Chapter 3

Linear and nonlinear multilayered structures with homogeneous interfaces

3.1 Introduction

Ferroelectric thin films and superconductors are commonly used materials for tunable microwave devices [71, 72]. The persistent demand for enhanced performance of communication and radar systems has led to the development of a class of microwave devices based on the combination of ferroelectric and superconducting thin films [15, 73, 74, 75]. In particular, a combination of ferroelectric materials and superconducting materials in a layered structure could be used in order to produce low loss frequency agile microwave transmission lines [5, 76]. Advantages of superconducting waveguides such as low dispersion, low loss and wide bandwidth...
etc. could be exploited in high frequency applications. Ferroelectric materials are highly compatible with high transition temperature superconducting (HTS) materials [14, 41, 72, 73]. Tunable permittivity of ferroelectrics makes it a suitable substrate material for superconducting waveguide structures.

In recent years, the propagation characteristics of electromagnetic waves in layered waveguide structure containing nonlinear ferroelectric has great interest in wide range of research areas [9, 77, 78, 79]. The nonlinear property of ferroelectric material can be used in nonlinear applications such as harmonic generators, parametric amplifiers, limiters, modulators etc [80, 81, 82].

Two typical waveguide structures are considered in this chapter: (i) a multi-layer structure in linear region composed of superconductor/ferroelectric/superconductor, and (ii) a nonlinear ferroelectric and superconductor interface. The objectives include the study of the tunable wave propagation in the waveguide structure with thin films of superconductors on either side of the ferroelectric thin film materials in linear regimes. In addition, the propagation characteristics of microwave through nonlinear ferroelectric/superconductor is also discussed. In this study, the waveguide structure is modeled by setting the boundary conditions at each interfaces using Maxwell’s equations, which lead to obtain closed form dispersion relation and is numerically solved. Finally the numerical results are discussed.
3.2 Superconductor / ferroelectric / superconductor waveguide in linear regimes of operation

3.2.1 Method of analysis

The multi-layered waveguide structure which is operating under linear regimes is studied in this section. Electromagnetic microwave propagation in a superconducting transmission line was theoretically studied by Swihart [20]. Later, field solutions for the Swihart waves in the case of a high temperature superconductor/dielectric film multilayerd structures were reported by C. J. Wu [21]. Here, the tunable wave propagation in the waveguide structure consisting of a pair of superconducting layers separated by ferroelectric thin film materials is studied. Geometry of the superconducting waveguide structure under study is depicted in Fig. 3.1.

Pair of superconducting layers of thickness $d_1$ is separated by the ferroelectric thin

![Figure 3.1: Geometry of waveguide structure.](image-url)
3.2 Superconductor / ferroelectric / superconductor waveguide in linear regimes of operation

film of thickness \( d_2 \). The propagation direction of TM wave is taken to be along \( z \) direction. The wave equation can be written as:

\[
\frac{\partial^2 E_z}{\partial x^2} - K^2 E_z = 0 \tag{3.1}
\]

In free space,

\[
K^2 = \beta^2 - \omega^2 \varepsilon_0 \mu_0 \tag{3.2}
\]

For superconducting layer,

\[
K_1^2 = K_2^2 = \frac{1}{\lambda^2} + \beta^2 - \omega \varepsilon_0 \mu_0 + i \mu_0 \omega \sigma \tag{3.3}
\]

And for the ferroelectric layer,

\[
K_3^2 = \omega^2 \varepsilon_r \mu_0 - \beta^2 \tag{3.4}
\]

Here \( \beta \) is the propagation constant along the \( z \) direction, \( \omega \) is the angular frequency, \( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and the permeability of vacuum respectively, \( \varepsilon_r \) is the dielectric constant of the dielectrics, \( \lambda \) and \( \sigma \) respectively the penetration depth and the conductivity of the superconductors.

The nonzero field components in different layers are derived from Maxwell’s equations as given below:

\[
E_z = -\frac{j}{\beta} \frac{\partial E_x}{\partial x} \tag{3.5}
\]
3.2 Superconductor / ferroelectric / superconductor waveguide in linear regimes of operation

and

\[ H_y = \frac{\omega \epsilon}{\beta} E_x \]  

(3.6)

Applying boundary conditions at each interfaces leads to the following dispersion relation

\[ \tan(K_3d) = \frac{\psi_1 + \psi_2}{\psi_1 \psi_2 - 1} \]  

(3.7)

where,

\[ \psi_1 = -\frac{K_2 \epsilon_f}{K_3 \epsilon_s} K_2 \epsilon_f \frac{\text{coth}(K_2 b_2) + \frac{K_3}{\alpha}}{\epsilon_s [\epsilon_f + \frac{K_2 \text{coth}(K_3 b_2)}{\alpha}]} \]  

(3.8)

\[ \psi_2 = -\frac{K_1 \epsilon_f}{K_3 \epsilon_s} K_1 \epsilon_f \frac{\text{coth}(K_1 b_1) + \frac{K_1}{\alpha}}{\epsilon_s [\epsilon_f + \frac{K_1 \text{coth}(K_1 b_1)}{\alpha}]} \]  

(3.9)

In applying the limiting condition for Swihart mode, the above dispersion relation reduces to

\[ \tan(K_3d) = -\psi_1 - \psi_2 \]  

(3.10)

(\( \psi_1 - \psi_2 = 0 \))

Since \( K_3d \) is very low, then \( \tan(K_3d) = K_3d \) which leads to the dispersion relation
3.2 Superconductor / ferroelectric / superconductor waveguide in linear regimes of operation

as follows:

\[ \beta^2 = K_0^2 \varepsilon_{rf} \left[ 1 + \frac{K_1}{d} F_2 + \frac{K_2}{d} F_1 \right] \]  \hspace{1cm} (3.11)

where,

\[ F_2 = \frac{(K_1^2 - \beta^2) \coth(K_1 b_1) - \frac{K_1 K_0^2}{\alpha}}{(K_1^2 - \beta^2) \left[ (K_1^2 - \beta^2) - \frac{K_1^2 K_0^2 \coth(K_1 b_1)}{\alpha} \right]} \]  \hspace{1cm} (3.12)

\[ F_1 = \frac{(K_2^2 - \beta^2) \coth(K_2 b_2) - \frac{K_2 K_0^2}{\alpha}}{(K_2^2 - \beta^2) \left[ (K_2^2 - \beta^2) - \frac{K_2^2 K_0^2 \coth(K_2 b_2)}{\alpha} \right]} \]  \hspace{1cm} (3.13)

The model developed by O.G. Vendik [83, 84] as given in the last chapter is used in modeling the microwave permittivity of SrTiO$_3$ (STO).

3.2.2 Numerical results

The propagation of Swihart waves which satisfy the condition \( K_0 d \sqrt{\varepsilon_r} \ll 1 \) through the ferroelectric/superconducting multi-layered waveguide is studied, where \( K_0 = \omega \sqrt{\mu_0 \varepsilon_0} \) is the wave number of free space, \( d \) is thickness of the dielectric slab, \( \varepsilon_r \) is the relative permittivity of the dielectric. YBCO is used as the superconducting material which has the typical material parameter values such as \( T_c = 92 K \), \( \lambda_0 = 140 \) nm. The dispersion relation is numerically solved using Muller’s method, an iterative method which converges quadratically and does not require the evaluation of
3.2 Superconductor / ferroelectric / superconductor waveguide in linear regimes of operation

the derivative of function [18].

Figure 3.2: Variation of phase constant with thickness of ferroelectric substrate for different electric fields.

Figure 3.3: Variation of attenuation constant with thickness of ferroelectric substrate for different electric fields.
3.2 Superconductor / ferroelectric / superconductor waveguide in linear regimes of operation

Figures 3.2 and 3.3 depict the variation of phase constant and attenuation constant of Swihart modes with the thickness of the ferroelectric substrate for different tuning electric fields values 55 KV/cm, 50 KV/cm and 45 KV/cm at a fixed temperature 46 K at a frequency 10 GHz. As seen from Fig. 3.2, the variation of phase constant decreases with thickness of substrate for different electric fields. In addition, it can be realized that increasing the electric field results in a decrease in the propagation constant.

In Fig. 3.2, variation of phase constant with the thickness of ferroelectric thin film layer for various biasing electric fields at a particular frequency is depicted. It is observed that the field coupling between the superconducting layers occurs as the thickness of the ferroelectric substrate reduces.

The attenuation constant also decreases with thickness of ferroelectric substrate at different electric fields which is shown in Fig. 3.3. As the biasing electric field increases, the attenuation constant decreases. Tunable permittivity of ferroelectric substrate undergoes substantial change with electric field is established. The quality factor of modes are given by $Q = 2\text{Re}(\beta)/\text{Im}(\beta)$ with substrate thickness is also calculated. Due to the small dimensions of superconducting film thickness as well as substrate, the $Q$ value increases with increase in thickness of substrate. From the above results it is clear that the propagation of Swihart modes in ferroelectric based superconducting waveguide structure shows considerable variation with the substrate thickness at different tuning electric fields.
3.3 **Nonlinear wave propagation in ferroelectric / superconductor interfaces**

In this section, we study a typical nonlinear wave supported by ferroelectric and superconductor interface. The ferroelectric medium is operating under nonlinear regime of electric field level.

### 3.3.1 Method of analysis

Geometry of the nonlinear ferroelectric/superconducting waveguide structure under study is depicted in Fig. 3.4. The nonlinear ferroelectric substrate occupies in the region $y > 0$ and the superconductor substrate in $y < 0$. The propagation direction of TE wave is taken to be along $z$ direction. For TE mode, the electric and magnetic field components are $E_x$, $H_y$, and $H_z$. Nonzero fields in different layers are derived.
3.3 Nonlinear wave propagation in ferroelectric / superconductor interfaces

from Maxwell’s equations and is given below:

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]  \hfill (3.14)

The proposed model of electric field dependent nonlinear permittivity of ferroelectric thin film is given in equation (2.14)

\[ \varepsilon_f = \varepsilon_0 a_0 + \varepsilon_0 a_1 E_x + \varepsilon_0 a_2 E_x^2 \]  \hfill (3.15)

From the above equations, the wave equations in the two regions can be formulated as:

\[ \frac{\partial^2 E_x}{\partial y^2} + (k_0^2 \varepsilon_0 a_0 - k^2) E_x + a_1 k_0^2 \varepsilon_0 E_x^2 + a_2 k_0^2 \varepsilon_0 E_x^3 = 0; \quad y > 0 \]  \hfill (3.16)

\[ \frac{\partial^2 E_x}{\partial y^2} - k_s^2 E_x = 0; \quad y < 0 \]  \hfill (3.17)

where, \( k_s = \sqrt{k^2 - \omega^2 \varepsilon_0 \mu_0 \mu_s} \). The negative relative dielectric constant of \( \varepsilon_s \) for the superconductor can be approximated by the two-fluid model and is given by

\[ \varepsilon_s = \left[ 1 - \frac{1}{\omega^2 \mu_0 \lambda_L^2 \varepsilon_0} \right] - i \frac{\sigma}{\omega \varepsilon_0} \]  \hfill (3.18)

where,

\[ \lambda_L^2 = \frac{\lambda_0^2}{\left[ 1 - \left( \frac{r}{r_c} \right)^4 \right]} \]  \hfill (3.19)
and $\lambda_0$ is the field penetration depth at temperature $T = 0$ K,

$$\sigma = \sigma_0 \left[ \left( \frac{T}{T_c} \right)^4 \right]$$ (3.20)

and $T_c$ is the critical temperature of the superconductor. The first integral of above nonlinear equation (3.16) can be written as

$$\frac{1}{2} \left( \frac{\partial E_x}{\partial y} \right)^2 = \frac{1}{2} k_x^2 E_x^2 - \varepsilon_0 k_0^2 \int \left( a_1 E_x^2 + a_2 E_x^4 \right) dE_x + C$$ (3.21)

where, $k_x^2 = k^2 - a_0 \varepsilon_0 k_0^2$, and $C$ is the first integration constant. This equation can also be written as,

$$\left( \frac{\partial E_x}{\partial y} \right)^2 = k_x^2 E_x^2 - \frac{2}{3} \varepsilon_0 k_0^2 a_1 E_x^3 - \frac{2}{4} \varepsilon_0 k_0^2 a_2 E_x^4 + C$$ (3.22)

In the equation (3.22), the integration constant $C$ is set to equal zero as no waves are leaving or entering the cladding.

Denoting $b_1 = \frac{2}{3} \varepsilon_0 k_0^2 a_1$ and $c_1 = \frac{2}{4} \varepsilon_0 k_0^2 a_2$, then equation (3.22) can be written as:

$$\left( \frac{\partial E_x}{\partial y} \right)^2 = E_x^2 \left( k_x^2 + b_1 E_x + c_1 E_x^2 \right)$$ (3.23)

Taking the square root of equation (3.23) and integrating by using the table of inte-
3.3 Nonlinear wave propagation in ferroelectric / superconductor interfaces

\[
\tanh(\lambda) = \frac{2k_2^2 + b_1E_x}{2k_2^2 \sqrt{k_2^2 + b_1E_x + c_1E_x^2}} \tag{3.24}
\]

where, \(\lambda = k_2(y + y_0)\). Squaring and rearranging the above equation,

\[
[k_2^2(1 - \tanh^2(\lambda))] + \left[ \frac{b_1^2}{4k_2^2} - c_1 \tanh^2(\lambda) \right] E_x^2 + [b_1(1 - \tanh^2(\lambda))] E_x = 0 \tag{3.25}
\]

This equation is in the quadratic form and can be solved for \(E_x\) as,

\[
E_{x1} = \frac{2k_2^2 - b_1 \text{sech}^2(\lambda) + \text{sech}(\lambda) \tanh(\lambda) \sqrt{4c_1k_2^2 - b_1^2}}{b_1^2 - 4k_2^2c_1 \tanh^2(\lambda)} \tag{3.26}
\]

Solving the magnetic field component \(H_{y1}\) by applying the Maxwell’s equation (3.14);

\[
H_{y1} = \frac{i}{\omega \mu_0} \frac{\partial E_{x1}}{\partial y} \tag{3.27}
\]

The solution of linear equation is given by

\[
E_{x2} = De^{k_s y} \tag{3.28}
\]

Solving the magnetic field component \(H_{y2}\) by applying the Maxwell’s equation (3.14);

\[
H_{y2} = \frac{i}{\omega \mu_0} \frac{\partial E_{x2}}{\partial y} \tag{3.29}
\]
Substituting the solution of $E_x$ in equation (3.29) as

$$H_{y_2} = \frac{i}{\omega \mu_0} D e^{k_y} k_s$$

Applying the boundary conditions at $y = 0$ in each interfaces yield the following dispersion relation;

$$\frac{k_2}{k_s} = \frac{XY}{Z}$$

where,

$$X = -b_1(1 - u^2) + u\sqrt{1 - u^2}\sqrt{4c_1k_2^2 - b_1^2}$$
$$Y = b_1^2 - 4k_2^2c_1u^2$$
$$Z = Y(2b(1 - u^2) + \sqrt{4c_1k_2^2 - b_1^2}(-\sqrt{1 - u^2}u^2 + (\sqrt{1 - u^2})^3)) + 8k_2^2c_1u(1 - u^2)X$$

where $u = \tanh(\lambda)$ is the nonlinear parameter.

### 3.3.2 Numerical results

The microwave propagation in nonlinear ferroelectric/superconductor multi-layered homogeneous waveguide structure is studied. The variation of the propagation
3.3 Nonlinear wave propagation in ferroelectric / superconductor interfaces

The propagation constant with frequency is depicted in Fig. 3.5. The curves 1 and 2 correspond to the two values of the nonlinear term $u = \tanh(\lambda)$, i.e., 0.22, and 0.23. As seen in the Fig. 3.5, the propagation of electromagnetic wave in the structure is highly

![Figure 3.5: Variation of normalized propagation constant and with frequency for nonlinear waveguide structure at different values of nonlinear term.](image1)

![Figure 3.6: Variation of normalized attenuation constant with frequency for nonlinear waveguide structure at different values of nonlinear term.](image2)
nonreciprocal. In addition, it can be realized that the value of the nonlinear term increases, the propagation constant also increases.

From the Fig. 3.6, it can be seen that the attenuation constant increases with frequency for different values of the nonlinear terms. In addition, it displays the slight variation between the different values of nonlinear terms. This nonlinear property of the ferroelectric thin film can be used in many devices such as harmonic generators, modulators etc. which have a wide range of applications in many communication and radar systems.

3.4 Summary

The microwave propagation in linear and nonlinear ferroelectric/superconductor multi-layered waveguide structures was presented. Variation of the phase constant with the relative permittivity of the ferroelectric thin film material, which is a function of electric field is established. The dependence of permittivity on applied electric field is strong and results in high tunability. The dispersion relation in ferroelectric/ superconductor waveguide structure has been derived in closed form. In case of the nonlinear structure, increasing the nonlinear parameter results in an increase in the propagation constant.