Chapter 1

Introductory

Basics, Development and Brief Survey of Scheduling Problems

1.1 OVERVIEW OF OPERATIONAL RESEARCH

The subject matter of the thesis entitled ‘Optimal / Heuristic Approach in solving certain Production Scheduling Problems in Fuzzy and non Fuzzy Environment’ deals with some special type of scheduling problems in an important area of Operation Research (O.R.). The roots of O.R. can be traced back many decades when early attempts were made to use a specific approach in management of organizations. However the beginning of the activity called operation research has been attributed to the military services during World War II, because of the war affect. There was an urgent need to allocate scarce resources to the various military operations and to the activities with in each operation in an effective manner. Therefore, the British and then the U.S. military management called upon a large number of scientists to apply a scientific approach to deal with this and other strategic and technical problems. In fact, a team of scientists from various disciplines was asked to do research on (military) operations. By developing effective methods, using the new tool of radar, better manage convey and antisubmarine operations, they also played a major role in winning the battle of North Atlantic. These teams of
scientists were the first O.R. teams. Though operational research (O.R.) by this time is hardly over seven decades old, yet the realm of O.R. applications is as wide as to cover all human efforts where quantitatively a purposeful decision making is involved. The trends and techniques initially developed in O.R. with defense problems have now been applied in system of engineering, banking, business concern, telegraph department, agriculture sector, marketing & financial management, industries production and planning etc. Keeping various aspects of O.R. in mind the different authors have articulated the objectives, definition and scope of O.R. using different words. However, three key aspects seem to figure in common, one is application of scientific and mathematical models, second is the study of operational system as a whole and third is to focus on decision making which is in the best interest of organization. The dramatic development and refinement of techniques of operation research and the advent of digital computers are the two prime factors that have contributed to the growth and application of O.R in the present period. In the field of business and industry, operations research helps the management to determine their tactical and strategic decisions more scientifically.

1.2 FEATURES OF OPERATION RESEARCH APPROACH

The significant features of O.R to any decision and control problem can be summarized as

(a) Inter disciplinary Approach – O.R is inter disciplinary in nature and requires a bland of people with expertise in area of mathematics, statistics,
engineering, economics, management, computer science and so on in order to find the solution of the problem.

(b) **Scientific Approach** - O.R is the application of scientific methods, techniques and tools to problems involving the operation of systems so as to provide those in control of operations with optimum algorithms to the problem. The data so obtained is used to decide whether the hypothesis should be accepted or not.

(c) **Wholeistic Approach** – An O.R team examines the relative importance of all the conflicting and multiple objectives and the validity of claims of various departments of the organizers from the perspectives of the whole organization.

(d) **Objectivistic Approach** - An O.R seeks to obtain an optimal solution to the problem under analysis. For this, a measure of desirability is defined, based on the objective and is used to compare with their outcomes.

### 1.3 PHASES OF OPERATION RESEARCH

The O.R. approach to problem solving consists of the following phases.

(a) Define the problem of interest and gather the relevant data.

(b) Develop a mathematical model keeping all assumptions, constraints and goals in mind.

(d) Make an attempt to solve the model and interpret the result.
(e) Develop a computer based procedure deriving the solutions to the problem

(f) Test the model and modify it as needed.

(g) Implement the solution obtained.

However, one or more of the steps may be modified to some extent before the final results are implemented. First thing to recognize is that an O.R. team normally works in an advisory capacity. The team members are not just given a problem and to solve it however they see fit. The team performs a detailed technical and mathematical analysis of the problem and then presents recommendations to the management. The decision maker evaluates the study and their recommendation takes into account a variety of intangible factors and makes the final decision based on its best judgment.

1.4 INTRODUCTION TO SCHEDULING THEORY WITH REALISTIC SITUATIONS

Operations Research problems fall into any of the categories i.e. Allocation, Routing, Replacement, Inventory, Queuing, Reliability, scheduling etc. The problem in which the facilities are fixed and the sequence of servicing the waiting customers is subject to control, the investigations to deal with such problems have given rise to an elegant theory of sequencing or scheduling. Sequencing simply refers to the determination of order in which the jobs are to be processed on various machines while scheduling refers to the time table that includes the start time and completion time of jobs on machines. Scheduling
problems arise from situations that require the assignment of resources (machines) over a time to perform a set of tasks (jobs). Competition in the international market requires industrial production to become structured more and more efficient. In particular, in the organization of production process with conflicting goals, production costs have to be kept as low as possible while specific customer demands should be satisfied to a maximum degree. Consequently, modern production management aims to satisfy high standards of product quality as well as to meet delivery due dates under economic conditions requiring progressively smaller lot of sizes. Frequently management goals also depend considerably on fluctuations of the international economy. The continuously changing objectives take place to increase emphasis on the clear formulation and flexibility of production scheduling strategies.

Large scale production is organized by means of production planning systems, delivery capacity plans as well as time windows for single orders. In a future step, scheduling derives definite assignments of the orders contained in a production plan to the available resources over time. These assignments, of course are subject to multitude of shop floor details ignored at higher planning levels. For instance, aspects like set up time of machines or breakdown of machines and compatibility relation between orders have to be taken into account. Eventually, the success of manufacturing policies depends on the extent to which schedulers are taking care of the actual shop floor dynamics.

The first approach to the scheduling problem in mathematical models was developed in the context of operation research. Although being theoretically sound and yielding optimal solutions, these mathematical/analytical models are applied seldomly in the realistic applications. The
difficulty to express all the scheduling constraints and inherent complexity and the uncertainties present in the most scheduling applications prevent the use of these models. As a consequence, the models have been enriched with heuristics to tackle the problems.

Another approach that claims to cope with these problems is the use of knowledge based techniques. These try to represent explicitly to all constraints of a scheduling problem even if some of available information is uncertain. Several techniques have been developed to overcome the complexity of resulting problem representations.

Some of the realistic situations in which scheduling problems exist are:

- Technological planning of how the jobs should be completed in a manufacturing unit.
- Scheduling problems related to health services making available the patients to the large hospitals.
- Scheduling of aircrafts waiting for landing clearance.
- Ordering of jobs for processing in a manufacturing plant.
- Scheduling of jobs under rental conditions in non deterministic environment.
- Scheduling of patients waiting in a hospital for different types of tests.

In a production industry or business activity, scheduling techniques are more useful and give scientific system approach to utilize machines idle time or operators time to cut down or reduce the cost of production. The scheduling techniques can also suggest to a decision maker as the interval of time for
which operators idle time can be utilized for alternate purpose for the industry or in a business unit in an economical manner.

In many situations, the scheduling problems are solved casually or automatically or on first come first serve basis without explicit recognition that the problem even exists. Such casual treatment of scheduling problems is not necessarily applicable to inanimate jobs on a factory floor as the machine idle time varies with different sequences of jobs, while informal scheduling methods have been used since long ago. In most of the daily activities the formal mathematical models of scheduling problems started appearing after Johnson’s research in 1954 for two stage production schedules of n jobs. Since then interest in this area has dramatically increased and the promising lines of research have been initiated as evidence for numerous articles on scheduling theory appeared in research journals.

1.5 FEASIBILITY OF CONSTRAINTS

Feasibility of constraints on jobs in the scheduling problems occurs due to technological restriction or internally imposed policy. Feasible schedule is one which is compatible with the feasible constraints and the schedule which is not compatible with feasible constraints is termed infeasible schedule. The effect of feasible constraints is that every possible sequence of jobs is not executable. In fact, the feasible constraints may be, in which manner the jobs are at a shop, the order in which the jobs should be processed or whether the jobs are completed by their due dates and so on. The process of decision making involves selecting one or more alternatives satisfying the constraints. If there is more then one
alternative then we need some criterion to decide which one is the best. In a classical decision theory, one can choose the alternative which has maximum utility.

The nature of jobs’ arrival provides the difference between static and dynamic problems. In a static problem, a certain number of jobs arrive simultaneously in a shop that is idle and is immediately available for work. No further jobs will arrive; hence attention can be focused on scheduling the completely known and available set of jobs. The problem in which all numerical quantities are known and fixed in advance and there is no uncertainty, the number of jobs, their starting times, their completion times and all such types of parameters are known and fixed are called deterministic. The problems in which jobs arrive randomly over a period of time are called dynamic. In a dynamic problem, shop is continuous process. Jobs arrive intermittently at times that are predictable only in statistical sense and arrivals will continue indefinitely. Problems in which the processing times etc. is uncertain called stochastic. Thus the basic difference between the static and dynamic problem is that in the static situation, all the jobs considered are available simultaneously at the beginning of the scheduling and in dynamic case the jobs arrive intermittently during the period.

Usually, the situation under which a decision is made is classified in two categories: deterministic and non-deterministic. In deterministic situations, the outcome for each action can be determined and ordered precisely. Where as, in the non-deterministic situations, this is not so. Further the non-deterministic situation can again be divided into two groups: probabilistic measure and possibility measure. The probability and possibility measure are two different kind of uncertainty. In the probabilistic situation, we require the knowledge of
probability distribution and in possibility measure we need a possibility distribution. This is the case where fuzzy set theory may be expected to be useful by providing new methodologies. In general, where we assign a fuzzy logic, a variable whose value we do not know exactly and the assignment introduces an imprecise constraint on the variable’s value, we call such a constraint a possibility distribution, because it specifies the degree of possibility for the variable to take a certain value. The constraint imposed by an internal value assignment is a special kind of possibility distribution. Possibility measures the degree of case for a variable to take a value while probability measures the likelihood for a variable to take a value. Therefore, they deal with two different types of uncertainty. Possibility theory handles imprecision and probability theory handles likelihood of occurrence. This fundamental difference leads to different mathematical properties of their distribution. Being a measure of occurrence, the probability distribution of a variable must add up to exactly one. The possibility distribution however is not subject to this restriction since a variable can have multiple possible values. However, possibilities and probabilities can be combined to deal with problems in which both kind of uncertainties (i.e. imprecision and likelihood) exist.

The deterministic form of classical decision making theories can also be grouped in several different ways:

1. Based on stages: Single stage, multiple stages.
2. Based on number of decision makers: Single person, more then one person.
3. Based on presence or absence of constraints: Simple
4. Optimization, optimization under constraints, optimization based on multiple criteria.
If $X$ is a collection of objects denoted by $x$ then a fuzzy set $A$ is defined as: $A = \{(x, \mu_A(x)) : x \in X\}$. $\mu_A(x)$ is called the membership function of $x$ in $A$ which maps $X$ to the membership space $M$.

In this thesis, we emphasis on how the fuzzy logic theory can be applied in solving various type of scheduling problems. Different kinds of knowledge co-operate in the determination of feasible schedules using rules w.r.t. facts and goals expressed by concept of fuzzy set theory. In the scheduling problem, for a given set of machines and technological constraints, production requirements expressed in terms of quantities, product quality and time constraints expressed by means of earliest starting times and due dates for jobs.

A set of jobs must be preferred by a set of $m$ machines. Each job $k$ is characterized by a set of operation assigned to machines on which they have to perform. A schedule is described by a set of pair $(O_i, O_j)$ denoting that $O_i$ must precede. 

This section is divided into following sub sections

- **Classification of Scheduling Problems.**
- **Basic Assumptions/ constraints taken in Scheduling.**
- **Various Concept / concepts used in scheduling.**
- **Performance Measures of scheduling problems.**
- **Components of Production Cost.**
- **Methodology Used in Solving Scheduling Problems.**
- **Historical Development and Brief Survey of Scheduling Problems.**
1.5.1 Classification of Scheduling Problems

The different types of scheduling problems are as:

(i) Flow Shop Scheduling Problem

In these types of problems the order of processing of jobs on the various machines is the same throughout i.e. the sequence in which the jobs go through the first machine as to be the same as the sequence in which the jobs go through any of subsequent machine and a job can not pass to another job while waiting for processing on a machine.

(ii) Job Shop Scheduling Problem

In job shop Scheduling Problems we have n jobs and m machines and each job has its own machine order specified. Order of jobs on different machines can be different as per the requirement of scheduling.

(iii) Open Shop scheduling problem

In these types of problems, any given job requires an execution on each of the m machines. The order of the jobs to be processed is immaterial.

(iv) Project Scheduling Problem

It is a type of project where all the resources are brought to the job.

(v) Parallel machines Scheduling Problem

In a flow shop scheduling problem there are more than one machine of same type is available. The allocation of the machine is completely fixed by the sequence of the jobs. In case there is more than one machine available of the same type for processing a set of jobs, the scheduling requires
sequencing of jobs as well as allocation of machine for processing the job. The general category is known as parallel machine scheduling.

There are three classes of problems depending on whether parallel machines are (1) identical (2) Uniform (3) Unrelated. When the machines are identical, the processing time of job $i$ is the same on all machines. When all the machines are uniform, the processing time of job $i$ on one of the parallel machine is $\frac{P_i}{S_k}$, where $p_i$ is constant for job $i$ and $S_k$ is speed factor associated with machine $M_k$. When the machines are unrelated, the processing time of job $i$ vary between parallel machines in an arbitrary manner.

*In this thesis we have mainly focussed at the flow shop and parallel machine Scheduling Problems.*

### 1.5.2 Basic Assumptions in Flow Shop Scheduling Problem

**(i) Assumption regarding machines**

- Each machine processes one operation at a time.
- An operation once started on a machine must be performed till its completion.
- Each operation takes finite time and it must be completed before any other operation begins.
- There is only one machine of each type.
• Each machine operates independent of the other.

(ii) Assumption regarding jobs

• All jobs are available for processing at time zero.
• All jobs allow the same sequence of operations.
• No job is processed more than once on any machine.
• Each job consists of a specified number of operations and each operation is done by only one machine.
• The processing times of the jobs are independent of the order in which jobs are performed.
• Each job once started must be processed till completion.

(iii) Assumptions regarding operating process

• Each job is processed as early as possible.
• Each job is considered as individual identity even though it may consists of a number of individual units.
• Each machine is provided with sufficient waiting space for allowing jobs to wait before starting their processing.
1.5.3 Various Concept/Concepts Used in Scheduling

(i) Concept of Equivalent Job Block in Flow Shop

A situation in which some sets of specified jobs are required to be processed together as a block in a sequence either by virtue of technological constraint or some externally imposed restrictions or due to priority of one job over the other is known as a group technology. This concept has a very wide application to a variety of production systems for the purpose of improving the productivity.

Rule for determining the equivalent job in n x 2 flow shop problem (Equivalent Job-Block Theorem)

Maggu and Das [62] in 1977, proved an equivalent job block theorem which is stated as: Let there be n jobs in a sequence s to be processed on two machines A and B in the order AB. Let the equivalent job b be of jobs i and j. Then

\[
A_\beta = A_i + A_j - \min(A_j, B_i)
\]

\[
B_\beta = B_i + B_j - \min(A_j, B_i)
\]

Where, \(A_\beta\) and \(B_\beta\) denote the processing time of equivalent of job block \(\beta\) on machines A and B respectively. The job block concept can be extended for n jobs 3 machines and n jobs & m machines scheduling.
(ii) Concept of Transportation Time of Jobs

In many practical situations of scheduling, sometimes it has been seen that the machines are distantly placed and therefore definite time is taken in transferring the job from one machine to another in the form of

(a) Loading time of jobs
(b) Moving time of jobs
(c) Unloading time of jobs

The sum of these times has been designated by various researchers as transportation time of job. Hence transportation time is the amount of time required to dispatch the job i after it has been completed on machine A, to the next succeeding machine B for its onward processing. It is denoted by $t_i$.

Sometimes in scheduling problems, a transporting agent is used e.g. carrying sugar canes by some of the transportation means in the sugar industry. After transporting the jobs from first machine to the second, the time taken by the agent to bring the mode of transport back to first machine to carry the next job is called transporting agent. This concept of transporting agent has been taken in account while scheduling.

In chapter 7 of the thesis, we have taken this concept of single transporting facility on two machines and double transporting facility on three machines.

(iii) Concept of Breakdown Interval of Machines

Generally in scheduling problems, it has been assumed that the jobs are continuously working i.e. no machine fails and hence no disturbance occurs in
the processing of jobs. But many a times it is practically observed that the machines do not work.

- Due to failure of electric supply from the mains
- Due to failure of one or more components suddenly
- Due to excessive heating or some other external cause, machines are required to stop for certain interval of time.
- Due to resource breaks down and all the operations assigned to a machine during estimated time interval, are necessary for repairing must be unscheduled and then classified by an urgency criterion.

Therefore we have considered the effect of breakdown interval on machines in flowshop problems. Singh TP (1985)[86], Singh T.P. and Indra Vij (2006) [109] discussed the effect of breakdown interval of n x 2 and n x 3 flow shop using a heuristic algorithm.

(iv) **Concept of Minimizing Total ‘In Process Waiting’ Time for Jobs**

In a two machines flow shop problem if the first machine is dominated by the second machine, then there occurs In Process Waiting of each job in the sequence. The structural time process condition is, \( \max A_i \leq \min B_i \), Where \( A_i \), \( B_i \) represent processing time of job i on machine A and B respectively.

It may be noted that though minimizing ‘In Process Waiting’ time of jobs may increase some other costs like machine waiting costs or penalty cost of jobs, yet the objective of minimizing in process waiting time/cost may be economical aspect from flow shop manager’s view point, especially when he
has minimum time contact with a customer party to complete a group of certain jobs.

**(v) Concept of Group Job Restriction in Flow Shop involving Setup Times of Jobs.**

In this concept it is assumed that:

- The production time required for completion of each job on a machine consists of set up time and the processing time. Set up for the processing of a job on machine B can be done before completion of operation of the job on A, if there exists some idle time on machine B.

- It is necessary to sequence k jobs \((i_1,i_2,\ldots,i_k)\) as a block or a group job in order \((i_1,i_2,\ldots,i_k)\).

Suppose \(A_i, B_i\) are processing times of job \(i\) on machines A and B. Let \(S_i^A, S_i^B\) be the set up time of job \(i\) on machine A and B. Since there is a group job restriction, it is possible to replace block \((i_1, i_2,\ldots,i_k)\) by single job. Let \(\beta_1 = (i_1,i_2)\) then

\[
A_{\beta_i} = A_{i_1} + A_{i_2} - \min(S_{i_1}^A + A_{i_2} - S_{i_2}^B, B_{i_1})
\]

\[
B_{\beta_i} = B_{i_1} + B_{i_2} - \min(S_{i_1}^A + A_{i_2} - S_{i_2}^B, B_{i_1})
\]

\[
S_{\beta_i}^A = S_{i_1}^A + S_{i_2}^B
\]

\[
S_{\beta_i}^B = S_{i_1}^A + S_{i_2}^B
\]

Consider \(\beta_2 = (\beta_1,i_3)\), then processing and set up times of job \(\beta_2\) are obtained using the above technique and continue in this way.
(vi) Concept of weightage in job

In real sense it has been observed that the values of all the jobs have not equi-importance in a workshop. Some jobs are more important and some are less. Hence weightage in job concept applied in scheduling is very useful concept.

(vii) Concept of Fuzzy Sets and Fuzzy Logic

Fuzzy logic refers to a logical system that generalizes classical valued logic for reasoning under uncertainty. A mathematical formulation within which various types of uncertainty can be properly characterized and investigated is now available in terms of the theory of fuzzy sets, fuzzy arithmetic and fuzzy measures. In fuzzy logic, a statement is true to various degrees, ranging from completely truth through half-truth to completely false. Fuzzy logic gives a new approach to mathematics of thinking. Fuzzy logic has been applied successfully to solve many engineering and management problems ranging from consumer products to industrial process control to automotive applications. Fuzzy logic offers a better way of representing reality.

A fuzzy number \(<a, b, c>\) represents a number in different environments. A fuzzy number is simply an ordinary number whose precise value is somewhat uncertain. A very convenient way to describe fuzzy numbers is to use modifying words, for example, fuzzy two as shown in figure 1.7.3.1 could be specified by “roughly 2” other modifying words available are nearly, about and crudely with progressively larger uncertainties. These words are called hedges in fuzzy mathematics circles.
For the defuzzification, average high ranking \(<AHR>\) of the number in fuzzy environment can be calculated as $$\frac{3b + (c - a)}{3}$$.

Fuzzy make span \(C\) can be calculated by considering all of its \(\alpha\)-cuts, i.e., \(C^\alpha\), \(0 \leq \alpha \leq 1\). To calculate a make span at \(\alpha\)-cut, we will need two interval operations i.e. addition and multiplication. The partial results derived from all \(\alpha\)-cuts can be merged to obtain a solution to the original problem formulated with fuzzy sets.

**SOME PRACTICAL CONSTRAINTS**

(i) **No Idle Constraint**

Under ‘No Idle Constraints’ machines work continuously without idle interval i.e. each machine after starting the processing of first job works continuously without break till the last job is completed on it.

(ii) **No Wait Constraint**

The ‘No Wait Constraint’ prescribes for a job to work continuously without waiting times between consecutive machines i.e. a job needs to go
through a sequence of machines without any delay in successive operations of the job. This means the difference between the completion times of the job on last machine and starting time of it on first machine is equal to the sum of its operation time on all machines.

(iii) Non Availability Constraint

Most of the research in scheduling problems has been done on the assumption is not true in real life situations. e.g. the machines may not be available during some interval of time due to some breakdown (stochastic) or due to some preventive maintenance (deterministic). The machines may not be available at the beginning of the planned period if machines continue to process unfinished jobs that were scheduled in the previous planning period.

(iv) Preemption

It is the situation in which the processing is interrupted. Preemption is said to occur when the processing of a job is stopped before its completion, if preemption is considered then a sequence of n jobs can not be described as a schedule of n jobs. Since in case an individual job may appear two or more times in the schedule and one must satisfy how much time is to be allowed at each appearance.

(v) Feasible Schedule

Schedules obeying all imposed constraints are termed as feasible.
1.5.4 PERFORMANCE MEASURE OF SCHEDULING PROBLEMS

Schedules obeying all imposed constraints are termed feasible. In addition to more feasibility, schedules may be evaluated according to some criterion of efficiency. These planning goals can be expressed either explicitly in a quantifying objective function or in terms of preference rules ordering possible schedule alternatives according to some measures of utility schedule. Performance refers to production time such as overall completion time of a set of orders, mean lateness of finished orders, number of late orders and also to other resources like unfinished orders and late orders. In theoretical analysis particularly two types of objective criterion dominate problems without due dates, the make span (i.e. the total time used to finish a set of orders) is minimized whereas in problems with due dates, a performance measure depending on lateness (or, sometimes tardiness) is taken e.g. the average lateness per order.

In this thesis, these performance measures have been studied in detail.

Let there are n jobs identified as 1, 2, 3….n and the m machines as M_1, M_2, ....M_m. Then the following are the performance measures:

(i) **Inserted Idle Time**

In this scheduling problem, a machine is held idle. Although the job is ready to be processed, yet the waiting is to be done.
(ii) **Shortest Processing Time**

Sequencing the jobs in non-decreasing order of processing times is known as shortest processing time (SPT).

(iii) **Release Time** ($r_i$)

This is the time at which a job (i) is released to the shop for some external job generating process. It is the earliest time at which the processing of the first operation of the job begins. It is also known as ready time or arrival time of job.

(iv) **Due Date** ($d_i$)

It is the promised delivery date for a job $i$.

(v) **Completion Time** ($c_i$)

It is the time at which processing time of last operation at the job $i$ is completed.

(vi) **Flow Time** ($F_i$)

It is the total time of job $i$ spend on the shop. $F_i = c_i - r_i$. $F_i$ is also known as manufacturing interval of the short time. If $r_i=0$, $F_i=c_i$ i.e. flow time of the job $i$ is equal to the completion time.

(vii) **Lateness** ($L_i$)

It is simply the difference between the completion time of job $i$ and its due date. $L_i = c_i - d_i$. 
(viii) **Tardiness (T_i)**

The tardiness of job i is equal to max (0, L_i)

(ix) **Earliness (E_i)**

Earliness of job i is equal to max (0,-L_i)

**Note**: Lateness, tardiness and earliness are the three different ways of comparing the actual completion time with the desired completion time. Lateness considers the algebraic difference of each job regardless of the sign of the difference. Tardiness considers only positive difference i.e. jobs which are completed after their due dates and earliness considers only negative difference i.e. Jobs completed before their due date. When a job is completed before the due date, L_i is negative and when a job is completed after due date, L_i is positive and non zero when the job is tardy.

(x) **Total Elapsed Time (C_{max})**

It is defined as total completion time at which the sequence of all jobs completes processing on all the machines. It is also known as makespan. C_{max} = max \{C_i\}, i=1, 2, 3\ldots n

(xi) **Idle Time (I_k)**

It is defined as idle time on machine M_k. I_k = C_{max} - \sum_{i=1}^{n} p_{ik} \text{ where } p_{ik} \text{ is the processing time of job } i \text{ on machine } M_k.
(xii) Mean Completion Time (C)

It is average completion time of any job. \( C = \frac{\sum_{i=1}^{n} C_i}{n} \)

(xiii) Mean Flow Time (F)

It is the average time a job spends in the shop. \( F = \frac{\sum_{i=1}^{n} F_i}{n} \).

Two performance measures are equivalent if a schedule which is optimal with respect to one performance measure is also equivalent to other measure and visa-versa. The performance measures C and F are equivalent while the measures \( C_{\text{max}} \) and \( F_{\text{max}} \) are not equivalent.

In this thesis, we are mainly considered with total elapsed time, mean flow time, makespan, idle time of machines, due dates, satisfaction level of the demand maker, lateness and the earliness of the jobs etc. as the performance measure for a class of scheduling problems.

1.5.5 COMPONENTS OF PRODUCTION COST

Following are the components of production cost
(i) **Operational Cost**

The component of cost which represents the cost incurred in an actual production and which may be treated as processing time of the cost of jobs on all machines is defined as the operation cost.

(ii) **Job Waiting Cost**

The component of cost which is also called as the process inventory reflects the opportunity cost due to the waiting of the semi finished jobs in the shop for processing on some machines. The jobs which wait in the shop in the form of capital could have been utilized to produce additional return on capital.

(iii) **Machine Idle Cost**

When machines are idle, some opportunity is lost. But by utilizing this idle capacity of the machines, some return machines could be obtained. Determination of machine idle cost may be divided in to two categories.

(a) The idle time of the machine can be utilized to perform some other work, which may not be as profitable as the existing work.

(b) The idle time of the machines can not be utilized to perform any other useful work.

For category (a) the machine idle cost is the difference between the expected rates of return from the machine which is obtained by utilizing the idle time of the machine on a subordinate job. For category (b) the idle cost of the machine is the expected rate of return.
(iv) Penalty Cost of Machines

If the jobs are not completed by their due dates, certain cost, called penalty cost is incurred.

(v) Total Production Cost

The total production cost of a schedule is the sum of the component operation cost, job waiting cost, machine idle cost and penalty cost.

(vi) Minimization of Total Rental Cost

When machines are taken on rent, the following rental policies generally exists

**PL1** - All machines are taken on rent at the same time and are return also at the same time.

**PL2** - All machines are taken on rent at the same time and are returned as and when they are no longer required for processing the jobs.

**PL3** - All machines are taken on rent as and when they are required and are returned as and when they are no longer required for processing the jobs.

When policy PL1 is adopted then the sequence which minimizes the makespan will be the optimal sequence as it minimizes the total rental cost of the machines. Under policy PL2, for machine 2, the sequence which minimize the total elapsed time, will minimize the total rental cost of the machines.

*In this thesis we have adopted rental policy PL-3 and the work is carried out in chapter 4 for two stage, three stages and general m stage flow shop under different parametric constraints, which have wider applications.*
1.5.6 Methodology Used in Solving Scheduling Problems.

(i) Branch and Bound Technique

Branch and Bound technique is most widely used in scheduling. It is an enumeration technique and is applied to optimization. It is useful method for solving many combinatorial problems and is a general purpose strategy for curtailed enumeration. According to its name, the approach consists of two fundamental procedures.

Branching: It is the process of partitioning a large number into two or more problems.

Bounding: It is the process of calculating a lower bound on the optimal solution of sub problems.

The branching procedure replaces an original problem into a set of new problems that are:

- Mutually exclusive and exhaustive sub problems of the original problem.
- Partially solved versions of the original problem.
- Smaller problem then the original problem.

Furthermore, the sub problems can themselves be portioned in a similar fashion.

Branching and bounding form a branching tree with vertices and nodes representing partial sequence of jobs, which have been assigned position in optimal sequence, that is sought to optimize the given criterion. The partial sequences represent the subject of the sub problems formed by partitioning. The
first level node (P) corresponds to not having assigned any job to any position in the required sequence. From this node (P), n different branches emanate with a job fixed in the first position. Lower bounds, on the performance measure for the given problem are calculated separately for the partial sequence starting with 1, 2,…,n respectively, where lower bound for any partial schedule is the lowest value of the objective function irrespective of the order in which the remaining jobs are to be processed. The node or the vertex with the lowest lower bound is branched further in (n-1) nodes with the second position being occupied by the remaining (n-1) jobs. Again lower bounds are calculated with the partial sequences with the first two positions being filled. The vertex with the lowest lower bound among the unbranched vertices is branched further. This process continues till the vertex with complete sequence is reached. If the lower bound attached with this node is less than or equal to the lower bounds attached with unbranched vertices, the complete sequence represented by this node is an optimal sequence. Otherwise, the node with a lesser lower bound is branched further and the optimal sequence is obtained.

The minimum number of nodes in Branch and Bound technique, which can be created is n(n+1)/2. This happens when each stage of further branching, only one node is chosen for the purpose and the value of the complete sequence so obtained is less than all the lower bounds. The maximum number of nodes in Branch and Bound technique, for being created is 1+ n + n (n-1) + n (n-1)(n-2) ………n!. Which is when all the vertices are branched further, where n is the number of jobs.
(ii) Heuristic Approaches

The Branch and Bound approach and the elimination approach have two inevitable disadvantages, which are typical of implicit enumeration methods. First the computational requirement will be severe for large problems. Second, even for relatively small problems there is no guarantee that the solution can be obtained quickly, since the extent of the partial enumeration depends on the data in the problem. Heuristic algorithms avoid these two drawbacks, they can obtain solutions to large problems with limited computation efforts and their computations requirements are predictable for problem of given size. The drawback of heuristic approaches is that it gives no guarantee for optimality and in some instances it may even be difficult to judge about its effectiveness. The procedure is a full-fledged iterative algorithm where each of the iteration involves to conducting a search for new solution that might be better than the best solution found previously. When the algorithm is terminated after a reasonable time, the solution it provides is the best one that was found during any iteration.

Heuristic methods often are based on relatively simple common sense ideas for how to search for a good solution. These ideas need to be carefully tailored to fit the specific problem under consideration. Thus heuristic methods tend to be adhoc in nature, i.e., each method usually is defined to fit a specific problem type rather than a verity of applications. Heuristic or approximation algorithm are found to perform fairly well if it is assumed that heuristic methods will not be used either a constructive, polynomial time solution exists or when implicit enumeration is computationally feasible. In other words, approximations are not accepted when optimal solutions may be easily found.
(iii) **Metaheuristic**

The problems faced by researchers in Branch and Bound techniques and heuristic methods came to an end with the development of powerful metaheuristic. A metaheuristic is a general solution method that provides both a general structure and strategy guidelines for developing a specific heuristic method to fit a particular kind of problem. What is needed to provide a better chance of reaching a global optimum is to use a metaheuristic that will enable the process to escape from a local optimum. Metaheuristic is classified into the following three categories:

(a) **Tabu Search**: Tabu search is widely used metaheuristic that uses some common sense ideas to enable the search process to escape from a local improvement procedure in the usual way (i.e. only accepting an improved solution at each iteration) to find a local optimum. A key strategy of tabu search is that it continues the search by allowing non improving moves to the best solutions in the neighborhood of the current trial solution; the local improvement procedure is reapplied to find a new local optimum.

The drawback with this approach is that after moving away from a local optimum the process will cycle right search temporarily forbids moves that would return to a solution recently visited.

(b) **Simulated Annealing**: It is another widely used metaheuristic that enables the search process to escape from a local optimum. To introduce the concept of finding the global optimal of a complicated maximization problem is analogous to determine which of a number of hills is the tallest hill and then climbing to the top of that particular hill. One approach adopted into tabu search, is to climb the current hill in the steepest direction until reaching its top and then start
climbing slowly downward while searching for another hill to climb. The drawback is that a lot of time (iteration) is spent climbing each hill encountered rather than searching for the tallest hill.

Instead the approach used in simulated annealing is to focus mainly on searching for the tallest hill. Since the tallest hill can be anywhere in the feasible region, the early emphasis is on taking steps in random directions (except for rejecting some, but not all that would go downward rather than upward), in order to explore as much of the feasible region as possible. Because most of the accepted steps are upward, the search will gradually gravitate towards those parts of the feasible region containing the tallest hills. Given enough time the process often will reach and climb to the top of the tallest hill.

(c) Genetic Algorithm (G.A.): Genetic algorithms provide a third type of metaheuristic that is quite different from the first two types. This type tends to be particularly effective at exploring various parts of the feasible region and gradually evolving toward the best feasible solutions.

As simulated annealing is based on an analogy to a natural phenomenon (the physical annealing process) genetic algorithms are greatly influenced by another form of a natural phenomenon. In this case analogy is to the biological theory of evolution formulated by Charles Darwine in the mid of 19th century. Each species of plants and animals has great individual with variations that impart a survival advantage through improved adoption to the environment are most likely to survive to the next generation which is referred as the survival of the fittest.

The modern field of genetics provides a further explanation of this process of evolution and natural selection involved in the survival of the fittest.
These ideas transfer over to the dealing with optimization problems in a rather natural way. Feasible solution for a particular problem corresponds to members of a particular species where the fitness of each member is measured by the value of the objective function. Rather than processing a single trial solution at time, (as with basic forms of tabu search and simulated annealing) we now work with as entire population of trial solution. Genetic Algorithm searching mechanism starts with a set of solutions called a population. One solution in the population is called chromosome. The search is guided by the survival of the fittest principle. The search proceeds for a number of generations, for each generation the fittest solution (based on the fitness function) will be selected to form a new population. It could terminate after a fixed number of generations, or after a chromosome with a certain high fitness value is located or after a certain simulated time.

*In this thesis we have mainly used Branch and Bound techniques and Heuristic approach.*

**P and NP type problems in scheduling**

(a) **Polynomial Time Approximation Scheme:** The approximation scheme that produces algorithm having polynomial time complexity i.e. polynomial both in the problem size and the desired accuracy is called polynomial time approximation scheme.

(b) **The Classes P and NP:** The class P consists of all problems for which algorithms with polynomial time behavior have been found while the class NP is essentially the set of problems for which algorithm with exponential behaviors have been found. Clearly P is contained in NP. If one has a polynomial time algorithm for a problem it can always be inflated inefficiently
so that it takes exponential time. Also, occasionally, a problem originally in NP but not in P, is moved into P, polynomial time algorithm.

(c) **NP- Complete**: The NP- Complete problem form a subclass of NP. For if one finds a polynomial time algorithm for NP complete problem. Then one can answer all the problems in NP in polynomial time.

(d) **NP- Hard**: When optimization problem can’t be solved in polynomial time, it is called NP- hard, which arises more often in scheduling. There are at present no easy solutions. The only methods available are those of implicit enumeration, which may take a prohibitive amount of computations. Certainly large NP hard scheduling problems are all practical purposes solvable. If a problem is large and NP hard, then one must consider using heuristic methods. But the emphasis is that NP hardness of a problem alone is not sufficient reason to resort to the heuristic methods.

### 1.5.7 HISTORICAL DEVELOPMENT & BRIEF SURVEY OF SCHEDULING PROBLEMS

The result obtained by Johnson in 1954 [46] is standard fundamentals in the theory of sequencing. With Johnson’s paper, a great interest in the subject has grown and several promising research directions in the field of scheduling have been initiated by many researchers. Johnson considered n-jobs 2-machines flow-shop problem with the objective of minimization of total elapsed time. Johnson’s algorithm for 2-machine gives the Johnson rule: “Job i precedes job j in an optimal sequence with regard to minimum total elapsed
time if $\text{Min } (p_{i1}, p_{i2}) \leq \text{Min } (p_{j2}, p_{j1})$” Johnson’s algorithm for the 2- machine flow- shop problem was extended to special cases of 3- machines flow- shop problems. When

$$\text{Min } p_{i1} \geq \text{Max } p_{i2} : \forall i$$

or

$$\text{Min } p_{i3} \geq \text{Max } p_{i2} : \forall i \text{ or both gives optimal sequence in each case.}$$

Fennel Burns and Rooker(1976)[34] relaxed Johnson’s using following conditions:

(i) $p_{i1} \leq p_{j1} : \forall i & j ; i \neq j$

(ii) $p_{i2} \leq p_{j3} : \forall i & j ; i \neq j$

Where $p_{i1}$, $p_{i2}$ and $p_{i3}$ are taken as the processing times of job i on machines $M_1, M_2$ and $M_3$; where $i = 1, 2, \ldots, n$

Szwarc [81] in 1974 considered the following special cases on n- jobs, 3- machines flow- shop problems;

(i) $\text{Min } p_{i2} \geq \text{Max } p_{i1} : \forall i$

(ii) $\text{Min } p_{i2} \geq \text{Max } p_{i3} : \forall i$

(iii) $\text{Min } p_{i2} \leq \text{Min}( p_{i1}, p_{i3}) : \forall i$

and developed the efficient procedures for these cases.

Dudek and Teuton (1964)[27] gave a solution for the general n-jobs, m- machines sequencing problem. William Karush (1965)[116] demonstrated, by a counter example, that the algorithm provided by Dudek and Teuton (1964)

The basic Branch and Bound technique of n- jobs, 3- machines was developed by Ignall and Scharge, (1965)[43] and independently by Lomnicki(1965)[53]. They obtained the expressions of the lower bounds LB \[J_r] with the objective being minimization of total elapsed time as follows:

\[
LB \[J_r] = \begin{cases} 
\sum_{i=1}^{n} p(i,1) + \text{Min}[p(i,2) + p(j,3)] \\
t(J_r,2) + \sum_{i \in J_r} p(i,2) + \text{Min}[p(i,3)] \\
t(J_r,3) + \sum_{i \in J_r} p(i,3)
\end{cases}
\]

Where

\[J_r\] = Partial schedule containing r scheduled jobs

\[J'_r\] = Complement set of \[J_r\] containing (n-r) jobs not contained in \[J_r\]

t(I_r, k)= completion time of the last job of scheduling \[J_r\] on machine \(M_k\); k= 1,2,3.

Further research on Branch and Bound technique was carried out by Brown and Lomnicki (1966) [4], Gupta (1971)[37], Mc-Mahon and Burton (1967)[61], S.Miyazaki and N.Nishiyama (1980)[83] and Panwalkr & Khan (1975)[75]. They modified the Branch and Bound technique by improving the bounds or reducing the computations. Under unavailability constraint, Lee (1991)[56] studied the problem of parallel identical machine scheduling with non- simultaneous machine available times and with the objective of
minimizing the total elapsed time. He studied the machine scheduling problem with an availability constraint under different performance measures. Also pseudo-polynomial dynamic programming algorithm to solve the problem optimal was provided.

In several practical situations, the jobs are required for immediate use as soon as these come out from the machines, because waiting for job result in a significant and delay in bringing into use involves loss. In such situations, one has to be interested in minimizing the sum of completion times (total flow time) of all the jobs, rather than minimizing the completion time of the last job alone.

Ignall and Scharge [43] in 1965 provided the Branch and Bound technique of finding a sequence which minimizes mean flow time in n-jobs, 2-machines flow shop sequence problem. The lower bound $LB[J_r]$ of the partial schedule $J_r$ of $r$ jobs for minimizing total flow time is computed by the following formula

$$LB[J_r] = \sum_{i=1}^{n-r} T(i,b) + \text{Max}[g_1, g_2] : \text{Where}$$

$$g_1 = \sum_{p=1}^{n-r} T(J_r, A) + \sum_{q=1}^{p} (A_q + B_q) \text{ such that } A_1 \leq A_2 \leq \ldots \leq A_{n-r}$$

$$g_2 = \sum_{p=1}^{n-r} T(J_r, B) + \sum_{q=1}^{p} B_q \text{ such that } B_1 \leq B_2 \leq \ldots \leq B_{n-r}$$

$A_i$, $B_i = \text{Processing times of } i^{th} \text{ job on machine A and B respectively.}$

$T(i,A), T(i,B) = \text{Completion time of } i^{th} \text{ job on machine A and B respectively.}$
T( J_r,X) = Completion time of the last job of the partial schedule J_r of r jobs on machine X (X=A,B)

J_r = Set of n-r jobs in J_r.

A_p, B_p = Processing time of p^{th} job of J_r on machine A and B respectively.


Maggu and Dass(1977)[62] established equivalent job block theorem which proved a pioneering work in the scheduling field. The work was further extended by singh T.P(1985), [86] taking into account the various parameters as transportation time, break down interval, arbitrary time and to associate probabilities with processing time etc. applied in more realistic situations. L. Narain [58] in 2003 studied some special cases of flowshop models by using Branch and Bound technique. Recently Singh T.P et al(2006) [96] applied idle/waiting operator O_{i,w} in multistage flow shop scheduling.
(a) Optimization of Rental Cost in Scheduling


(b) Work on Parallel Machines in Scheduling

To complete the work on due time or to minimize the flow time parallel machines concept is very useful. There has not been much work reported in the case of parallel machine bicriteria problems. Garey and Johnson (1977)[38] proposed an algorithm for two independent identical processors, bicriteria scheduling problems. The criterion considered were the start times and dead lines. The algorithm did not give the solution to all the problems, that is, the job did not meet start times and dead lines for some problems. Latter, Paul Bratley et al (1971)[74] used the above two criterion as constraints to give an optimal completion time for a similar problem. Many more researchers have worked on parallel machines. Cenna et al (1991) [19] has worked on parallel machines on job shop scheduling problems. Singh T.P & Sunita (2008)[112] considered the parallel machines concept on various parameters in deterministic environment. Divya Prakash (1997) [33] extends the work of parallel machines in wider sense, developing an algorithm of bi criteria scheduling on parallel machines taking one unit processing time of each job.

In this thesis we have used parallel machines concept with various parameters in chapter 3. More over we have developed an algorithm
for the optimization of bicriteria scheduling taking the processing time in fuzzy environment.

(c) Due Date and the Analysis of Satisfaction Level of Demand Maker

Due Date is an important factor in production scheduling. The flow time of jobs should remain less than the due date of the jobs otherwise a penalty can be imposed on the demand maker. Chong, T.C.E (1955)[14] determined on optimal due date by taking sequence of n jobs on single machine. S.S. Lam and X. Cai (2000)[93] studied single machine scheduling with nonlinear lateness cost function and fuzzy due dates. Abdelaziz Hamid et al (2002), [2] worked on single machine taking common due date for each job in scheduling problems using neural network.

In this thesis, chapter 2 extends the idea of due date and analysis the satisfaction level of demand maker taking the concept of earliness and lateness of jobs in scheduling.

(d) Fuzzy Scheduling

More and more work has been done in deterministic environment and many important results have been developed by the researchers. Nowadays the environment is very much fluctuating and scheduling in fuzzy environment has become an open field for the researchers. Many researchers have associated the processing time with probability, but taking the processing time in fuzzy environment is more realistic than the deterministic environment or by associating the processing time with probabilities. Zadeh, L.A (1965)[122] studied fuzzy sets and system. He further extended the idea of fuzzy logic and approximate reasoning (1975)[124] and discussed the role of fuzzy logic in the management of uncertainty in expert system. Yager, R.R.(1981) [120] has

**In chapter 4 of this thesis, we have considered various parameters in scheduling under fuzzy environment. In chapter 5 of this thesis, a new approach of α-cut is introduced and developed an algorithm taking the concept of α-cut, which shows an improvement in results of scheduling and the approach in fuzzy environment gives a new direction to the schedulers.**

(e) **Bi-criteria in Scheduling**

bicriteria in scheduling for parallel machines. Singh T.P. (2006) [96] studied the bicriteria in scheduling under specific rental policy, processing time associated with probability including job block concept.

In chapter 3 (section 3.4) we have studied ‘bicriteria on parallel machines flow shop under various constraints’.

Conclusion

The mathematical analysis of the scheduling algorithm was stimulated by various problems annealing from the field of computer science, like resource management in operating systems, processor scheduling etc. These investigations resulted in a large number of more or less efficient (i.e. polynomially bounded in run time) algorithms. In response of computational complexity of the combinatorial & deterministic optimization, the focus shifted to the investigation of tractable methods which probably find optimal solution only most of the time i.e. algorithms which converge to a global optimal for most inputs but without any definite guarantee to do so in any particular input. The class of method compresses several related algorithms like simulated annealing (Van Laahovon et al, 1988[115]); Tabu Search (de Werra & Hertz, (1989) [31],Taillard,E,(1990)[108])& Genetic Algorithms (Davis L,(1985) [29]). Another mode of coupling integrates numerical heuristic symbolic scheduling approaches by lairing both components. Kusiak, 1988[50] presents scheduling problem from flexible manufacturing in which, first a set of part is produced, in the second step the parts are mounted into various assemblies.

Now if production and mounting times for the parts and assemblies are given by the scheduling problems on an outer layer corresponds formally to two-machine flow shop (Machine 1= part production, Machine 2= part assembly);
after orders are sequenced on flow shop level, part production can be scheduled dynamically on the inner layer of the problem by using various heuristic rules for priority queuing of parts.

Currently…. in the scheduling discipline, we are facing an impressing multitude of research lines. In addition of the efforts concerning synthesis of different approaches, two particular trends can be supported: on one hand new methodology are continuously introduced to the field while on the other hand, the modeling aspects are gaining dominance. In scheduling model of O.R., the structure of planning is modeled by a set constraint; which is the set of feasible solution. The scheduling yielding, the optimum or some close approximation of a defined objective function. The objective function expresses the overall economic interest of the planner but reduces practically to a single, uni-dimensional performance measure based directly on the decision variable of optimization model.

In our thesis, we have made an assessments regarding

(1) The modeling of scheduling constraints.

(2) The design of comprehensive scheduling system able to manage real life problem.

(3) Aimed towards reacting scheduling applications by making assessable pre defined scheduling algorithm, scheduling heuristics, scheduling evaluation facilities etc.