Chapter 4

A Filter for Restoration of Images Degraded by Atmospheric Turbulence

4.1 Introduction

The aim of any restoration procedure is to restore the image without loss of information of the object. In case of astronomical imaging, the limits on resolution is set by the intervening atmosphere.

Even with bigger and bigger telescopes it is not possible to get the object information in entirety. This is because of the gradients in refractive index in the atmosphere. The gradients in refractive index are due to thermal gradients in the atmosphere and these gradients fluctuate in time. Random phase delays introduced in the wavefront degrade the image recorded using ground based telescopes. The statistics of the behavior of these gradients have been studied, both theoretically and experimentally (Bouricius & Clifford 1970; Clifford et al. 1971; Buser 1971).

In any imaging, the information recorded depends on the intervening medium between the object and the imaging instrument, the psf of the imaging instrument, and the inevitable noise associated with any measurement. The psf of the imaging instru-
frequencies compared to the short exposure MTF. Hence noise contamination is higher at higher frequencies in the case of long exposure images. The functional form of the long exposure atmospheric psf is such that even though the MTF falls off rapidly it does not go to zero. This property of the long exposure MTF has been exploited to obtain the Fried’s parameter $r_0$ in the earlier chapter.

4.3 Image restoration

Conventional image degradation model is given by,

$$i(x, y) = o(x, y) * h(x, y) + n(x, y)$$ (4.1)

in the physical space, and in the Fourier space,

$$I(f) = O(f)H(f) + N(f)$$ (4.2)

where $f$ is a function of $u$ and $v$ (spatial frequency variables), $o(x, y)$ is the true object intensity distribution, $h(x, y)$ is the intensity psf, $n(x, y)$ is the noise, and $i(x, y)$ is the degraded image; $*$ denotes convolution; $x$ and $y$ are the coordinates in the physical space; $O(f), H(f), N(f)$ and $I(f)$ are their respective Fourier transforms.

The problem with direct deconvolution procedures like inverse filtering is that the deconvolution becomes ill-conditioned as $H(f) \to 0$. The general rule is to reduce large inverse values at high spatial frequencies since it is only at these high spatial frequencies the noise is high which leads to severe noise amplification by large inverse values. To counter this one can tune the inverse filter and reduce any large inverse values at high frequencies and hence control the noise amplification. These kinds of filtering does not lead to exact inverse and instead an approximate inverse is performed. In the next section we discuss the different kinds of noise associated with CCD imaging.
4.4 Kinds of noise

The restoration of images in the presence of noise poses a challenge even with advanced techniques available for image restoration in the presence of noise. If the noise frequencies and object frequencies are present at different frequency intervals then one can use a window and retrieve the true object information. The problem of image restoration becomes severe when the noise spectrum occupies the same spectral interval as the object information.

We will discuss few kinds of noise (Newberry, 1994) which is of importance to us and discuss their respective power spectrums.

1) Thermal or Johnson Noise
2) Shot Noise
3) 1/2 Noise
4) Quantizing Noise

Thermal or Johnson noise occurs because of the variations in electron density caused due to the thermal motion of the electron gas. The fluctuation in density of electrons lead to variations in potential difference between the electrical contacts of the recording instrument. The possible frequencies are unlimited and is termed as White Noise. The term white noise refers to the spectral shape of the noise power spectrum.

Shot noise is characteristic of any system where the charge collection occurs statistically. For example, in minority carriers flowing across a junction diode, or across the base of a junction transistor to the collector. These fluctuations can also occur at any frequency and hence the noise is termed as White Noise.

1/2 noise is determined by unknown effects at surfaces, contacts and barriers. It is associated with generation and recombination of minority carriers in semiconductor devices.

Quantizing noise occurs because of the limited capabilities of digital equipments. The digital equipments have a certain number of digits to display. These instruments
can be no more accurate that the number of digits they can handle.

In case of CCD imaging, the kinds of noise introduced in the image are,

1) Readout Noise
2) Dark Count
3) Background Noise
4) Processing Noise

**Readout Noise:** When the signal generated by light falling on a CCD is collected, amplified and converted to a digital value one or more of the above discussed noise degrades the signal. The noise added to each of the pixel by reading the signal is called readout noise.

**Dark Counts:** Even when light is not falling on the CCD electrons accumulate in the CCD. This adds to the signal generated when light from source falls on it. The rate at which this dark current is produced increases with increasing temperature.

**Background Noise:** A most commonly overlooked source of noise is this Background noise. Light, from natural skyglow, moonlight, or light pollution contributes to the signal collected by the CCD. This background adds to the photons generated by the true signal. Also any photon measurements have inherent uncertainty which adds to the noise. The presence of this background noise severely degrades the observation.

In general, the image we record can be written as,

\[ i(x, y) = i_o(x, y) + i_{\text{bgd}}(x, y) + i_{\text{dark}}(x, y) + i_{\text{bias}}(x, y) + i_p(x, y) \] (4.3)

where \( i_o(x, y) \) is the blurred image intensity distribution, \( i_{\text{bgd}}(x, y) \) is the intensity contribution from background radiation, \( i_{\text{dark}}(x, y) \) is the contribution of thermo electrons, \( i_{\text{bias}} \) is the contribution from the CCD and \( i_p(x, y) \) is the gaussian distributed read out noise.

The noise model can thus be modeled as,

\[ n(x, y) = i_{\text{bgd}}(x, y) + i_{\text{dark}}(x, y) + i_{\text{bias}}(x, y) + i_p(x, y) \] (4.4)
Figure 4.1: Power spectrum of noise obtained from a CCD image. A small area free from object information is selected and used for noise power estimation.

To model $n(x,y)$ in the real image we do the following. In the real image we take a part of the image where object information does not exist. We find the power spectrum of this small portion of the image. The power spectrum has been obtained for a two dimensional image. For the sake of clarity we plot a cut across the power spectrum. Fig (4.1) is an example of the power spectrum obtained from a small area of a CCD image obtained at Vainu Bappu Observatory, Kavalur, India. Bias subtraction and flat fielding has been done on the entire image before the power spectrum is estimated.

The power spectrum of noise in this real image gives us an idea of the noise level in the image. Also we get an idea of the FWHM of the noise power which can be used as an input for constructing the filter.

We see in Fig (4.1) that the low frequency power is higher than the high frequency power. Hence in the Wiener filter we use this information as an input for the Wiener parameter.
4.5 Filter for image restoration

The image we record is blurred by the point spread function of the medium and noise added to this blurred image. The task is to measure the image by removing the blur and the noise. Our aim is to find a filter which when applied to the blurred noisy image will give as output an object estimate which is as close to the true object as possible. We will estimate the true object from the degraded image such that the least square error between the estimated object and the true object is minimised.

If \( \hat{O} \) is the object estimate, then

\[
\int \int |\hat{O}(f) - O(f)|^2 \, du \, dv
\]

is minimised.

From this we obtain a filter (Goldman 1953),

\[
\phi(f) = \frac{\phi_o(f)H^*(f)}{\phi_o(f)H^2(f) + \phi_n(f)}
\]

We need information on the power spectrum of the true object and the noise to build this filter. The noise power spectrum can be modeled by assuming that beyond a certain cut off frequency the Fourier components of the image does not contribute and hence those Fourier frequencies beyond the cut off frequency are noise contribution. We can then extrapolate the power spectrum of noise into the lower frequency domain and hence model the noise.

In deriving this filter function we try to minimise the least square error in the estimated object to the true object. Additional constraints could be placed on the error estimate and different filter functions can be obtained. (Peter A. Jansson 1997)
4.6 Object sharpness as a criterion for image restoration

Frieden (1975), considers the above problem where he introduces a criterion called the sharpness criterion,

\[
\langle \int_{-\infty}^{+\infty} \left| \frac{d\hat{O}(x)}{dx} \right|^2 dx \rangle = \langle \int f^2|O(f)|^2 df \rangle \quad (4.7)
\]

where \( f \) is a function of \( u \) and \( v \). The integration is performed from \(-f_c\) to \(+f_c\) where \( f_c \) is the cut off frequency. Above this cut off frequency the data contain no information about the object \( o(x) \). The term \( |\frac{d\hat{O}(x)}{dx}|^2 \) is based on the assumption that the object intensity distribution is a smooth function of \( z \) and does not have sharp discontinuities. We seek to minimise the equation which includes both the mean square error criterion used earlier in the conventional Wiener filter and the sharpness of the solution such that,

\[
\langle \int \phi(f)[O(f) = H(f) + N(f)] - O(f) \rangle^2 df + \beta \langle \frac{dO(f)}{df} \rangle^2 dx \rangle \quad (4.8)
\]

Setting the integrand to 0, we get

\[
\phi(f) = \left( \frac{H(f)^2 \phi_n(f)}{H(f)^2 \phi_n + \phi_n} \right) \left( \frac{1}{1 + \beta f_c^2} \right) \quad (4.9)
\]

The efficiency of these filters depends on the noise level in the recorded image.

For efficient implementation of the above filters discussed we need the estimate of the ratio of the noise power to the signal power. This ratio is referred to as the Wiener parameter. The noise power spectrum might be flat, smooth or tilted. A reasonable noise model has to be assumed to get an image restored with minimum least square error between the restored and the true object. The image restoration procedure is highly dependent on the noise model we assume. But any reasonable hypothesis of the noise model close to the true noise power spectrum will yield restorations with minimum least square error between the true object and the estimated object.

The power spectrum of noise in CCD images have higher low frequency power than at high frequencies. Hence we will try to model the low frequency power spectrum of
noise. We model the low frequency part of the power spectrum of the noise and the object using a gaussian function with varying width. Let the low frequency Fourier spectrum of the object be,

$$\phi_o' = e^{-\left(f/\sigma_o'\right)^2}$$  \hspace{1cm} (4.10)

and the low frequency spectrum of the noise be,

$$\phi_n' = e^{-\left(f/\sigma_n'\right)^2}$$  \hspace{1cm} (4.11)

where $\sigma_o'$ and $\sigma_n'$ are the widths of the low frequency part of the power spectrum of the object and the noise respectively.

Hence the ratio of these two will be,

$$\frac{\phi_n'}{\phi_o'} = e^{-f^2\left[\left(\frac{1}{\sigma_o'}\right)^2 - \left(\frac{1}{\sigma_n'}\right)^2\right]}$$  \hspace{1cm} (4.12)

It is reasonable to assume that $\sigma_o'$ is always greater that $\sigma_n'$.

Panel 1 in Fig (4.2) is a test object. Panel 2 is the blurred noisy image with a signal to noise ratio equal to 200. The noise added to the image is a gaussian distributed random noise. If we assume the signal to noise ratio a constant at all Fourier frequencies, then the Wiener parameter is a constant at all frequencies. Fig (4.3) is the power spectrum of a gaussian distributed noise. Panel 1 in Fig (4.4) is the plot of the mean square error versus different values of Wiener parameter for an image with signal to noise ratio equal to 200. Panel 2 in Fig (4.4) is the plot of the mean square error versus different values of Wiener parameter for an image with signal to noise ratio equal to 50. Panel 1 in Fig (4.5) is mean square error estimated for an image with signal to noise ratio equal to 200. Panel 2 in (4.5) is an estimation of mean square error for an image with signal to noise ratio equal to 50.

4.7 Deconvolution in the presence of noise

The recovery of signal from blurred noisy image demands a good idea about the blurring function and a good statistical knowledge of the noise in the image.
noise. We model the low frequency part of the power spectrum of the noise and the object using a gaussian function with varying width. Let the low frequency Fourier spectrum of the object be,

$$\phi_o = e^{-(f/\sigma_o)^2} \quad (4.10)$$

and the low frequency spectrum of the noise be,

$$\phi_n = e^{-(f/\sigma_n)^2} \quad (4.11)$$

where $\sigma_o^L$ and $\sigma_n^L$ are the widths of the low frequency part of the power spectrum of the object and the noise respectively.

Hence the ratio of these two will be,

$$\frac{\phi_n}{\phi_o} = e^{-f^2[2(\frac{1}{\sigma_o^L})^2-(\frac{1}{\sigma_n^L})^2]} \quad (4.12)$$

It is reasonable to assume that $\sigma_o^L$ is always greater than $\sigma_n^L$.

Panel 1 in Fig (4.2) is a test object. Panel 2 is the blurred noisy image with a signal to noise ratio equal to 200. The noise added to the image is a gaussian distributed random noise. If we assume the signal to noise ratio a constant at all Fourier frequencies, then the Wiener parameter is a constant at all frequencies. Fig (4.3) is the power spectrum of a gaussian distributed noise. Panel 1 in Fig (4.4) is the plot of the mean square error versus different values of Wiener parameter for an image with signal to noise ratio equal to 200. Panel 2 in Fig (4.4) is the plot of the mean square error versus different values of Wiener parameter for an image with signal to noise ratio equal to 50. Panel 1 in Fig (4.5) is mean square error estimated for an image with signal to noise ratio equal to 200. Panel 2 in (4.5) is an estimation of mean square error for an image with signal to noise ratio equal to 50.

4.7 Deconvolution in the presence of noise

The recovery of signal from blurred noisy image demands a good idea about the blurring function and a good statistical knowledge of the noise in the image.
Figure 4.2: Panel 1 is the true object's Fourier spectrum and Panel 2 is the Fourier spectrum of the blurred noisy image of the object in panel 1. The signal to noise ratio is 200.

Figure 4.3: Power spectrum of a typical gaussian distributed noise used in the simulations
Figure 4.4: Panel 1: Plot of mean square error versus Wiener parameter (S/N = 200) Panel 2: Plot of mean square error versus Wiener parameter (S/N = 50)

Figure 4.5: Panel 1: Plot of mean square error for different values of the FWHM of the ratio of the noise to signal power. (S/N = 200) Panel 2: Plot of mean square error for different values of FWHM of the ratio of the noise to signal power. (S/N = 50)
In any imaging the signal or the image we record are all corrupted by more than one kind of noise. In case of ground based astronomical imaging the background noise plays an important role in image degradation. This background noise is a combination of the natural skyglow and the inherent noise associated with the photon measurements. Added to this could be the moonlight or light pollution in some cases.

The noise which gets added to the image is a combination of different kinds of noise which degrades the original image at every stage. Hence it is appropriate to consider noise as a function of frequency and try to restore images.

Now, the image degradation model in the Fourier space as,

\[ I(f) = O(f)H(f) + N(f) \]  \hspace{1cm} (4.13)

where \( H(f) \) is the point spread function of the medium, in our case it is the atmospheric point spread function and \( N(f) \) is the frequency dependent noise.

\[ \phi(f) = \frac{H(f)}{H^2(f) + \frac{\sigma_n(f)}{\sigma_o(f)}} \]  \hspace{1cm} (4.14)

here \( \frac{\sigma_n(f)}{\sigma_o(f)} \) is not replaced by a constant as was done in the case of a conventional Wiener filter.

We consider an object which has sharp discontinuities in the physical space so that it has higher Fourier frequencies. In Fig (4.6), the 3 different panels shows the frequency response of the Wiener filter, the frequency response of the filter obtained by including the sharpness in the object in the minimizing criterion and the filter derived by modeling the noise power spectrum and hence the Wiener parameter is not held a constant but rather is modeled from a small area in the observed image itself, as described earlier.

To test the restoration of the filter we construct an object which has high frequency components. Panel 1 in Fig (4.1) is an example of such an object. The blurring function is taken to be a long exposure point spread function. Frequency dependent noise is added to this blurred image. The signal to noise ratio in this example is roughly 200. To check for the dependence of the filter’s frequency response
Figure 4.6: Frequency response of the filters.
to the Wiener parameter, the restorations were carried out for various values of the Wiener parameter. We also tested for the dependence of the restoration with error in estimation of the width of the true point spread function. In all these simulations the noise added to the image is gaussian distributed.

In the next section we show with some examples the restoration of images using these filters, their advantages over the others at different signal to noise ratios. The performance of these filters also depend on our a priori knowledge about the psf which has blurred the true object.

4.8 Simulation of objects and restoration using different filters:

We will use the Fourier spectrum of the true object shown in panel 1 in Fig (4.1) in our simulations. We try to recover the true object's Fourier spectrum from the Fourier spectrum of blurred noisy images generated with different signal to noise ratios. We will confine ourselves to restoration of images at signal to noise ratios 50 and 200. To determine the sensitivity of the restoration procedure we plot the mean square error between the restored Fourier spectrum and the Fourier spectrum of the true image as a function of the free parameter in the filters. The free parameter in conventional Wiener filter is the Wiener parameter and the free parameter in the noise modeled Wiener filter is the FWHM of the noise power spectrum. We see that the overall mean square error is less in the case of restoration using noise modeled Wiener filtered Fourier spectra compared to the ones restored using conventional Wiener filtering.

The restorations were tried out for different values of Wiener parameter and different widths for the noise power spectrum. Fig (4.7) is one such of restorations. These restorations were performed on images with signal to noise ratio equal to 200. Panel 1 is the Fourier spectrum obtained using the conventional Wiener filter for filtering the noisy spectrum, panel 2 is the Fourier spectrum of the noisy image obtained
using the Wiener filter with object sharpness criterion and panel 3 is the Fourier spectrum obtained using the Wiener filter where noise is modeled. We see that the recovery of the high frequency information is more pronounced in the case of noise modeled Wiener filter compared to the other two filters. The plots of mean square error versus Wiener parameter in the conventional Wiener filtering and the plot of mean square error versus the FWHM of function describing the noise to signal ratio are given in Fig (4.4) and Fig (4.5) respectively.

For a realistic simulation of images degraded by atmospheric turbulence the following was done.

An object intensity distribution is simulated and is convolved with the short exposure atmospheric psf. The short exposure psf was generated the following way. We assume a power spectral density of the form (Tatarski, 1961),

$$\rho(f) = \frac{L_o^{11/3}}{(1 + f^2 L_o^2)^{11/6}}$$

(4.15)

where $L_o$ is the aperture size or the outer scale of turbulence and $k$ the spatial frequency is given as $f = \sqrt{(u^2 + v^2)}$. This is taken to be the power spectral density of the turbulence. This is now multiplied with $\exp(i \phi)$, where $\phi$ is the random phase generated uniformly between $-\phi$ and $+\phi$. This resulting pattern in $u,v$ space is Fourier transformed to yield one realisation of the wavefront. The object intensity distribution is convolved with the simulated short exposure psf. The result is a short exposure image of the object. A gaussian noise is added to this degraded image. Several such noisy degraded images are realised. These noisy degraded images are used as test images for processing using the Wiener filter with frequency dependent Wiener parameter for image recovery and restoration.

For different values of signal to noise ratio and also for varying values of outer scale of turbulence $L_o$ simulations were done. The results are presented here. Fig (4.8) is the true object intensity distribution used in the simulations. Fig (4.9) is the short exposure image of the object with noise and Fig (4.10) is the restored image. Fig (4.8) was generated by an power spectral density with outer scale of turbulence equal to 250 cm. The width of the filter used in the Fourier space is 0.3. The number of
Figure 4.7: Fourier spectrum of the image restored using different filters. In panel 2 $\beta=0.01$ and in Panel 3 the width of the gaussian representing the ratio of noise to signal power is 0.7
short exposures added is 50. The signal to noise ratio in the image is 100.

In the restored image we see that the high frequency information is restored to a great extent. The sharp edges are clearly seen and the four squares are distinctly

4.9 Application of Filters on real images

4.9.1 Images of Globular cluster

We used the image of Globular cluster NGC1904. (observed by Prof. RamSagar and Mr. Alok Gupta using the 2.34m telescope at the Vainu Bappu observatory, Kavalur, India). The mean wavelength of observation is 5656Å. The image is roughly 60 arcseconds × 60 arcseconds. This image is cut into small portions of 5 arcsecond × 5 arcseconds. Here we show a restoration of one portion of the image. Fig (4.11) is a contour map of a part of the Globular cluster NGC1904. Fig (4.12) is the image restored using the Wiener filter in which the noise to signal ratio is modeled. A Wiener parameter of 0.01 is used in both the filters.

4.9.2 Comet images

To test the filter restoration was performed on images of comets observed at the Vainu Bappu observatory, Kavalur, India. The observations were carried out in 1m and 2m telescopes. The details of observations are,

<table>
<thead>
<tr>
<th>Telescope</th>
<th>1m</th>
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<tbody>
<tr>
<td>Filter</td>
<td>R</td>
</tr>
<tr>
<td>Central Wavelength</td>
<td>6000Å</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>1000</td>
</tr>
<tr>
<td>Plate Scale</td>
<td>16&quot; / mm</td>
</tr>
</tbody>
</table>
The details of the CCD camera which was used for imaging are as follows. In 1m telescope the CCD camera used had the following characteristics. It is a Tek1024 CCD camera. The CCD camera used in the 2m telescope had a gain of 5.9e/ADU measured with 16 bits. Noise at 7.2 e^- rms. At 4X setting, the gain is 1.4 e/ADU. In the 1m telescope for a gain of 13 e/ADU the noise is 7 e^- rms. The data we get is 16 bit data. It is a Thomson 512 X 378 pixel chip with a plate scale of 16" per pixel.

4.9.3 Results

The object was observed using a CCD camera. This image is then flat fielded and bias subtracted. This flat fielded and bias subtracted image is then Fourier transformed and multiplied in the Fourier space with the Wiener filter. We can see structures in the restored image all the way upto 40 arcseconds. The central region is blocked so that the other fainter features are seen when the image is displayed.

Fig (4.13), Fig (4.14) and Fig (4.15) are the original images and the restored images of the comet. The image on top is the original image and the images displayed at the bottom are the restored image. The bow like structures far away from the core and finer features near the core are seen in the restored image.

4.10 Discussions and Conclusion

In this chapter we have attempted to highlight the problem of image restoration in the presence of noise. Here we have considered the noise introduced when an image is observed using a CCD camera. Several attempts have been made and are being
Figure 4.8: Simulated object intensity distribution

Figure 4.9: Blurred and Noisy image

Figure 4.10: Image obtained by successive addition of short exposure image filtered using noise modeled Wiener filter
Figure 4.13: On the left is the image of comet Hale Bopp as observed at the 2.3 m optical telescope at Vainu Bappu Observatory, Kavalur, India. The image on the right is the restored image.

Figure 4.14: Contour map of 5" X 5" portion of the original image of the globular cluster NGC1904.
Figure 4.15: Contour map of the image filtered using Wiener filter constructed using noise model. The dotted line corresponds to negative values. Wiener parameter $W$ is 0.01.

made to alleviate the problem of image restoration in the presence of noise. Since noise is not a deterministic process we try to model the ratio of the noise to signal power. We have shown that any reasonable hypothesis for obtaining the Wiener parameter can restore some of the high frequency features. Noise contributes over a wide range of Fourier frequencies hence, any attempt to remove noise removes some of the true object's information.

If the seeing is poor, then the higher frequencies in the Fourier spectrum of the observed object is attenuated more. In this chapter we have shown that even with a signal to noise ratio of 200 it is difficult to recover the higher Fourier frequency information in entirety. Therefore we need especially good sites for astronomical observations.

The technique we have discussed in this chapter is applicable only to long exposure images. If we are looking for changes in the object with time scales of few milliseconds then we necessarily have to go in for short exposure or speckle imaging. There are techniques like blind iterative deconvolution which restores images using a single
frame. In these techniques Fourier modulus error is an important constraint. Since the short atmosphere psf is not symmetric in the physical space the atmospheric phase corrupts the phase of the object. Restoration of these short exposure images introduces spurious phase information in the restored image. Instead of specklegrams if long exposure images are used, phase information is retained to a large extent to constrain the ambiguities of rotation and translation which one encounters with phase retrieval (Jefferies, & Christou 1993)

A stable sky for few seconds is a prerequisite to obtain maximum information from a long exposure image. We have seen that noise plays an important role in image restoration. Any attempt to remove noise tampers with the true object information since there is no specific frequency interval in which noise alone exists. Measurement noise effects on image restoration are quite important under low light level conditions (Roggemann, Welsh & Fugate 1997).

Artifacts get produced in the restored image when the signal to noise ratio is low. Each technique has got its own limitations when it comes to recovery of true object information from the blurred noisy image. The classical inverse filtering and the Wiener filtering are linear filtering methods. There are nonlinear image restoration algorithms 1) Maximum-likelihood estimation is a method where the object and image are treated as probability density functions and we try to estimate the most likely object to have resulted in the measured image. 2) Blind deconvolution algorithm is a constrained iterative method which results in joint estimation of the true object and the psf. 3) CLEAN algorithm - This iterative algorithm which uses successive subtraction of properly weighted and located dirty psf's located at the brightest point in the image. This process is completed until the residual image in the original array reaches the rms noise level of the data, 4) Maximum entropy algorithm - This method is based on maximizing a specialised measure of error based on the concept of entropy used in information theory. 5) Super resolution algorithms - Here the is is to obtain an accurate estimate of the object spectrum within the measured passband and to reconstruct the object's spectrum outside the passband.

Any linear processing method cannot generate nonzero values at unmeasured spatial
frequencies and cannot extrapolate into noise contaminated regions of the Fourier frequency plane. Here we do not attempt for information beyond the resolution limit of the telescope. If \( D \) is the diameter of the telescope then the spatial cut off frequency of the telescope is \( f_c^{\text{teles}} = D/\lambda \). The Fried’s parameter \( r_o \) characterising the atmospheric psf limits the cut off frequency to \( f_c^{\text{atm}} = r_o/\lambda \). It is this information lying between the atmospheric cut off spatial frequency \( f_c^{\text{atm}} \) and the telescope cut off spatial frequency \( f_c^{\text{teles}} \) which we are trying to recover. We have tried to show that simple linear filtering methods with reasonable estimate of the blurring function and noise can restore images with information about the object lying between \( f_c^{\text{atm}} \) and \( f_c^{\text{teles}} \). The restoration of comet images and the globular cluster images using linear filtering techniques shows the possibility of obtaining information beyond \( f_c^{\text{atm}} \).

Fig (4.16) is the plot of amplitude of the Fourier spectrum at a spatial frequency as a function of the Fried’s parameter. This is to highlight the problem of contrast reduction in the observed image with changing atmospheric conditions. We can see that as the Fried’s parameter changes the strength of the higher spatial frequency falls at a faster rate than the the rate at which the lower frequency strength changes.

The main motivation of this work is towards understanding the linear image restoration methods. Iterative processing methods like maximum entropy method, blind iterative deconvolution etc., are computationally intensive procedures and the processing needs to be done offline. Our future plan is to try and implement the linear filtering method discussed in this chapter on the telescope to produce high resolution images online.

Optical disturbances like the photometric stability of the sky, scattered light from the atmosphere and the telescope, image motion, and size of the isoplanatic patch all play a crucial role in the resolution of the final image.

In Fig (4.16) we see that at smaller values of Fried’s parameter the reduction of the value of the amplitude is more pronounced in the case of all the frequencies. As we go to higher values of \( r_o \) we see that the attenuation of high frequency component is high
Figure 4.16: Plot of amplitude of the Fourier frequencies with changing Fried's parameter. + - low spatial frequency component, * - intermediate spatial frequency component, - High frequency component

compared to the low frequency component. As the value of the Fried's parameter $r_o$ becomes smaller then the value of the Fourier amplitude will reach the value of the noise at those frequencies.

CCD system noise level is photon-shot noise limited in the case of high light levels and at low light levels the CCD system is readout noise limited (Mackay 1986). Intrapixel nonuniformity has been largely ignored. Although CCD's are flat some of them are thin membranes suspended by their edges. They show surface nonuniformities of typically $70\mu$ peak to peak. The electronic hardware systems driving the CCD themselves have significant effect on the data taken at a telescope. Ground loop problems between the dewar and the driver electronics, within the driver electronics or between the driver electronics and the computer system can cause interference with the readout electronics, giving rise to patterns that are synchronized with the line frequency. Electrical noise from motors, light dimmers, or computer parts can be picked up by the detector system. Electronic drift can cause background levels to change during the readout of one frame over a longer period. This problem can be overcome by reading more pixels in each row than are physically there. In this
process we get dark pixels at the end of each row of data which can be used to monitor variations of dark level and readout noise. It is very important that the astronomer is on the lookout for these problems since the above discussed problems can significantly contribute to the noise level in the data (Mackay 1986). Hence it is imperative that we model the noise in the data. In this chapter we have shown that a simple hypothesis of the ratio of the noise power spectrum to the signal power spectrum results in superlative results.

Apart from the availability of good observational site and good detector systems we need good computing facilities to support the observations. For implementing image restoration techniques we need fast computing machines. New techniques like phase diverse speckle restoration have been used to produce good quality images. This technique will be discussed in some detail in the next chapter. The requirements to use techniques like phase diverse speckle restoration are several workstations working in parallel over many weeks to produce 178 frames of high quality data (Lofdahl 1997). It is necessary to have image selection algorithms which would select the best images online. If we are looking for features which are undergoing changes every few seconds then the frames have to be acquired at those speeds.