CHAPTER VI

SUMMARY OF RESULTS

Acceptance sampling provides statistical methods for taking decision upon the disposition of incoming raw materials or outgoing finished products. The decision is made based on the information obtained from the inspection of items sampled from the lots or batches of items submitted for taking decision. Single sampling plan by attributes is the simplest acceptance sampling tool which prescribes the number of items to be sampled for inspection, n, maximum number of nonconforming items in the sample, c. When the number of sampled items not conforming to the specified quality standards is maximum of c, the lot is accepted. Otherwise the lot is rejected. Determination of n and c for a given production condition under various considerations is the main problem of statistical research in designing single sampling plans by attributes.

Statistical distributions are used to model the production conditions. Hypergeometric, binomial and Poisson distributions are the frequently used statistical distributions in this respect. When the production of items in a manufacturing industry is done in more than one unit / stream / shift, the lots formed with the items
may be heterogeneous with respect to quality variation in the items. When items are produced in two different units / streams / shifts, each lot may consist of two homogeneous sublots. The production condition in this situation may be represented appropriately by a mixture of two distributions. The component distributions in the mixture distribution may belong to different families of distributions.

Sampling plans designed for inspecting homogeneous lots cannot be implemented for inspection of heterogeneous lots. This work has considered design and evaluation of single sampling plans by attributes for the inspection of heterogeneous lots when the production condition is described by a mixture of two binomial distributions defined in (3.5) as

\[
P(X = x| φ, n_1, n_2, a_1, a_2) = φ \binom{n_1}{x} a_1^x (1-a_1)^{n_1-x} + (1-φ) \binom{n_2}{x} a_2^x (1-a_2)^{n_2-x},
\]

The plan parameters under the conditions of this binomial bi-mixture distribution are determined for three different scenarios:

(i) decisions upon the lots are made based on sample information,

(ii) decisions upon the lots are made based on sample information and prior information about \( a_1 \) and \( a_2 \) when \( φ \) is known, and

(iii) decisions upon the lots are made based on sample information and prior information about \( a_1, a_2 \) and \( φ \).

For Scenario-1, the sampling plans are designed in the classical or non-Bayesian method whereas in the other two scenarios the plan parameters are determined applying Bayesian method.
In Scenario-1, the derivation of the OC function of the SSP given in (3.7) is straightforward. But, evaluation of its values for each given set of values of n, c, φ and p is difficult. The lot proportion nonconforming, p, is a function of φ, a1, and a2, where a1 and a2 are unknown. An algorithm is developed to determine the values of a1 and a2 based on their relationship with p and φ. The value of a1 can be chosen for each set of p and φ from anyone of the three possible ranges

(i) (0, 1) when \( L_1 < 0 \) and \( L_2 > 1 \)

(ii) (0, \( L_2 \)) when \( L_1 < 0 \)

(iii) \( (L_1, L_2) \)

where \( L_1 = \frac{p - (1 - \phi)^2}{\phi^2} \) and \( L_2 = \frac{p}{\phi^2} \). It is found from empirical studies that a1 may be chosen as the mid-point of its range so that the resultant sampling plan will protect the interests of both producer and consumer. Another component parameter a2 may be calculated as \( a_2 = \frac{p - \phi^2 a_1}{(1 - \phi)^2} \).

Empirical analysis of behaviour of OC curves drawn for several combinations of values of n, c, φ and p highlights the following properties of the SSP

(i) the lot acceptance probability decreases when the lot quality deteriorates,

(ii) the sampling plans with smaller sample size protect the producer against rejection of good quality lots and the plans with large values of n safeguard the interests of consumer,

(iii) in contrast to the influence of n, the sampling plans with large acceptance number increases the lot acceptance probability safeguarding the interests of producer. But, the sampling plans with smaller values of c are consumer-oriented plans,
(iv) the lot acceptance probability corresponding to \( \phi \) and 1-\( \phi \) are equal when other characteristics are fixed,

(v) the sampling plans when \( \phi \) is close to 0.5 is both producer-oriented as well as consumer-oriented, and

(vi) the lot acceptance probability at \( p \approx c/n \) of SSPs with same \( n \) and \( c \) is same for all values of \( \phi \).

An algorithm is developed to determine the values of \( n \) and \( c \) for given values of AQL (\( p_1 \)), LQL (\( p_2 \)), producer’s risk (\( \alpha \)) and consumer’s risk (\( \beta \)) applying the search procedure proposed by Guenther (1969) so that the resultant plan will provide protection to both producer and consumer at prespecified levels. Tables of \( n \) and \( c \) are constructed for different combinations of (\( p_1, \alpha, p_2, \beta \)) and \( \phi = 0.1 (0.1) 0.5 \). The plan parameters remain same for \( \phi \) and 1-\( \phi \). The exact producer’s risk due to binomial bi-mixture SSP is smaller than the risk due to binomial SSP. Also, the total risk of producer and consumer under binomial bi-mixture plan is smaller than the respective risk under binomial plan.

There may occur errors while classifying the items into conforming or nonconforming during sampling inspection. Occurrence of both kinds of errors may affect exact amount of producer’s and for consumer’s risks. When the probabilities for the occurrence of both kinds of inspection errors are known, SSPs may be designed taking into account of occurrence of inspection errors. In this respect, the OC function of binomial bi-mixture SSP considering the occurrence of inspection errors is derived. Empirical analysis of OC function indicates that the increase in the chances for the occurrence of the first kind of inspection error reduces consumer’s risk whereas the occurrence of the second kind of inspection error favours producer. In
general, inclusion of probabilities of occurrences of inspection errors in design of SSP increases the number of items to be sampled for inspection in order to ensure the specified level of protection to both producer and consumer.

In Scenario-2, it is considered that the parameters $a_1$ and $a_2$ are random variables and decisions upon the lots are taken based on sample information and the information obtained about these parameters prior to sampling. When variation in the quality of products between lots is considerably more than variation within lots, the above consideration becomes meaningful. Employing independent beta prior distributions to $a_1$ and $a_2$ as $a_1 \sim \text{Beta}(g, h)$ and $a_2 \sim \text{Beta}(s, t)$, the predictive distribution of the number of nonconforming in the sample is found to be a mixture of two Polya distributions. The OC function of the Bayesian SSP under the conditions of Polya bi-mixture distribution is a function of $n, c, \varphi, p, g$ and $s$.

Empirical studies of behaviour of OC curves drawn for various combinations of $n, c, \varphi, p, g$ and $s$ show that the pattern of influence of $n, c, \varphi$ and $p$ is similar to the kind of influence observed for binomial bi-mixture SSP. It is interesting to note that the Polya bi-mixture sampling plans are both producer-oriented as well as consumer-oriented plans when $g$ and $s$ increase individually. The OC curves drawn for various values of $g$, when the values of other characteristics are fixed, intersect at $p \approx 2\varphi c/n$. But, the OC curves drawn for various values of $s$ intersect at $p \approx 2(1-\varphi)c/n$.

An algorithm is developed to determine the plan parameters $n$ and $c$ applying the search procedure for specified values of other characteristics which influence the values of OC function. The plans designed in this manner will protect both producer and consumer at specified levels of risks. When $g$ and $s$ are equal, the sampling plans are same for $\varphi$ and $1-\varphi$. Polya bi-mixture SSP reduces the exact amount of risks of
both producer and consumer when compared to Polya SSP. Similarly, Polya bi-
mixture SSP provides lesser risks to the producer and reduces the total sampling risk
when compared to binomial bi-mixture SSP. Hence, Polya bi-mixture SSP is more
suitable for implementation under the conditions of binomial bi-mixture distribution
with known mixing proportion when prior information is available about the model
parameters.

Behaviour in the performance of Polya bi-mixture SSPs designed considering
the occurrences of both kinds of inspection errors is as similar to the case of binomial
bi-mixture SSPs.

As a generalization of Scenario-2, Bayesian sampling plans are designed under
the conditions of binomial bi-mixture distribution in Scenario-3, employing
beta (b, m), beta (g, h) and beta (s, t) prior distributions to $\phi$, $a_1$ and $a_2$ respectively.
The predictive distribution of the number of nonconforming items in the sample is
derived as a mixture of two beta-Polya distributions. The OC function of the
corresponding SSP is a function of $n$, $c$, $p$, $b$, $m$, $g$ and $s$.

A detailed empirical analysis of OC curves shows that $n$, $c$, $p$, $g$ and $s$ have
similar kind of influence upon the OC function as observed in the preceding scenarios.
But, the OC curves drawn for various values of $g$ intersect at $p \approx 2bc/n(b+m)$. The
curves drawn for different values of $s$ intersect at $p \approx 2mc/n(b+m)$. Interestingly, the
OC curves drawn for various values of $b$ intersect at $p \approx c/n$ and the plans with large
values of $b$ protect both producer and consumer. Another hyperparameter $m$ also has
similar impact upon the OC function as like ‘b’.

An algorithm is evolved to determine the plan parameters for given
specifications applying the search procedure. When the hyperparameters $g$ and $s$ are
equal, the sampling plans are same for the interchanging values of b and m. The implementation of Beta-Polya bi-mixture SSP reduces the risk of producer as well as total risk compared to the binomial and Polya bi-mixture SSPs.

Empirical studies conducted on studying the impact of occurrence of both kinds of inspection errors reveal that the effect of inspection errors is similar to the cases of binomial bi-mixture plans and Polya bi-mixture plans. Design of beta-Polya bi-mixture SSP considering the occurrence of inspection errors show that more number of items are to be inspected for providing ensured protection to producer and consumer.

Computer programs developed in MATLAB and R are very useful to determine plan parameters and to generate points for drawing OC curves in all the three scenarios.

**Scope for Further Research**

Bayesian plans may be determined employing other suitable informative prior distributions and non-informative prior to the unknown parameters in the binomial bi-mixture distribution.

Attempts on determination of plan parameters may be made subject to minimization of Bayes risk when the loss function is defined using variations costs involved in carrying out sampling inspection.

Applications of the results obtained in this are limited with mixture of two distributions. Separate studies are required when the production condition is represented by a mixture of three or more number of distributions. The present
methodology for designing the sampling plans may be followed. However, a separate algorithm has to be developed for computing the values of the OC function.

Further the work may be extended to design double sampling plans under the conditions of binomial bi-mixture distributions.