CHAPTER 5

STRONG TERAHertz RADIATION GENERATION BY BEATING OF TWO X-MODE SPATIAL TRIANGULAR LASERS IN A MAGNETIZED PLASMA

Chapter 5 is devoted to study a scheme of resonant THz radiation generation by beating of two extraordinary lasers (with triangular envelopes in space) copropagating in a plasma having periodically modulated density ripples. Again, the issue of high amplitude and power of THz radiation are addressed by employing triangular profiles and x-mode polarization of lasers. Both the beating lasers and generated THz radiation have same state of polarization because the terahertz emission is maximized when the polarization of laser beams and the terahertz are aligned [42]. Applied dc static magnetic field in transverse direction plays an important role to improve the directionality and tunability of THz radiation. Two copropagating x-mode triangular shaped \([\omega_1, \vec{k}_1]\) and \([\omega_2, \vec{k}_2]\) (Figure 5.1) exert a nonlinear ponderomotive force at frequency \(\omega = \omega_1 - \omega_2\) and wave vector \(\vec{k} = \vec{k}_1 - \vec{k}_2\) on plasma electrons. Velocity perturbation due to ponderomotive forces couples with density ripples of appropriate periodicity and excites a nonlinear current. The nonlinear current can excite THz radiation if resonance conditions are simultaneously satisfied. In Sec. 5.1, expressions of ponderomotive force, density perturbation and nonlinear current density are derived. The amplitude and efficiency of THz wave are calculated in Sec. 5.2. Results are discussed in the Sec. 5.3.

5.1. Nonlinear current due to beating of lasers

Consider a laser produced rippled plasma of density \(n \equiv n_0 + n', n' = n_{\alpha 0} e^{i\alpha z}\) with static magnetic field \(\vec{B}_0\) in \(\hat{x}\) direction, where \(n_{\alpha 0}\) is the amplitude of ripple and \(\alpha\) as the wave vector of density...
Figure 5.1: Schametic diagram of terahertz radiation generation by beating of two x-mode lasers.

ripples. Two x-mode spatial triangular lasers co-propagate in plasma along $\hat{z}$ - direction having electric field profile [42]:

$$
\vec{E}_j = \left( \hat{y} - \frac{\varepsilon_{jy}}{\varepsilon_{jz}} \hat{z} \right) E_{00} e^{-i(\omega_j - k_j z)}
$$

(5.1)

where $E_{00}^2 = A_0^2 \left( 1 - |y/a_0|^2 \right)$, for $|y/a_0| < 1$ and zero otherwise; $j = 1, 2$;

$k_j = \frac{\omega_j}{c} \left( 1 - \frac{\omega_j^2}{\omega_p^2} \right)^{1/2} / \omega_p$;

$a_0$ is the beam width parameter of lasers, $\omega_h^2 = \omega_p^2 + \omega_c^2$,

$\omega_p^2 = 4m_0 e^2 / m$ and $\omega_c = eB_0 / m$ are upper hybrid frequency, electron plasma frequency and cyclotron frequency, respectively; $-e$ and $m$ are the electronic charge and mass; and

$\varepsilon_{jz} = \left[ 1 - \omega_p^2 / (\omega_j^2 - \omega_c^2) \right]$ & $\varepsilon_{jy} = -i[\omega_c \omega_p^2 / \omega_j (\omega_j^2 - \omega_c^2)]$ are components of the dielectric tensor $\varepsilon_j$. Extraordinary mode or x-mode is the natural electromagnetic mode of magnetized plasma. If the electric field of propagating laser $\vec{E}_j = \hat{y} E_{00} e^{-i(\omega_j - k_j z)}$ in a plasma is perpendicular to applied dc magnetic field $\vec{B}_0 = B_0 \hat{x}$, then the plasma electron motion will also be affected by $-e(\vec{v} \times \vec{B}_0)$.
force due to applied magnetic field and the dispersion relation will be changed. As a result, a longitudinal component develops and laser becomes partly longitudinal and partly transverse. These two components are related to each other by the expression $E_z = -(\varepsilon_{jy} / \varepsilon_{jz})E_y$. This expression has the signature of applied dc magnetic field [68]. These extraordinary modes of electromagnetic waves are extensively utilized in magnetized plasma for various purposes like parametric instabilities, electron acceleration, harmonic generation etc. Here, lasers impart oscillatory velocity to plasma electrons in $\hat{y}$ and $\hat{z}$ direction, given by

$$v_{jy} = \frac{e}{m(\omega_j^2 - \omega_e^2)} \left[ i \omega_j + \omega_e \frac{\varepsilon_{jy}}{\varepsilon_{jz}} \right] E_{0y} e^{-i(\omega_j - k_z) z}$$  \hspace{1cm} (5.2a)$$

$$v_{jc} = \frac{e}{m(\omega_j^2 - \omega_e^2)} \left[ \omega_e + i \omega_j \frac{\varepsilon_{jy}}{\varepsilon_{jz}} \right] E_{0e} e^{-i(\omega_j - k_z) z}$$  \hspace{1cm} (5.2b)$$

Lasers beat together and exert a ponderomotive force $\vec{F}_p = F_{py} \hat{y} + F_{pz} \hat{z}$ on plasma electron at frequency $\omega = \omega_1 - \omega_2$ and wave vector $\vec{k} = \vec{k}_1 - \vec{k}_2$. The components of the ponderomotive force $F_{py}$ and $F_{pz}$ are as follows:

$$F_{py} = \frac{e^2}{2m} \left[ \frac{1}{(\omega_1^2 - \omega_e^2)(\omega_2^2 - \omega_e^2)} \left( \frac{2}{a_0} \left( i \omega_1 + \omega_e \frac{\varepsilon_{1y}^2}{\varepsilon_{1z}} \right) \left( -i \omega_2 + \omega_e \frac{\varepsilon_{2y}^2}{\varepsilon_{2z}} \right) - ik_2 \left( \omega_e + i \frac{\varepsilon_{1y}}{\varepsilon_{1z}} \right) \left( -i \omega_2 + \omega_e \frac{\varepsilon_{2y}^2}{\varepsilon_{2z}} \right) \right) \right]$$

$$+ \frac{k_2 \left( \omega_e + i \frac{\varepsilon_{1y}}{\varepsilon_{1z}} \right) \left( \omega_e - i \frac{\varepsilon_{2y}}{\varepsilon_{2z}} \right)}{\omega_2 (\omega_1^2 - \omega_e^2)^2} F_{0y} e^{-i(\omega - k_z z)}$$

$$+ \frac{k_1 \left( \omega_e - i \frac{\varepsilon_{2y}}{\varepsilon_{2z}} \right) \left( \omega_e - i \frac{\varepsilon_{2y}}{\varepsilon_{2z}} \right)}{\omega_1 (\omega_2^2 - \omega_e^2)} F_{0e} e^{-i(\omega - k_z z)}$$

$$F_{pz} = \frac{e^2}{2m} \left[ \frac{1}{(\omega_1^2 - \omega_e^2)(\omega_2^2 - \omega_e^2)} \left( \frac{2}{a_0} \left( i \omega_1 + \omega_e \frac{\varepsilon_{1y}^2}{\varepsilon_{1z}} \right) \left( -i \omega_2 + \omega_e \frac{\varepsilon_{2y}^2}{\varepsilon_{2z}} \right) + ik_2 \left( \omega_e + i \frac{\varepsilon_{1y}}{\varepsilon_{1z}} \right) \left( -i \omega_2 + \omega_e \frac{\varepsilon_{2y}^2}{\varepsilon_{2z}} \right) \right) \right]$$

$$+ \frac{k_2 \left( \omega_e + i \frac{\varepsilon_{1y}}{\varepsilon_{1z}} \right) \left( \omega_e - i \frac{\varepsilon_{2y}}{\varepsilon_{2z}} \right)}{\omega_2 (\omega_1^2 - \omega_e^2)^2} F_{0y} e^{-i(\omega - k_z z)}$$

$$+ \frac{k_1 \left( \omega_e - i \frac{\varepsilon_{2y}}{\varepsilon_{2z}} \right) \left( \omega_e - i \frac{\varepsilon_{2y}}{\varepsilon_{2z}} \right)}{\omega_1 (\omega_2^2 - \omega_e^2)} F_{0e} e^{-i(\omega - k_z z)}$$
\[ F_{pz} = \frac{e^2}{2m} \left[ \frac{1}{(\omega_1^2 - \omega_c^2)(\omega_2^2 - \omega_c^2)} \left\{ \frac{1}{a_0(a_0 - 1)} \left( i\omega_1 + \omega_c \frac{e_{1yz}}{e_{1zz}} \right) \left( \omega_c - i\omega_2 \frac{e_{2yz}}{e_{2zz}} \right) + \frac{1}{a_0(a_0 - 1)} \right\} \right. \]

\[
\left. \left( -i\omega_2 + \omega_c \frac{e_{2yz}}{e_{2zz}} \right) \left( \omega_c + i\omega_1 \frac{e_{1yz}}{e_{1zz}} \right) - ik \left( \omega_c + i\omega_1 \frac{e_{1yz}}{e_{1zz}} \right) \left( \omega_c - i\omega_2 \frac{e_{2yz}}{e_{2zz}} \right) \right] \right) \] (5.4)

The ponderomotive force drives space charge oscillation at \( \omega = \omega_1 - \omega_2 \) and wave vector \( \vec{k}_1 - \vec{k}_2 \).

Assuming the potential of space charge mode to be \( \phi \), the oscillatory velocity of electron due to space charge mode along with ponderomotive force in the presence of static magnetic field can be expressed as follows:

\[ v_y = \frac{1}{m(\omega^2 - \omega_c^2)} \left[ e\omega_y \nabla \phi - \omega_c F_{py} + i\omega F_{pz} \right] \] (5.5a)

\[ v_z = \frac{1}{m(\omega^2 - \omega_c^2)} \left[ e\omega_z (\nabla \phi) + \omega_z F_{pz} + i\omega F_{py} \right] \] (5.5b)

The nonlinear velocity given by Eq. (5.5) along with continuity equation provide density perturbation \( n = n^L + n^{NL} \), where

\[ n^L = \frac{1}{4\pi e} k^2 \chi \phi, \] (5.6)

\[ n^{NL} = \frac{n_0 k}{m\omega(\omega^2 - \omega_c^2)} \left[ i\omega F_{pz} - \omega_z F_{py} \right] \] (5.7)

and \( \chi = -\omega_p^2/(\omega^2 - \omega_c^2) \). Linear density perturbation (\( n^L \)) is induced self consistently by space
charge field and nonlinear density perturbation \( n^{NL} \) is the consequence of ponderomotive force. Here, density perturbation is assumed to be small as compared to the density ripple. Substituting \( n = n^L + n^{NL} \) in the Poisson’s equation \( \nabla^2 \phi = 4\pi ne \), we obtain

\[
\varepsilon \phi = -\frac{4\pi e}{k^2} n^{NL} \tag{5.8}
\]

where, \( \varepsilon = 1 + \chi \). Combining Eqs. (5.6-5.8), we have,

\[
\phi = -\frac{4\pi ne}{m k \omega (\omega^2 - \omega_h^2)} \left[ i \omega F_{pc} - \omega \chi F_{py} \right] \tag{5.9}
\]

Substituting this value of \( \phi \) in Eq. (5.5), we obtain the oscillatory velocity components of plasma electrons:

\[
v_y = \frac{1}{m(\omega^2 - \omega_h^2)} \left[ -\omega_e F_{py} + i \omega F_{pc} \right] \tag{5.10a}
\]

\[
v_z = \frac{\omega_e}{m(\omega^2 - \omega_h^2)} F_{pc} + i \frac{(\omega^2 - \omega_p^2)}{m(\omega^2 - \omega_h^2)} F_{py} \tag{5.10b}
\]

Oscillations at \( (\omega, \tilde{k}_1 - \tilde{k}_2) \) in the presence of density ripple \( n_{\omega_0} e^{i\alpha} \) excite nonlinear current at \( (\omega, \tilde{k}_1 - \tilde{k}_2 + \tilde{\alpha}) \) which can be written as

\[
\vec{J}^{NL} = -\frac{1}{2} n_{\omega_0} \vec{e}_\omega \vec{e}^{i\alpha} \tag{5.11}
\]

This oscillatory current is the source for the emission of THz radiation at the beating frequency \( \omega \) which is the same as that of the ponderomotive force but its wave number is different. For
strong THz radiation, plasma density ripples should be periodic, otherwise \( \vec{k}(= \vec{k}_1 - \vec{k}_2 + \vec{a}) \) will exhibit non-periodic behavior; resonance condition can’t be achieved and maximum energy transfer will not take place and consequently a weak field THz radiation will be generated.

### 5.2. THz radiation amplitude

Following wave equation is solved to find the amplitude of the THz wave

\[
- \nabla^2 \vec{E} + \hat{\nabla} \cdot (\hat{\nabla} \vec{E}) = \frac{4 \pi i \omega}{c^2} \vec{J} + \frac{\omega^2}{c^2} (\vec{E} \cdot \vec{E})
\]  
(5.12)

where, \( \varepsilon \) is the plasma permittivity tensor at \( \omega \). Taking fast phase variations in the electric field profile of terahertz radiation as \( \vec{E} = \vec{A}(z)e^{-i(\omega t - kz)} \), the wave equation [Eq. (5.12)] governing the propagation of terahertz waves can be splitted into \( \hat{y} \) and \( \hat{z} \) components as follows:

\[
A_z = - \frac{4 \pi i}{\omega \varepsilon_{zz}} J_{z}^{NL} \frac{\varepsilon_{zy}}{\varepsilon_{zz}} A_y 
\]  
(5.12a)

\[
k^2 A_y - 2ik \frac{\partial A_y}{\partial z} - \frac{\partial^2 A_y}{\partial z^2} = + \frac{4 \pi i \omega}{c^2} J_y^{NL} + \frac{\omega^2}{c^2} \left( \varepsilon_{yy} A_y + \varepsilon_{yz} A_z \right)
\]  
(5.12b)

The excited THz mode will have same polarization state as the beating lasers. It will have both \( y \)- and \( z \)- components of electric field corresponding to its X-mode nature. These components will be related by Eq. [5.12(a)]. Xie et al. [118] also observed that THz emission will be maximized when the polarization of laser beams and the THz are aligned. By rearranging Eq. [5.12(a)] and Eq. [5.12(b)], we obtained the equation governing transverse component \( A_y \) of THz radiation. Longitudinal component of THz can be calculated by Eq. [5.12(a)]
From Eq. (5.13), one can observe that resonant terahertz radiation generation demands to satisfy the following dispersion relation for exact phase matching condition in rippled magnetized plasma which provides the periodicity of rippled structure and suggests that the maximum energy transfer from beating lasers to THz radiation will take place at resonance condition:

\[
k^2 - \frac{\omega^2}{c^2} \varepsilon_{yy} + \frac{\omega^2}{c^2} \frac{\varepsilon_{yz} \varepsilon_{zy}}{\varepsilon_{zz}} = 0
\]  

(5.14)

Substituting the phase matching condition in Eq. (5.13), we obtain the amplitude of THz radiation as follows:

\[
|A_y| = \frac{1}{4k} \left( \frac{n_{a0}}{n_0} \omega \frac{\omega_p^2}{\omega^2 - \omega_h^2} \right)^{1/2} \left[ \left( \frac{\omega^2}{\omega} - \frac{1}{\varepsilon_{zz} \varepsilon_{yy}} \right) F_{py} + \left( \frac{\varepsilon_{yz} \varepsilon_{zy}}{\varepsilon_{zz}} \right) F_{pz} \right]
\]

(5.15)

The normalized amplitude of terahertz radiation can be written as follows:

\[
\frac{|A_y|}{A_0} = \left( \frac{1}{4k} \frac{n_{a0}}{n_0} \frac{1}{\omega (\omega^2 - \omega_h^2)} \right)^{1/2} \left[ \left( \frac{\omega^2}{\omega} - \frac{1}{\varepsilon_{zz} \varepsilon_{yy}} \right) f_{py} + \left( \frac{\varepsilon_{yz} \varepsilon_{zy}}{\varepsilon_{zz}} \right) f_{pz} \right]
\]

(5.16)

where, \( f_{pc} = iF_{pc} \) and \( f_{py} = iF_{py} \). Eq. (5.16) reveals that the normalized terahertz amplitude is directly proportional to the normalized amplitude of density ripples \( n_{a0}/n_0 \), thus, THz field increases on increasing ripple amplitude. Its explanation lies in Eq. (5.11); higher the ripple amplitude, greater the number of electrons involving in the generation of oscillatory nonlinear current. Higher number of charge carriers results into higher nonlinear current (\( \tilde{J}^{NL} \)) leading to
more efficient THz radiation. The phase matching condition
\[ \alpha = (\omega/c) \left| \varepsilon_{yy} + (\varepsilon_{xy} \varepsilon_{yx})^{1/2} - 1 \right| \]
for resonant excitation of THz radiation provides the estimate of periodicity of rippled structure for maximum energy transfer in this process. In Figure 5.2, normalized periodicity factor \((c \alpha/\omega_p)\) of density rippled structure is plotted as a function of normalized THz frequency \((\omega/\omega_p)\) and normalized cyclotron frequency \((\omega_c/\omega_p)\). Periodicity factor \((\alpha)\) decreases with THz wave frequency, increases with applied magnetic field and achieves maximum value as THz frequency \((\omega)\) approaches toward resonance value \(\omega \sim \omega_c\). The wavelength \((\lambda = 2\pi/\alpha)\) is inversely proportional to \(\alpha\), thus, one can conclude that steep ripples at closer distances should be constructed for resonant excitation of THz radiation. The reason behind the requirement of smaller periodicity factor \(\alpha\) (or larger ripple wavelength \(\lambda\)) for the excitation of high frequency THz waves can be traced in resonance condition \(\vec{k} = \vec{k}_1 - \vec{k}_2 + \vec{\alpha}\) and \(\omega = \omega_1 - \omega_2\). To increase \(\omega\), one has to increase \(\omega_1\) by keeping \(\omega_2\) fixed i.e. \(\Delta \omega = \Delta \omega_1\). Wave vectors \(\vec{k}\) and \(\vec{k}_1\) change corresponding to changes in \(\omega\) and \(\omega_1\) respectively [keeping \(\vec{k}_2\) fixed]. Thus, \(\Delta \vec{k} = \Delta \vec{k}_1 + \Delta \vec{\alpha}\). We know that for x-mode electromagnetic wave, we have
\[ \left. \frac{dk}{d\omega} \right|_{\text{high frequency}} > \left. \frac{dk}{d\omega} \right|_{\text{low frequency}} \]
Thus, for equal change in frequency, change in wave vector for high frequency x-wave will be greater than change in wave vector for low frequency x-mode wave. Since, beating laser \((\omega_1, \vec{k}_1)\) and excited THz radiation \((\omega, \vec{k})\) have same state of polarization [x-mode polarization]. In the present scheme, thus, for equal change in frequency \(\Delta \omega = \Delta \omega_1\) we have \(\Delta \vec{k} < \Delta \vec{k}_1\). So, we can
conclude that periodicity factor $\alpha$ should be reduced [corresponding to $\Delta \vec{k} = \Delta \vec{k}_i + \Delta \vec{\alpha}$] to enhance the frequency of excited THz radiation.

Figure 5.2: Plot of the normalized ripple factor $\alpha c / \omega_p$ as a function of the normalized THz wave frequency $\omega / \omega_p$ and normalized cyclotron frequency $\omega_c / \omega_p$.

Figure (5.3a) exhibits the variation of normalized THz wave amplitude ($A_y / 0.15A$) as a function of THz frequency ($\nu$) and applied magnetic field $B_0$. Two mountain ranges of high THz amplitude excitation are observed which are corresponding to two propagating frequency regimes of extra ordinary electromagnetic wave in a plasma. These two frequency regime are given by (i) $\omega_h < \omega < \omega_h$ [Region II
of Figure (5.3b)] and (ii) \( \omega > \omega_R \) [Region IV of Figure (5.3b)], where 
\[
\omega_L = 1/2[ -\omega_c + \sqrt{\omega_c^2 + 4\omega_p^2}] \quad \text{&} \quad \omega_R = 1/2[ \omega_c + \sqrt{\omega_c^2 + 4\omega_p^2}].
\]
These two mountains are separated by a regime of frequency [Region III of Figure (5.3b)] in which \( \omega^2/k^2 \) becomes negative and propagation of wave stopped.

Figure 5.3a: Plot of the normalized THz amplitude \( A_y / 0.15 A_0 \) as a function of the THz frequency \( \nu(THz) \) and applied static magnetic field \( B_c \). Other normalized parameters are \( a_0, \omega_p/\omega_c = 5, z, \omega_p/\omega_c = 1, \)
\[
y/a_0 = 0.05, v_1 = 0.3 c, n_{a_0}/n_0 = 0.3.
\]
Peaks of both amplitude mountains are corresponding to resonance conditions which occur at \( \omega \sim \omega_h \) and \( \omega \sim \omega_R \), respectively. The frequency of THZ radiation having maximum amplitude shifts towards higher end of frequency as applied magnetic field increases due to enhanced value \( \omega_h \) and \( \omega_R \). Similar observations can be drawn from Figure (5.4) and Figure (5.5) in which
normalized amplitude of THz radiation is plotted as a function of THz frequency and normalized distance travelled in z-direction. THz radiation of frequency 1 THz, (1.1 THz & 1.2 THz), (1.2 THz & 1.3 THz) and (1.3 THz & 1.5 THz) are achieved by applying dc magnetic field ($B_0$) 0 kG, 107 kG, 178 kG and 285 kG, respectively. Thus, maximum energy transfer from beating lasers to THz radiation takes place at resonance condition and frequency of THz can be tuned by changing applied magnetic field. As the amplitude of THz radiation enhances in z-direction thus amplitude of excited THz radiation can also be tuned by varying plasma length.
Figure 5.4: Plot of the normalized THz amplitude \( \frac{A_y}{0.15 A_0} \) as a function of the THz wave frequency \( \nu \) (THz) and normalized transverse distance \( z\omega_p / c \) at different normalized cyclotron frequency (a) \( \omega_c / \omega_p = 0.0 \), (b) \( \omega_c / \omega_p = 0.3 \). Other normalized parameters are \( y/a_0 = 0.05, \nu_c = 0.3c, n_{e0}/n_0 = 0.3 \).
Figure 5.5: Plot of the normalized THz amplitude \( \frac{A_y}{0.15 A_0} \) as a function of the THz wave frequency \( \nu \) (THz) and normalized transverse distance \( z \omega_p / c \) at different normalized cyclotron frequency (c) \( \omega_c / \omega_p = 0.5 \), (d) \( \omega_c / \omega_p = 0.8 \). Other normalized parameters are \( y/a_0 = 0.05, v_i = 0.3c, n_{\omega_0}/n_0 = 0.3 \).
It can be seen from Figure 5.6 that THz amplitude has maximum value on the axis (\( y/a_0 = 0 \)) and decreases as one moves off axis which can be attributed to the maximum value of ponderomotive force on the axis. Thus, radiation emitted in the present scheme using x-mode triangular lasers is more collimated as compared to other schemes [118]. The amplitude of excited THz radiation is depending on the ponderomotive force exerted by the laser beams in the plasma [as shown in Eq.(15)]. As laser beams propagating in z direction has transverse spatial dependency (triangular shaped) along y direction, the ponderomotive force will also have transverse dependency.

Figure 5.6: Plot of the normalized THz amplitude \( (A_y/0.15A_0) \) as a function of the normalized transverse distance \( z\omega_p/c \) and normalized beam width \( y/a_0 \). Other normalized parameters are

\[
v_1 = 0.3c, \omega_1/\omega_p = 12, \quad n_{a0}/n_0 = 0.3, \omega_c/\omega_p = 0.3, a_0\omega_p/c = 5, \omega/\omega_p = 1.1.
\]
Hence this behavior is appearing in our result showing the profile of THz radiation propagating in z direction and having transverse dependency. It can also be explained in another way: if we choose a particular value of y, and solve Eq.(15), we will have the variation of $A_y$ as a function of longitudinal direction z. Now, if we choose another value of y, then we will again find different 1 D variation of $A_y$ because of different value of ponderomotive force (which is dependent on transverse direction y). Hence due to direct effect of laser profiles and ponderomotive force, excited THz radiation amplitude $A_y$ will also exhibit the transverse dependency.

Finally, we calculated the efficiency of the present scheme based on the energies of the lasers and emitted radiation. The average electromagnetic energy stored in unit volume in electric and magnetic fields [42, 111] yields

$$W_{pump} = \frac{1}{16\pi} \varepsilon_0 a_0 E_0^2$$  \hspace{1cm} (5.17)

$$W_{THz} = \varepsilon_0 \left[ \frac{1}{4k} \frac{n}{n_0} \frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_h^2} ec^2 \left( i \left( \frac{\omega^2 - \omega_p^2}{\omega} - \frac{\varepsilon_{xy}}{\varepsilon_{xx}} \omega \right) F_{py} + \left( \omega - \frac{\varepsilon_{zy}}{\varepsilon_{zz}} \right) F_{pz} \right) \right]^2$$  \hspace{1cm} (5.18)

Here, $W_{pump}$ is the average energy density of the pump lasers of triangular shape. The ratio of THz energy to pump energy gives the efficiency $\eta$ as below

$$\eta = \frac{W_{THz}}{W_{pump}} = \frac{|A_z|^2}{|A_0|^2}$$

$$\eta = \left[ \frac{1}{4k} \frac{n}{n_0} \frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_h^2} ec^2 \left( i \left( \frac{\omega^2 - \omega_p^2}{\omega} - \frac{\varepsilon_{xy}}{\varepsilon_{xx}} \omega \right) F_{py} + \left( \omega - \frac{\varepsilon_{zy}}{\varepsilon_{zz}} \right) F_{pz} \right) \right]^2$$  \hspace{1cm} (5.19)
The efficiency of present scheme is proportional to the square of ripple amplitude, which can be explained on the basis of more charge carriers involved in excitation of nonlinear current responsible for the generation of THz radiation. The efficiency is also proportional to $z^2$ which can be explained as follows: as beating lasers propagate in $z$-direction, they encounter more and more periodic density ripples; higher the number of encountered periodic density ripples better will be the efficacy of three wave coupling required for THz radiation generation due to efficient momentum transfer (from ripple periodicity to nonlinear current $\vec{J}_{NL}$ to overcome the mismatch in momentum). At the same time, larger number of electrons due to higher number of density ripples will produce larger nonlinear current. The efficiency of THz radiation generation is plotted as a function of THz frequency and applied magnetic field in Figure 5.7.

In the present scheme, the efficiency of THz radiation generation $\sim 2.5\%$ can be achieved for the frequency range $1-1.3\ THz$ by applying magnetic field $\sim 100 \ kG$. It can be noticed that, the efficiency of the present scheme is better than reported by other investigators. For example, Malik et al. [41] have reported the conversion efficiency $\sim 0.002$ by beating of two spatial-Gaussian lasers; conversion efficiency $\sim 0.006$ is achieved by Malik et al. [42] by using super Gaussian lasers; whereas present scheme achieved the conversion efficiency $\sim 0.02$ by using two triangular shaped x-mode lasers. Hamester et al. [63, 115] also obtained the efficiency $\sim 10^{-5}$ with a single laser that is Gaussian in space and time which is two order lesser as compared to the conversion efficiency of present scheme. Wu et al. [117] reported theoretical as well as numerical simulation of powerful THz emission using inhomogeneous plasma density. In their model, the maximum energy conversion efficiency at peak intensity $5.48 \times 10^{12} W/cm^2$ is 0.0005, which is much lower than present model. Kim et al. [114] proposed a model for the
generation of THz radiation by irradiating different gases with a symmetry-broken laser field composed of the fundamental and second harmonic laser pulses. In that model, the energy conversion efficiency was \( \approx 10^{-5} \), which is three order lesser as compared to present model.

Figure 5.7: Variation of efficiency of THz radiation \( \eta \) as a function of the THz frequency \( \nu \) (THz) and applied static magnetic field \( B_c \). Other normalized parameters are \( \nu_i = 0.3 \), \( \omega_i / \omega_p = 12 \), \( n_{a0} / n_0 = 0.3 \), \( \omega_c / \omega_p = 0.3 \), \( a_0 \omega_p / c = 5 \), \( z \omega_p / c = 1 \), \( y / a_0 = 0.5 \).
Effect of electron collision frequency $(\nu)$ on the efficiency of present scheme as a function of electron temperature is shown in Figure 5.8. Electron collision frequency depends upon electron temperature by the expression given by

$$\nu = (z\omega_p / 10N_p) \ln(9N_p / z)$$

where $N_p = 4\pi n_0 \lambda_{De}^3 / 3$, $\lambda_{De}$ is the electron Debye length and $n_0$ is the electron density in plasmas. As plasma electron temperature increases, electron collision frequency $(\nu)$ decreases; as a result of which percentage change in efficiency $\left[\left(\eta - \eta_\nu \right) / \eta \right] \times 100$ of the present THz generation scheme decreases. Here, $\eta_\nu$ and $\eta$ are efficiencies of the scheme with and without electron collisions respectively. The feasible laser and plasma parameters used in the present scheme are tabulated below:

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Parameters</th>
<th>Corresponding realistic parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\omega_1 / \omega_p = 12$</td>
<td>$\omega_1 = 24\pi \times 10^{12} \text{rad/sec}$</td>
</tr>
<tr>
<td>2</td>
<td>$n_{a0} / n_0^0 = 0.3$</td>
<td>$n_{a0} = 5.34 \times 10^{18} \text{m}^{-3}$</td>
</tr>
<tr>
<td>3</td>
<td>$\omega_c / \omega_p = 0.0-0.8$</td>
<td>$B_s = 0 \text{kG} - 285 \text{kG}$</td>
</tr>
<tr>
<td>4</td>
<td>$v_1 = 0.3c$</td>
<td>$v_1 = 0.9 \times 10^8 \text{m/s}$</td>
</tr>
<tr>
<td>5</td>
<td>$z\omega_p / c = 1$</td>
<td>$z = 0.05 \text{mm}$</td>
</tr>
<tr>
<td>6</td>
<td>$a_0 \omega_p / c = 5$</td>
<td>$a_0 = 0.25 \text{mm}$</td>
</tr>
</tbody>
</table>

**Table 5.1: Laser and plasma parameters used in the present scheme.**

All the dimensionless parameters are chosen for CO$_2$ laser ($\lambda = 1.06 \times 10^{-5} \text{m}$) having frequency $\omega_1 = 2 \times 10^{14} \text{rad/s}$ and intensity $I_L = 2 \times 10^{15} \text{W cm}^{-2}$. 
Figure 5.8: Percentage change in efficiency of THz radiation generation as a function of plasma temperature $T$ (eV).

5.2 Conclusions

Dynamics of THz radiation generation by beating of two $x$-mode laser beams in a rippled magnetized plasma is studied, (only ponderomotive nonlinearity is operative). Triangular envelope of $x$-mode laser beams is utilized to enhance beat ponderomotive force acting on plasma electrons. The extra momentum required to excite THz wave resonantly by beating of two $x$-mode lasers of frequencies in upper hybrid range is provided by the periodicity of density ripples. The required ripple wave number depends upon THz frequency and applied magnetic field. It decreases as the THz frequency increases and increases as the magnetic field increases. Thus, frequency of THz radiation can be easily tuned by varying plasma density and applied magnetic field. The nonlinear mechanism which generates the THz radiation can be understood as follows: the non linear interaction of laser beams with the plasma having static magnetic field
generates the velocity perturbation which leads to density perturbation. Static magnetic field imparts extra longitudinal component to oscillatory velocity of plasma electrons resulting in to transverse ponderomotive force. Both the components of ponderomotive force [Eq. (5.3) and Eq. (5.4)] are responsible for density perturbation. As a result, a nonlinear current at beat wave frequency is generated due to coupling between density ripples and velocity components of electron as shown in Eq. (5.11). Since the difference in laser beam frequencies is in the range of THz and phase matching conditions are satisfied, the nonlinear current generated at beat frequency generates the desired THz wave. The THz amplitude can be controlled by laser plasma parameters and magnitude of applied static magnetic field as shown in Eq. (5.16). In this scheme, magnetic field plays two roles (i) it controls the phase velocity and group velocity of beating lasers and (ii) the polarization of generated THz wave. The THz amplitude scales directly to the amplitude of density ripples and maximizes as frequency (ω) approaches to resonance frequency (≈ ωₘ). THz radiation emitted by beating of two triangular shaped lasers is more collimated (Figure 5.5) as compared to plane lasers and Gaussian shaped lasers. In conclusion, we can say that the efficiency, amplitude and tunability of the present THz generation scheme is better than the schemes reported in literature.