CHAPTER 3
THEORY RELATED TO TAGUCHI'S METHOD

3.1 TAGUCHI'S BRIEF BIOGRAPHY

When Japan began its reconstruction efforts after World War II, Japan faced an acute shortage of good quality raw material, high quality manufacturing equipment and skilled engineers. The challenge was to produce high quality products and continue to improve quality product under these circumstances. The task of developing a methodology to meet the challenge was assigned to Dr. Genichi Taguchi, who at that time was manager in charge of developing certain telecommunication products at the electrical communication laboratories (ECL) of Nippon Telecom and Telegraph Company (NTT). He worked with NTT from 1949-1961. Through his research in the 1950’s and the early 1960’s. Dr. Taguchi developed the foundation of robust design and validated his basics philosophies by applying them in the development of many products. In recognition of this contribution, Dr Taguchi received the individual Deming award in 1962, which is one of the highest recognition in the quality field (Phadke, 1989).

Genichi Taguchi was born in Japan in 1924 and studied textile engineering in Kiryu Technical College. From 1942 to 1945, he served in the Astronomical Department of the Navigation Institute of the Imperial Japanese Navy. After that, he was started work in the Ministry of Public Health and Welfare and the Institute of Statistical Mathematics, Ministry of Education and there he was educated by Matosaburo Masuyama on the use of orthogonal arrays and also on different experimental design techniques. Thereafter in 1950, he worked for more than 12 years at the newly formed Electrical Communications Laboratory of the Nippon Telephone and Telegraph Company.
He served as a visiting Professor at the Indian Statistical Institute from 1954 to 1955. While he was there, Taguchi met Sir R.A. Fisher and Walter A. Shewhart and published the first edition of his two-volume book on Experimental design in 1958. In 1962, he was awarded his PhD from Kyushu University.

In 1970, he developed the concept of the Quality Loss Function and also published the third and most current edition of his book on experimental designs.

When Dr. Taguchi was first brought his ideas to America in 1980, he was already well known in Japan for his contribution to quality engineering. But in U.S., he got first recognition in 1984 when Ford Motors Company sponsored the first supplier Symposium on Taguchi Methods.

After years of research, Dr. Taguchi introduced some new ideas to make the design of experiment (DOE) technique to make familiar to the industrial engineers and to offer a much simpler and standardized methods for experimental designs and analysis of results. By applying the Taguchi’s techniques, the performance of products or processes can be improved by improving the consistency of performance and increasing insensitivity towards uncontrollable factors which is also known as robustness.

Professor Genichi Taguchi was the director of the Japanese Academy of quality and four times receipt of the Deming Prize. The term Taguchi Methods was coined in the United States. Recently western and also asian based industries have began to recognize the Taguchi’s methodology as a simple and highly effective way to improve the product or process quality and utilizes it effectively.
3.2 DIFFERENT DEFINITIONS ON THE QUALITY

- “Fitness for use”
  
  Dr. Juran (1964)

- The totality of features and characteristics of a product and service that bear on its ability to satisfy a given need.
  
  European Organization for Quality Control Glossary (1981)

- The totality of features and characteristics of a product or service that bear on its ability to satisfy given needs.
  
  The American Society for Quality Control (1983)

- Quality should be aimed at needs of consumer, present and future.
  
  Dr. Deming

- The leading promoter of the “zero defects” concept and author of Quality is Free (1979) defines quality as” Conformance to requirements.
  
  Philips Crosby

- The aggregate of properties of a product determining its ability to satisfy the needs it was built to satisfy.
  
  Russian Encyclopaedia
3.3 THE TAGUCHI’S APPROACH TO THE QUALITY

Traditional Definition:

The more traditional "Goalpost" mentality of what is considered good quality says that a product is either good or it isn't, depending on whether or not it is within the specification range (between the lower and upper specification limits -- the goalposts). There is good or bad products as per the limits. In this approach, the specification limits are more important than the target value. But, is the product as good as it can be, or should be, just because it is within specification. This traditional approach is shown in figure 3.1.

Taguchi’s Definition:

Taguchi did not accept the above traditional definition of quality. He defines the ‘quality’ as deviation from on-target performance. He has given an unusual definition to the product quality. According to him, “quality of a manufactured product is total loss generated by that product to society from the time it is shipped, other than any losses caused by intrinsic functions”. By loss Taguchi refers to the loss caused by the variability of the function and loss caused by harmful side effects. When a product moves from its target value then it will cause the loss even if the product lies or not within the specification limits. This Taguchi’s approach is shown in figure 3.2.
Fig. 3.1 Traditional approach to the definition of quality
Fig. 3.2 Taguchi’s approach to the definition of quality


3.4 TAGUCHI'S QUADRATIC QUALITY LOSS FUNCTION

Taguchi developed the concept of quality loss function in 1970. According to the Taguchi, the loss due to the performance variation is proportional to the square of the deviation the performance characteristics from its nominal value. Taguchi defined financial loss or Quality loss mathematically as:

\[ L(y) = k (y - m)^2 \]  

where, \( y \) = objective characteristic or attribute

\( L \) = loss associated with \( y \)

\( m \) = specification target value

\( k \) = constant called quality loss coefficient depending upon target and specification limit

Suppose \( m \pm \Delta \) be the specification limits and loss at \( m \pm \Delta \) is \( A \). Therefore,

\[ k = A/\Delta^2 \]  

Here, \( A \) = cost of defective product or scrap or rework

\( \Delta \) = Tolerance

Therefore equation (1) can be rewritten as

\[ L(y) = A/\Delta^2 (y - m)^2 \]  

[33]
3.5 TAGUCHI'S PHILOSOPHY OF ROBUST DESIGN

Robustness may be defined as a very small or negligible variation in the performance of a product which provides consistent customer satisfaction.

Taguchi introduces his approach in using experimental design for designing a process or product in order to be robust to the environmental conditions, component variation and minimizing variation around a target value (Ross, 1988). His philosophy has far reaching consequences, yet it is founded on three very simple and fundamental concepts (Roy, 1990). These concepts are:

(1) Quality should be designed into the product and not inspected into it.

(2) Quality should be best achieved by minimizing the deviation from a target. The product should be so designed that it is immune the uncontrollable environmental factors.

(3) The cost of quality should be measured as a function of deviation from the standard and the losses should be measured system-wide.

In first concept, Taguchi believed that better way to improve quality was to design and built it into the product. Quality improvement starts at very beginning from the design stages of a product or process which continues through the production phase. He proposed an “off-line” strategy for developing the quality improvement in place of an attempt to inspect the quality into a product on the production line. He observed that the poor quality cannot be improved by process of inspection, screening and salvaging. He emphasizes that quality is what one designs into a product (Roy, 1990).

His second concept deals with the actual method of affecting the quality. He contended that quality is directly related to the deviation of design parameter from the target value, not to conformance to some fixed specifications (Roy, 1990).
His third concept calls for measuring deviation from a given design parameter in terms of the overall life cycle cost of the products. These costs would include the cost of scrap, rework, inspection, warranty service calls and product replacement. These costs provide the guidance regarding the major parameters to be controlled (Roy, 1990).
3.6 TAGUCHI’S THREE STAGE PROCESS TO THE ROBUST DESIGN

The main objectives of the Taguchi’s methods are to minimize the variation in a product response while keeping the mean response on the target and to made products robust to the changes in operating and environmental conditions.

To achieve the desirable product quality by design, Dr. Taguchi recommends a three stage process (Roy, 1990).

   (1) Systems design
   (2) Parameter design
   (3) Tolerance design

System design is the process of applying the scientific and engineering knowledge to determine the proper working levels of selected design parameters. It includes the designing and testing a system based on the engineer’s judgement of selected materials, parts and nominal product or process parameters based on current technology. Most often it involves innovation and knowledge from the applicable fields of science and technology (Roy, 1990).

Parameter design is an investigation conducted to identify the settings of design parameters that optimize the performance characteristics and reduce the sensitivity of engineering design to sources of variation or noise (Enright and Prince, 1983). So, the parameter design requires some form of experimentation for the evaluation of the effect of noise factors on the performance characteristics of the product. With the help of this experimentation optimum condition is obtained so that the influence of noise factors will be minimized and which leads to the robust design.

In this, the last step is the tolerance design which is the process of determining tolerances around the nominal setting which is already identified in the parameter design process. Tolerance design is a step used to fine tune the results of parameters design by tightening the tolerance of factors with significant influence on the product (Roy, 1990).
3.7 TAGUCHI’S METHODOLOGY FOR SINGLE OBJECTIVE OPTIMIZATION

The following steps have been used in Taguchi Methodology (Fig. 3.3):-

- **Planning the Experiment**
  - Define the problem
  - Selection of factors and no. of levels.
  - Selection of Orthogonal Array (OA).

- **Performing the Experiment**

- **Analyzing the Experiment Results**
  - Statistical analysis and interpretation of results
  - Determination of Optimal condition

- **Confirmation Run**

Fig. 3.3 Steps in Taguchi Methodology for single objective optimization
(1) DEFINING THE PROBLEM

Well said “Well begun is half done”.

It is the first step of whole process and planning phase. It involves setting the objectives of the study, defining the problem, defining the areas of concern and finally the selection of response variable or quality characteristics. The preparation required before beginning the experiment will depends on the problem under study. So careful, good and concrete planning can help in avoiding the problem that would occur in the execution of actual experimental performance. A well defined objective ensure that experiment answers the right questions and get the better results. So, at the very first step, goals of the experiment have to be identified.

The objective is to identify those control parameters or factors settings which optimize the objective function. Developing a good and proper problem statement decides whether experiment is going in the right way or not.

At this stage it is necessary to find out the answers to these questions which are:

Why there is a need to study this problem?

Is this study would be beneficial from economical point of view or not?

Whether this study will improve the quality of the products at minimum cost or not?

After answering the above questions with the help of brainstorming and cause and effect diagram identify the objective function or quality characteristics. The objective of the experiment is to improve the performance characteristics of the product or process according to customer’s needs and expectations.

The objective of experimentation should be to reduce the variation and control the variation of a product or a process and subsequently decisions must be made which parameters affect the performance of a product or process.
In the design process, a number of factors or parameters can influence the quality characteristics or response variable of the product (output of the product) which can be classified into three classes for which the block diagram for product or process design is shown in the figure 2. These three classes are:

(1) Signal factors (M): These are the parameters set by the users or operator of the product to express the intended value for the response of the product. For example, the speed setting of a fan is the signal factor for specifying the amount of breeze, the steering angle is a signal factor that specifies the turning radius of an automobile. The signal are selected by the design engineer based on engineering knowledge of the product being developed (Phadke, 1998).

Robust design projects can be classified on the basis of nature of signal factor and quality characteristics. The problems in which signal factor takes a constant value are called static problems while the other problems are called dynamic problems.

(2) Noise Factors (X): Certain parameters cannot be control by the designer and are called noise factors. The levels of the noise factors change from one unit to another, from one environment to another and from time to time. The noise factors cause the response y to deviate from the target specified by the signal factor M and lead to quality loss (Phadke, 1998). Three types of noise factors can be classified as:

(i) Unit to unit noise factors: These are the random variations in a process. Examples are variation in manufacturing process, raw materials, machines, participation of operator etc.

(ii) Internal noise factors: These are the internal sources of variation in a product or process. Examples are wear and tear of components, raw material spoilage, aging, improper settings etc.
(iii) External noise factors: These are the external sources of variation in a product or process. Examples are external environmental factors like temperature, humidity, dust, load, human errors etc.

(3) Control Factors (Z): These are the parameters that can specified freely by the designer. In fact, it is the engineer responsibility to determine the best values of these parameters. Each control factor takes multiple values which are called levels. When the levels of certain control factors are changed, the manufacturing cost also changes (Phadke, 1998).

Thus, after defining the problem and selection of objective function, the next step is to identify the noise factors, testing condition and quality characteristics. The values of the various dimensions and design parameters need to be set during the initial development stages of product in order to improve the performance characteristics (Kadu et.al., 2009). List all possible parameters contributing to the problem with help of brainstorming or cause-effect diagram and the levels which each would be tested. Layout of cause and effect diagram is shown in figure 3.4.
Fig. 3.4 Layout of Cause and Effect Diagram
(3) SELECTION AND USE OF APPROPRIATE ORTHOGONAL ARRAY (OA) AS THE EXPERIMENTAL MATRIX

Any process is a combination of one or more parameters and each parameter can have any value. This process will give best possible result when all these parameters operate at the optimum values. In OA, all parameters are varied at the same time and their effect on performance interactions are studied simultaneously (Kadu et. al., 2009). OAs has a balanced property that every factor levels occurs the same number of times for every level of all other factors in the experiment. A Taguchi OA is denoted by $L_N (S^M)$. Here “$L$” denotes for Latin square, “$N$” denotes the number of experiments/trials, “$S$” denotes the levels at which these experiments is to be conducted and “$M$” denotes the number of variables/factors/parameters chosen or the number of columns in the matrix. A “rule of thumb” is to make sure that the number of degrees of freedom associated with experiment is always greater than or equal to the number of degrees of freedom required for studying the main and interaction effects. Here, degrees of freedom is the number of fair and independent comparisons that can be made from a number of levels (Antony et. al., 2001). The first step of selecting the appropriate Orthogonal Array (OA) is to count the total degrees of freedom (dof). This count fixes the minimum number of experiments that must be perform for studying the effect of parameters involved. In calculating the total dof, one dof will be given to the overall mean of the response under study; and degree of freedom for number of parameters running at different levels = Number of control parameters × (Number of Levels-1). Therefore, the total dof = dof of overall mean of the response + dof for number of parameters or factors running at different levels. That means, one must conduct at least experiments equal to total number of dof to obtain the desired results. Therefore, for selecting an appropriate OA the number of parameters and their levels to which these parameters should be run must be known. The example of $L_8(2^6)$ orthogonal array is shown in the Table 3.1 and Table 3.2 shows the rules for selecting the standard orthogonal array (OA).
Table 3.1: Example of $L_8(2^6)$ Orthogonal Array

<table>
<thead>
<tr>
<th>Treatment Condition</th>
<th>Factors</th>
<th>Response</th>
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<td></td>
<td>A</td>
<td>B</td>
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Table 3.2: Table for the Selection of Orthogonal Array (OA)

<table>
<thead>
<tr>
<th>No. of Levels</th>
<th>No. of Parameters</th>
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<td>2</td>
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<td>3</td>
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PERFORMING THE EXPERIMENTAL RUN

This is also known as implementing or conducting stage of the study. Conducting the matrix experiments as per the Taguchi’s orthogonal array design matrix and then recording the responses or performance characteristics from each trial. These trials have to be closely monitored to find any discrepancies while running the experiment.

The experiment requires a minimum of one run per condition of the experiment. But one run does not represent the range of possible variability in the results. Repetition of runs enhances the available information in the data. Taguchi suggests some guidelines for the repetitions (Roy, 1990).

Repetition offers several advantages which are

(i) The additional trial data confirms the original data points.
(ii) If the noise factors vary during the day then repeating trials through the day may reveal their influence.
(iii) Additional data can be analyzed for variance around a target value.

When the cost of repetitive trials is low, repetition is highly desirable. When the cost is high or interference with the operation is high then the number of repetition should be determined by means of an expected payoff for the added cost (Roy, 1990).

Repetition permits the determination of a variation index called the signal to noise ratios (S/N). The greater this value, the smaller the product variance around the target value. The signal to noise ratios concept has been used in the fields of acoustics, electrical and mechanical vibrations, and other engineering disciplines for many years (Roy, 1990).
Finding a correct objective function to maximize in an engineering design problem is very important. Failure to do so can lead to great inefficiencies in experimentation and also lead to the wrong conclusions about the optimum levels. The task of finding what adjustments are meaningful in a particular problem and determining the correct S/N ratio is not always easy (Phadke, 1989).

In this, the first step is to analyze the signal-to-noise (S/N) ratio, which measures the functional robustness of product or process performance in the presence of undesirable external disturbances (Kapur and Chen, 1988). The S/N ratio is a special kind of data summary which is able to combine two characteristics into the desired one and is often used in analyzing the data for parameter design (Rahman and Talib, 2008).

Taguchi suggests the transformation of the repetition output data in a trial into consolidated single value which is known as Signal to Noise Ratios (S/N). It may also be defined as the ratio of desirable value (signal) to the undesirable value (Noise) and it expresses the scatter around the desired value. The larger the value, the smaller will be the scatter.

The following S/N ratios are used for optimization of design parameters setting:

(1) **Nominal-the-best type (NB):**- In this type of problems, the quality characteristics is continuous and non-negative. For example, dimension of a part consistently achieved with modest variance.

Its target value is non zero and finite. For these problems when the mean becomes zero, the variance also becomes zero. Also, for these problems we can find a scaling factor that can serve as a adjustment factor to move the mean on the target (Phadke, 1989).

The S/N ratios for nominal-the-best response is given by the following equation

\[
S/N = 10 \log \left( \frac{\bar{Y}}{S} \right) \quad \text{.................. (1)}
\]
Where, \( \bar{Y} = \frac{\sum_{i=0}^{n} Y_i}{n} \)

\[ S^2 = \frac{\sum_{i=1}^{n} (Y_i - \bar{Y})^2}{(n-1)} \]

In some situations, the scaling factor can be identified by engineer expertise while in other conditions it can be identified through experimentation.

(2) **Larger-the-best type (LB):** In this type of problems, the quality characteristics has to be as large as possible and also it is continuous and non negative. Also, there is no need of any adjustment factor. For example, strength of a product to be maximized, life of a component to be maximized, material removal rate during machining process etc. Since, there is no adjustment or scaling factor in these type of problems, so quality characteristics should be maximized without any adjustment.

The S/N ratios for larger-the-best response is given by the following equation

\[ S/N = -10\log \left[ \frac{1}{n} \sum_{i=1}^{n} \frac{1}{\bar{Y}} \right] \] \hspace{1cm} \text{(2)}

(3) **Smaller-the-best (SB):** In this type of problems, the quality characteristics has to be as small as possible and also it is continuous and non negative. Also, there is no need of any scaling or any other adjustment factor. For example, the surface roughness during a machining process, number of defects in a product etc. Since, there is no adjustment or scaling factor in these type of problems, so quality loss should be minimized without any adjustment.

The S/N ratios for smaller-the-best response is given by the following equation

\[ S/N = -10\log \left[ \sum_{i=0}^{n} \frac{Y_i^2}{n} \right] \] \hspace{1cm} \text{(3)}

[47]
Here, $n$ represents the number of observations and $Y_1, Y_2, Y_3, \ldots, Y_n$ represents the values of a performance characteristic. Using one of the above equations according to the given condition that best fits the problem, the S/N ratio corresponding to each trial condition is computed. Now, the next step is to calculate the average S/N ratio at each level of each factor. In order to determine which of the factor/interaction effects are statistically significant, a powerful statistical technique called analysis of variance (ANOVA) is used. Using ANOVA, one is able to identify the active and inactive factor/interaction effects with statistical confidence (Logothetis, 1994).

ANOVA is a mathematical technique based decision tool for detecting any differences in average performance of groups of items tested. It breaks total variation into accountable sources i.e. total variation is decomposed into its appropriate components.

ANOVA is a statistical decision tool for detecting any differences in average performance of groups of items which has to be tested. It also investigates the design parameters which significantly affect the quality characteristics.

The analysis of variance computes the quantities known as degrees of freedom, sums of squares, mean squares etc. and organizes them in a tabular form. These quantities and their interrelationships are defined as shown below using the following notations (Roy, 1990):

\[
\begin{align*}
V &= \text{Mean of squares (Variance)} \\
S &= \text{Sum of squares} \\
S' &= \text{Pure sum of squares} \\
f &= \text{Degrees of freedoms} \\
e &= \text{Error (Experimental)} \\
F &= \text{Variance Ratios} \\
P &= \text{Percent contribution} \\
T &= \text{Total (of results)} \\
N &= \text{Number of experiments}
\end{align*}
\]
Variance: Variance may be defined as the ratio of sum of square of each trial to the degrees of freedom of each factor. Mathematically this can be express as:

\[ V_A = \frac{S_A}{f_A} \] (variance for say factor A)

\[ V_e = \frac{S_e}{f_e} \] (variance for error)

Variance Ratio (F Ratio): It is the ratio of variance of each factor to the variance for error. Mathematically this can be express as:

\[ F_A = \frac{V_A}{V_e} \]

Degree of freedom: Mathematically the rank of quadratic form is known as degree of freedom. The degree of freedom can be calculated by

\[ f_A = \text{Number of levels of (say) factor A} - 1 \]

And degree of freedom for error variance can be calculated by

\[ f_e = f_T - f_A \]
A sample ANOVA table is given below:

**Table 3.3: Sample ANOVA table**

<table>
<thead>
<tr>
<th>Source</th>
<th>Degree of freedom (f)</th>
<th>Sum of squares (S)</th>
<th>Variance (V)</th>
<th>Variance Ratio (F)</th>
<th>Pure sum of squares (S’)</th>
<th>Percent Contribution (P)</th>
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<td>Factor A</td>
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<td>Factor B</td>
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The optimal condition is the optimal parameters settings which yield the optimum performance. From the results of ANOVA, the optimal condition is obtained which yield the optimum performance. The optimal condition is obtained by identifying the levels of significant control parameters which yield the highest S/N ratios and a parameter level corresponding to the maximum average S/N ratio is called optimal level performance for that parameter and overall it is called optimal condition. Thus, the process is optimized under these conditions.

Next step is to predict the performance under these levels. With the help of optimal conditions as determined above, the optimum performance will be predicted with the help of following equation:

\[ Y_{Opt} = T + \sum_{i=1}^{k} (T_i - T) \]

Here,

\[ Y_{Opt} = \text{Predicted optimum performance} \]

\[ T = \text{Total mean of all experimental runs} \]

\[ T_i = \text{Mean of all experimental runs at optimum level for factor i} \]

\[ K = \text{Number of factors} \]
(7) CONFIRMATION EXPERIMENT

The last step of the Taguchi methodology is confirmation or verification experiment. The purpose of confirmation of experiment is to verify the optimum conditions so as to reduce the variation. It is very important specially when the optimum condition is not one of the trial runs already completed. Therefore confirmation experiment is conducted by using the levels of optimal setting parameters. This experimental combination of parameters resulted in substantial reduction in variation of performance characteristics and shows that the factors or parameters and levels chosen from the experiment do provide the desired results.

Confirmation experiment is the last and important step in the Taguchi process as it is the direct proof of the methodology. If the predicted and the observed values are close to each other then the used model is adequate for describing the effect of parameters on quality characteristics and if there is a large difference in observed values and predicted values then the used model is adequate.
3.8 THEORY RELATED TO THE MULTI-OBJECTIVE OPTIMIZATION THROUGH THE COMBINED APPLICATION OF TAGUCHI'S METHOD AND UTILITY CONCEPT

Utility concept

The performance evaluation of any machining process or a component depends upon a number of diverse output characteristics. To able to make a rational choice, these evaluation on different characteristics should be combined to give a composite index. Such a composite index represents the overall utility of the product. The overall utility of a process or component measures its usefulness in the eyes of evaluator (Kansal et al., 2006).

According to the utility theory (Kumar et al.2000), if \( X_i \) is the measure of effectiveness of an attribute or quality characteristics or response variable i and there are n attributes evaluating the outcome space, then the joint utility function can be expressed as (Derek, 1982):

\[
U(X_1, X_2, \ldots, X_n) = f(U(X_1), U(X_2), \ldots, U(X_n))
\]  
(1)

Here \( U_i(X_i) \) is the utility of \( i^{th} \) attribute.

The overall utility function is the sum of individual utilities if the attributes are independent and is given as follows:

\[
U(X_1, X_2, \ldots, X_n) = \sum_{i=1}^{n} U_i(X_i)
\]  
(2)

The attributes or response variables may be assigned weights depending upon the relative importance or priorities of the characteristics. Then the overall utility function after assigning weights to the attributes can be expressed as:

\[
U(X_1, X_2, \ldots, X_n) = \sum_{i=1}^{n} W_i \cdot U_i(X_i)
\]  
(3)
Here $W_i$ is the weight assigned to the given attribute or response variable $i$. The weights should be selected in such a way that the sum of all the weights must be equal to 1.

In order to determine the utility value for a number of different quality characteristics, a preference scale has to be constructed. Then these scales are weighted to obtain a composite number known as overall utility. The weighting is done to satisfy the test of indifference on the various quality characteristics. The minimum acceptable quality value for each quality characteristics is allotted a preference number of 0 and the best available quality value for each quality characteristics is assigned a preference number of 9. The preference number ($P_i$) was given by Gupta and Murthy in 1980:

$$P_i = A \log_{10} \left( \frac{Y_i}{Y_i'} \right)$$

(4)

Here $X_i$ is the value of any response variable or quality characteristics or attribute $i$, $X_i'$ is just an acceptable value of the attribute $i$ and $A$ is a constant. The value $A$ can be found by the condition that if $X_i = X^*$ (where $X^*$ is the optimal or best value), then $P_i = 9$.

Therefore,

$$A = \frac{9}{\log_{10} \left( \frac{Y_i^*}{Y_i'} \right)}$$

(5)

Where, $Y_i$ is the value of quality characteristics $i$

$Y_i'$ is the minimum value of quality characteristics $i$

$Y_i^*$ is the optimum value of $Y_i$

The next step is the calculation of overall utility value ($U$). For this a weighing factor ($W_i$) is assigned to each quality characteristics such that

$$\sum_{i=1}^{n} W_i = 1$$

(6)
Here, in this study the weights to the given quality characteristics were assigned as given below:

\[ W_{TFWW} = 0.4 \]  \hspace{1cm} (7)
\[ W_{SR} = 0.4 \]  \hspace{1cm} (8)
\[ W_{RN} = 0.2 \]  \hspace{1cm} (9)

The overall utility value can be computed as:

\[ U = \sum_{i=1}^{p} WiPi \]  \hspace{1cm} (10)

Among the various quality characteristics type i.e. lower the better, higher the better and nominal the best suggested by Taguchi, the utility function would be treated as higher the better type. Therefore, if the quality is maximised, the quality characteristics considered for its evaluation will automatically be optimized (Singh and Kumar, 2006 and Routara et al., 2010).
Methodology used for Simultaneous Multi-objective optimization based on the combined application of Taguchi’s Method and Utility concept

In this thesis, the following methodology is used based on Taguchi’s method and utility concept:

(1) Find the optimal values of selected response variables (tool flank wear width, surface roughness and roundness) through the single objective optimization using Taguchi’s method.

(2) Construct preference scale using equation no. 4 and 5 for each response variable with the help of experimental data of optimal values and minimum quality levels for the given response variables.

(3) Assign weights according to the requirements such that equation no. 6 will be satisfied.

(4) Using equation no. 10, calculate the utility values for each trial condition of the experiment.

(5) Now use these utility values as a response of each trial condition of the selected experimental run.

(6) With the help of Taguchi’s method analyze the results.

(7) Find the optimal setting of machining parameters for optimum utility and predict the values of response variables.

(8) Conduct the confirmation experiment and compare the predicted optimal values with actual ones.