Chapter 12

Discussion and Outlook
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In this final chapter, we summarise all the major results obtained by our investigation and comment on the scope of further studies that could be taken up in this field.

Quantal bound states are significant both from theoretical and experimental points of view and energy profile is a signature of the potential. In relativistic quantum mechanics, as is well known, a general interaction potential may be introduced in two ways following the minimal coupling scheme - one as the fourth component of a four vector field and the other as a Lorentz invariant. It is usually difficult to study the quantum dynamics of relativistic particles under short range interactions exactly and results generally depend on numerical or approximate methods.

We have obtained the analytical solutions of spinless particles in a one dimensional screened Coulomb potential studied by Dominguez Adame and show the existence of several genuine bound states, in both the coupling schemes. While the bare Coulomb potential in one dimension presents only scattering states, the screened Coulomb potential appears to be very confining and admits a rich set of solutions, non-relativistically and relativistically, both with the Klein-Gordon and Dirac equations. Interestingly, the energy profiles indicate a qualitatively different behaviour of bound states in the two cases. Screened Coulomb potential of the type we have considered can find applications in solid state physics, nuclear physics, quantum chemistry and biophysics.

Our study concerning the linear finite range potential is seemingly interesting, as such a potential is envisaged as a quark-confining potential. We have shown that both the relativistic wave equations yield true bound states with this potential. While in the Klein-Gordon case, the potential is introduced as the fourth component of a Lorentz vector field,
in the Dirac case it is treated as a Lorentz scalar. The relativistic version of the WKB method is indeed useful in obtaining the eigenenergies quickly without having to solve complicated differential equations.

While it is a common feature to address bound state problems in one and three dimensions, discussion in two dimensions is not commonly available even in the non-relativistic case. Since certain solid state devices like CCD behave like two dimensional systems, as well as for pedagogical reasons, we have addressed the quantal bound states in two dimensions. The relativistic and non-relativistic eigenenergies of a particle in a two dimensional square well potential are computed and compared. Significantly, we find that for the same potential strength, relativistic particles are more tightly bound than the non-relativistic particles. The problem of two dimensional square well potential is of vital importance and may find applications in opto-electronic devices such as semiconductor lasers and modulators.

The harmonic oscillator and rigid rotator which are well known problems of non-relativistic quantum mechanics, are addressed relativistically. Adapting the principle of Moshinsky, the Klein-Gordon and Dirac equations lead to exactly solvable equations having appropriate non-relativistic limits. Contrary to the non-relativistic oscillator, the relativistic oscillator has unevenly spaced levels. While discreteness is a property of bound systems, equispacedness is a characteristic feature of the non-relativistic oscillator. Such oscillator models are indeed important in nuclear shell model, quark model of hadrons and in the field theory.

Since much of rotational spectroscopic results rely on rigid rotator models, we have introduced the notion of 'relativistic rotator' and investigated the energy spectrum of spin-zero and spin half homonuclear and heteronuclear systems. It is seen that the level spacing which is equidistant for a non-relativistic rotator is no longer so for a relativistic rotor. Our results can find application in microwave spectroscopy.
We have also examined the relativistic generalisation of the Kronig-Penny model, that may be more aptly called the periodic potential. Explicit calculations reveal that relativistic effects cause a reduction of the width of the allowed energy bands. The relativistic version of the well-known WKB method and the relativistic analogue of the virial theorem are deduced in the classical and quantum domains and their non-relativistic limits are worked out. Extension of the critical binding conditions to relativistic domain have been made. Contrary to the formal Klein-Paradox, often addressed in the context of scattering situations, we have brought to light another form of Klein-Paradox that deals with bound states. A paradoxical situation emerges when bound states appear for a potential barrier when the strength of the latter exceeds $2mc^2$, with the 'hill' becoming a 'well' and scattering swapped by binding.

The newly developed relativistic version of the usual scattering problem using the Green's function formalism is employed to obtain the first order correction to Born scattering amplitude and the differential scattering cross-section. This novel method, apart from being elegant, is a straightforward extension of the non-relativistic approach to scattering in wave mechanics. The validity of the relativistic scattering formulation is examined with specific examples. For the pure Coulomb potential, which is the limiting case of the screened Coulomb potential, our results may be used in connection with Rutherford scattering for accurate comparisons.

In a nutshell, we have attempted to explore the relativistic generalisation of those phenomena and models, often encountered in non-relativistic quantum mechanics. Such an approach involves mathematical complexities and requires the use of modern computational tools like Mathematica. We have developed programs using Mathematica to solve intricate transcendental eigenvalue equations involving special functions like confluent hypergeometric functions, Bessel functions, Airy functions, parabolic cylindrical functions etc.,.
Relativistic studies, apart from having immense pedagogical value can also be of practical use in the light of ever increasing accuracies of modern experimental methods. Our studies also bring out the limitations of the single particle wave equations in the relativistic domain. Some parts of our research work have been published in refereed journals.

Studies on binding energy provide a wealth of valuable information on the nature of the interaction. Extension of this work to higher space dimensions, where spin and angular momentum play a significant role would be challenging. A critical study of bound states with more structured potentials will shed light on realistic models of three dimensional lattices. Bound states for particles with arbitrary spin would be exciting. Hence we believe that such studies are potentially significant and worth undertaking.
References


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