ increases, $\sigma$ and $\beta$ remaining the same, $R(2)$ decreases rapidly whereas there is slight increase in $R(3)$ which results in decrease in $R_3$. For example, when $k=0.1$, $\beta_1=1$, $\beta_2=1.0$ and $\sigma_1=0.5, \sigma_2=1.0, \sigma_3=2.0$ we find $R(2)=0.8290$, $R(3)=0.0048$ and system reliability $R_3=0.9857$ but $k=0.5$, for same set of parameter values $R(2)=0.4949$, $R(3)=0.0826$ and $R_3=0.7375$. When $\sigma_1$ increase $R(1)$ increases. Similar conclusion may be drawn for increase in $\sigma_2$ and $\sigma_3$. But when $\sigma_1$ and $\sigma_2$ increase we find increase in the values of $R(i)$; $i=1,2,3$ and consequently $R_3$.

From the Table 4.5, we see that $R(i)$, $i=2,3$ increase when $k$ increases. For instance when $a_1=0.1, b_1=4.0, a_2=0.2, b_2=4.0$, then $R(1)=0.7096$, $R(2)=0.1592$, $R(3)=0.0395$. When any of the upper limits increases, $R(i)$ ($i=2,3$) will decrease some times and similarly when any of the lower limits increases, $R(i)$, ($i=2,3$) will decrease.

From the Table 4.6, we notice that all the limits $(a_{ij}, b_{ij}), i=1,2,3$ and $j=1,2$ remaining the same $R_3$ increases when $k \leq 0.9$ and $R_3$ decreases when $k > 0.9$. Also it is observed that keeping all other parameters fixed, $R(i)$, $i=2,3$ can be decreased, by increasing $b_{ij}$ values and also $R(i)$, $i=2,3$ and $R_3$ can be decreased, by increasing $a_{ij}$ values.

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