Chapter 7

Superconducting coplanar waveguide structures

7.1 Introduction

Coplanar waveguide (CPW) has a wide range applications in RF and microwave integrated circuits due to low frequency dispersion, easy fabrication of shunt and series passive elements, easy surface mounting of external devices etc [95]. The coplanar waveguide structure introduced by C. P. Wen in 1969 consists of two slots printed on a dielectric substrate. The mode of electromagnetic wave propagation in coplanar waveguide structure is non-TEM at higher frequencies. Since coplanar waveguide can be constructed without a ground plane, this coplanar transmission geometry is widely used as characterization tool for ferroelectric thin film and substrate [88].

Ferroelectric materials are good candidate in microwave integrated circuits due to its tunability with electric field. [1, 2, 91, 96, 97]. Ferroelectric materials with
high dielectric constant can be incorporated in coplanar waveguide and therefore highly compact miniaturization can be designed with this combination. Moreover, its frequency agility, ultrashort response time, and the low cost of thin film samples are main features of the developed tunable microwave devices [98].

In recent years, the low surface resistance of superconducting thin films and applied dc field dependence of high dielectric constant of ferroelectric thin films have brought forth new class of high temperature superconducting coplanar waveguide with ferroelectric thin film as the substrate [4, 52, 54, 99]. High temperature superconducting coplanar waveguide as widely used circuit elements employed in filter application in combination with ferroelectric thin films. High dielectric constant of ferroelectric material such as SrTiO$_3$ is of particular important because of its crystalline compatibility with high temperature superconducting materials. It also provides the advantageous for power handling capability in microwave integrated circuits. In addition, CPW is a coupled slot and has large area of interface between metallization and dielectric. Due to the high dielectric constant of ferroelectric material, field enhances in regions closed to the ferroelectric thin film and hence losses in the metalization near such surface increases. This makes the replacement of normal metallic element with superconducting materials with very low loss is important. The high temperature superconducting thin film can easily be deposited on low loss tangent substrate MgO which provides good lattice matching to the perovskite ferroelectrics.

In this chapter, the microwave propagation characteristics for superconducting coplanar waveguide structure is studied using spectral domain method [17, 27, 28, 91]. The two fluid model is used for modeling superconducting materials where conductivity is a complex quantity with real part representing normal conductivity
and imaginary part for the superconducting electrons. The theory and method of analysis are explained in section 7.2. The numerical results are discussed in section 7.3.

### 7.2 Theory

![Figure 7.1: Cross sections of coplanar waveguide structure.](image)

The geometry of the superconducting coplanar waveguide structure under consideration is shown in Fig. 7.1. This structure consists of a superconducting strip of zero thickness, uniform and infinite in the $z$ direction, separated by a narrow gap with semi-infinite ground planes on either side of the strip. The coupled slot line is mounted on a bilayered substrate with an upper ferroelectric layer and a lower dielectric layer. The ferroelectric layer considered is SrTiO$_3$ having a dc electric field dependence for nonlinear permittivity given by Vendik et al. [2, 3, 9, 31, 82, 93]. The lower layer considered is MgO having dielectric constant $\varepsilon_1 = 10$ and thickness $500\mu$m. The upper cladding layer is assumed to be air. For taking the simplicity of
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derivation, a conductor backed CPW (ground plane) is considered for this analysis. Thickness of the substrate can be set large for obtaining the propagation properties of CPW’s with one side metalization (ground plane).

For the coplanar geometry, the electric and magnetic field components in the substrates and in the air media are hybrid in nature and can be expressed as superpositions of both TE and TM electromagnetic fields as explained in chapter 4. Incorporating the high temperature superconducting strip, the field equations can be written similar to the slotline structure discussed in previous chapter as:

\[
\begin{bmatrix}
\tilde{Y}_{11}^s & \tilde{Y}_{12}^s & \tilde{Y}_{13}^s \\
\tilde{Y}_{21}^s & \tilde{Y}_{22}^s & \tilde{Y}_{23}^s \\
\tilde{Y}_{31}^s & \tilde{Y}_{32}^s & \tilde{Y}_{33}^s \\
\end{bmatrix}
\begin{bmatrix}
\tilde{E}_{x1}^s \\
\tilde{E}_{x2}^s \\
\tilde{E}_{z1}^s \\
\end{bmatrix}
=
\begin{bmatrix}
\tilde{J}_{x1} \\
\tilde{J}_{x2} \\
\tilde{J}_{z1} \\
\end{bmatrix}
\] (7.1)

where

\[
\begin{bmatrix}
\tilde{Y}_{11}^s & \tilde{Y}_{12}^s & \tilde{Y}_{13}^s \\
\tilde{Y}_{21}^s & \tilde{Y}_{22}^s & \tilde{Y}_{23}^s \\
\tilde{Y}_{31}^s & \tilde{Y}_{32}^s & \tilde{Y}_{33}^s \\
\end{bmatrix}
\begin{bmatrix}
\tilde{P}_{11} \tilde{P}_{12} \tilde{P}_{13} \\
\tilde{P}_{21} \tilde{P}_{22} \tilde{P}_{23} \\
\tilde{P}_{31} \tilde{P}_{32} \tilde{P}_{33} \\
\end{bmatrix}
\begin{bmatrix}
\tilde{Y}_{11} \tilde{Y}_{12} \tilde{Y}_{13} \\
\tilde{Y}_{21} \tilde{Y}_{22} \tilde{Y}_{23} \\
\tilde{Y}_{31} \tilde{Y}_{32} \tilde{Y}_{33} \\
\end{bmatrix}
\] (7.2)

with matrix \( (P) \) defined as

\[
(P) \equiv \begin{bmatrix}
\tilde{P}_{11} & \tilde{P}_{12} & \tilde{P}_{13} \\
\tilde{P}_{21} & \tilde{P}_{22} & \tilde{P}_{23} \\
\tilde{P}_{31} & \tilde{P}_{32} & \tilde{P}_{33} \\
\end{bmatrix}
= \begin{bmatrix}
1 - \tilde{Y}_{11}^sZ_s & \tilde{Y}_{12}^sZ_s & \tilde{Y}_{13}^sZ_s \\
\tilde{Y}_{21}^sZ_s & 1 - \tilde{Y}_{22}^sZ_s & \tilde{Y}_{23}^sZ_s \\
\tilde{Y}_{31}^sZ_s & \tilde{Y}_{32}^sZ_s & 1 - \tilde{Y}_{33}^sZ_s \\
\end{bmatrix}
\] (7.3)

\( Y_{i,j}^s, i, j = 1, 2, \) are the admittance Green’s functions in the spectral domain, and \( Z_s \)
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is the surface impedance of the superconducting material given by \( Z_s = \left[ t_{sc} (\sigma_1 - j\sigma_2) \right]^{-1} \), with \( t_{sc} \) as the thickness of the strip, and \((\sigma_1 - j\sigma_2)\) the complex conductivity.

In order to obtain the dispersion properties, a set of basis functions are used, as detailed in chapter 4, to expand the unknown electric fields in the form

\[
\tilde{E}_z = \sum_{m=1}^{N} C_m \tilde{E}_{zm}(\alpha), \tilde{E}_x = \sum_{m=1}^{M} D_m \tilde{E}_{xm}(\alpha)
\]  

(7.4)

ensuring to satisfy the required edge boundary conditions described in chapter 4.

As before, the Galerkin method \([86]\) in the Fourier-domain along with Parseval’s theorem is used to solve the above set of equations. Using the root seeking process based on Müller method \([29, 30]\), the propagation parameters of the electromagnetic waves corresponding to the geometry considered is obtained.

7.3 Numerical results

The propagation characteristics of high temperature superconducting coplanar waveguide incorporated on a ferroelectric substrate is studied. The material parameters used on the typical values of high \( T_c \) superconducting system are critical temperature \( T_c = 92K \), penetration depth \( \lambda = 0.2 \times 10^{-6} m \), normal state conductivity (at \( T_c \)) \( \sigma_n = 3.9 \times 10^5 (\Omega m)^{-1} \).

The variation of normalized propagation constant and attenuation constant with the thickness of ferroelectric thin film at 12 GHz for various electric fields are shown in Fig. 7.2. The propagation characteristics shows considerable variation with the
thickness of the ferroelectric thin film. As the thickness of the ferroelectric thin film increases, both phase constant and attenuation constant increases. From Fig. 7.2a, it can be realized that the propagation constant decreases when the applied biasing electric field increases. This is due to the fact that the real part of permittivity of ferroelectric thin film decreases with increase in biasing field. The attenuation constant first increases with biasing electric field and the decreases with further increase in biasing field (Fig. 7.2b) following the behavior of imaginary part of permittivity with electric field.

The microwave propagation characteristics for superconducting coplanar waveguide structure as a function of frequency at slot width $s = 2\, \text{mm}$ is investigated. From the Fig. 7.3a, it can be realized that the propagation constant increases with frequency. With the increase in biasing electric field, the propagation constant decreases. The attenuation constant also increases with frequency as seen from Fig.
7.3 Numerical results

Figure 7.3: Variation of normalized (a) propagation constant and (b) attenuation constant with operating frequency for superconducting coplanar waveguide structure having slot-width \( s = 2 \text{mm} \) for different biasing electric field (in units of KV/cm)

7.3b. With the application of biasing electric field, the attenuation of the propagating wave can also be tuned as desired. Hence, it can be seen that the structure can be tuned over a considerable range of operating frequencies.

The variation of the normalized propagation constant and attenuation constant with temperature for superconducting coplanar waveguide structure at different electric fields is presented in Fig. 7.4. As seen in Fig. 7.4a, the propagation constant decreases with reduced temperature firstly and increases when the temperature approaches to critical temperature. As for the corresponding attenuation constant in Fig. 7.4b, it can be seen that the attenuation constant increases considerably near the transition temperature. The influence of ferroelectric material dominates at lower temperature while the effects of superconducting material dominates as the temperature approaches critical temperature.

The microwave propagation characteristics as a function of the superconducting
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Figure 7.4: Variation of normalized (a) propagation constant and (b) attenuation constant with reduced temperature for superconducting coplanar waveguide structure having slot-width $s = 2\text{mm}$ for different biasing electric field (in units of KV/cm).

Figure 7.5: Variation of normalized (a) propagation constant and (b) attenuation constant with thickness of HTS thin film for superconducting coplanar waveguide structure having slot-width $s = 2\text{mm}$ for different biasing electric field (in units of KV/cm).
film thickness at various electric fields is shown in Fig. 7.5. The thickness of the ferroelectric thin film is kept at $h_2 = 0.6 \, \mu m$. The propagation constant and attenuation constant slightly decrease with increase in the thickness of high temperature superconducting thin film. From, figures 7.5a and 7.5b, it can be realized that both propagation constant and attenuation constant decrease when the biasing electric field increases.

Figure 7.6: Variation of normalized (a) propagation constant and (b) attenuation constant with biasing electric field for superconducting coplanar waveguide structure having slot-width $s = 2mm$.

The dependence of propagation conditions on the electric field at constant strip width $w = 1mm$ and slot width $s = 2mm$ is given in Fig. 7.6. From the above results, it can be seen that the normalized propagation constant decreases continuously while the attenuation constant increases firstly and then decreases with electric field. Thus, the structure could find application in tunable microwave devices as the dependence of the permittivity of the ferroelectric materials on the applied voltage considerably influences the propagation characteristics of the device in the
microwave regime.

7.4 Summary

The electric field tuning characteristics of superconducting coplanar waveguide structure based on ferroelectric thin film has been theoretically analyzed using spectral domain method. Methodology of incorporating HTS film as the coplanar waveguide into the analysis is presented. The propagation constant and attenuation constant increases with operating frequency and thickness of the ferroelectric thin film. Dispersion of these propagation parameters on applied electric field has been established.