Chapter 6

General Theory of the Emitted Line Polarization by an Atom in the Presence of External Fields

6.1 Introduction

In recent years astrophysical spectropolarimeters have revealed a variety of details which have so far remained inaccessible. In this context, it is of interest to examine theoretically the nature of signatures in polarized line spectra which arise due to the possible presence of electric fields in addition to the well established presence of magnetic fields in astrophysical plasmas. As is well known, Oxygen has a prominent line at 844.6 nm corresponding to a spin-1 to spin-0 transition, while Nitrogen has
a prominent line at 868.6 nm which corresponds to a transition from a spin-3/2 to a spin-1/2 level. Recently, spectropolarimetric observations of the Na I 589 nm and 589.6 nm emission lines have been reported (Allen et al. 2002) from the exosphere of Mercury. The 589 nm emission line corresponds to a transition from spin-3/2 to spin-1/2 whereas 589.6 nm line involves a transition from spin-1/2 to spin-1/2. It is interesting to note that the spin-3/2 level is sensitive to the electric quadrupole as well as uniform magnetic fields, whereas the spin-1/2 levels get split only due to the magnetic field. It is also known that the linear polarization pattern of these Na I lines, observed in solar spectra, still poses a theoretical challenge (Nagendra & Stenflo 1999, Stenflo & Nagendra 1996 for several papers concerning this important line). It is desirable, therefore, to develop a formalism to describe the polarization of line radiation emitted by the atom while it makes a transition from an upper level with spin $J_u$ to a lower level with spin $J_l$, which is applicable to situations where the upper and lower levels are split, due to external fields, into $2J_u + 1$ levels $|\Psi_i\rangle$ with $i = 1, 2, \cdots, (2J_u+1)$ and $2J_l+1$ levels $|\Psi_f\rangle$ with $f = 1, 2, \cdots, (2J_l+1)$ respectively, where $|\Psi_i\rangle$ and $|\Psi_f\rangle$ are not necessarily identifiable with the magnetic substates. In such scenarios, the polarization of the atomic states may conveniently be represented by the Fano statistical tensors $t_q^k$. These tensors $t_q^k$ are governed completely by the details of the relative configuration of the external fields. The Stokes parameters $(I, Q, U, V)$ completely specify a general state of the polarized radiation.

In this chapter, using the density matrix formalism, we derive elegant formulae for the emergent Stokes parameters in terms of the Fano statistical tensors $t_q^k$ characterizing the upper and lower levels. We make a detailed comparison of the emergent
Stokes profiles arising $J = 1 \rightarrow J = 0$ as well as $J = 3/2 \rightarrow 1/2$ transitions in the presence of external electric and magnetic fields, with those in the case of Zeeman effect. Explicit formalae for the emitted Stokes parameters are presented. Specific features or signatures of the polarized line spectra are discussed as functions of the relevant physical parameters. The Stokes parameters are also analyzed in terms of the Zeeman term contributions and the cross term contributions (which arise due to quantum interference).

6.2 General theory of the line polarization when both the upper and lower atomic levels are split by the presence of external fields

We may express the upper (spin $J_u$) and lower (spin $J_l$) levels $|\Psi_i\rangle$ and $|\Psi_f\rangle$, with energy eigenvalues $E_i$, $E_f$ respectively, as

$$|\Psi_i\rangle = \sum_{m_u=-J_u}^{J_u} c_{m_u}^{i} |J_u, m_u\rangle , \quad i = 1, 2, \ldots, (2J_u + 1) ,$$  

(6.1)

with $\sum_{m_u} |c_{m_u}^{i}|^2 = 1$ and

$$|\Psi_f\rangle = \sum_{m_l=-J_l}^{J_l} c_{m_l}^{f} |J_l, m_l\rangle , \quad f = 1, 2, \ldots, (2J_l + 1) ,$$  

(6.2)

with $\sum_{m_l} |c_{m_l}^{f}|^2 = 1$.

The corresponding atomic spin density matrices $\rho^i$ and $\rho^f$ are given in terms of their elements, by

$$\rho_{m_i, m_i'}^{i} = c_{m_i}^{i} c_{m_i'}^{*i} ,$$
where the shorthand \[|J\] = (2J + 1)^{1/2} is used.

It may be noted that Eqs. (6.3) and (6.4) define the Fano statistical tensors \(t_{q}^{(i)}(f)\) characterizing the spin polarization of the initial and final atomic states, once the expansion coefficients \(c_{m_{u}}^{l} = \langle J_{u}, m_{u} | \Psi_{i} \rangle\) and \(c_{m_{l}}^{l} = \langle J_{l}, m_{l} | \Psi_{f} \rangle\) are known. These coefficients \(c_{m_{u}}^{l}\) and \(c_{m_{l}}^{l}\) are obtained by diagonalizing the Hamiltonian \(H_{int}\), characterizing the interaction of the atom with the external fields.

Let us denote by \(\langle f | T(k, \mu) | i \rangle\), the transition matrix element from the initial state \(|\Psi_{i}\rangle\) to the final state \(|\Psi_{f}\rangle\), when it is accompanied by the emission of radiation of frequency \(\omega\), momentum \(k\) in the direction \((\theta_{k}, \phi_{k})\) (see Fig. 6.1) and polarization \(\mu = \pm 1\) corresponding to the left and right circular polarization states as defined by Rose (1957). The transition matrix element is readily given by

\[
\langle f | T(k, \mu) | i \rangle = \frac{1}{(E_{i} - E_{f} - \omega) - i(\Gamma_{i} + \Gamma_{f})} \sum_{L=|J_{u}| - |J_{l}|}^{(J_{u} + J_{l})} (-i\mu)^{h(L)} J_{L} \sum_{m_{u} = -J_{u}}^{J_{u}} \sum_{m_{l} = -J_{l}}^{J_{l}} c_{m_{u}}^{l} c_{m_{l}}^{l} C(J_{l}, L, J_{u}; m_{l}, M, m_{u}) D_{h,\mu}^{L}(\phi_{k}, \theta_{k}, -\phi_{k})^{*},
\]

where \(\Gamma_{i}\) and \(\Gamma_{f}\) denote the natural widths of the levels \(|\Psi_{i}\rangle\) and \(|\Psi_{f}\rangle\) with energies \(E_{i}\) and \(E_{f}\) and \(h(L) = 1/2 \left[1 + \pi_{u} \pi_{l} (-1)^{L}\right]\). The \(\pi_{u}, \pi_{l}\) denote the parities of upper
and lower levels respectively. The total angular momentum and its projection of the emitted radiation along the quantization axis parallel to \(Z\)-axis of PAF are denoted by \(L\) and \(M\) respectively. The \(D_{\mu,\mu'}^{L}(\phi_k, \theta_k, -\phi_k)\) denote elements of the well known Wigner-rotation matrix as defined in Rose (1957). The multipole transition strength \(\mathcal{J}_L\) is proportional to the corresponding reduced matrix element and may be displayed in the form

\[
\mathcal{J}_L = g(L)M_L + h(L)E_L ,
\]

with \(g(L) = \frac{1}{2} \left[ 1 - \pi \pi(-1)^L \right]\). The above equation indicates that \(\mathcal{J}_L\) is either a magnetic transition \(M_L\), or an electric transition \(E_L\), when the atomic levels are good eigenstates of parity.

The density matrix \(\rho\) describing the state of polarization of the emitted radiation is then given in terms of its elements

\[
\rho_{\mu,\mu'}^{J}(f, i) = (f|T(k, \mu)|i) \cdot p_i \cdot (f|T(k, \mu')|i)^* ,
\]

where \(p_i\) are given, under thermodynamic equilibrium conditions, by

\[
p_i = P_u^{-1} e^{-E_i/k_B T} , \quad P_u = \sum_{i=1}^{(2J+1)} e^{-E_i/k_B T} .
\]

In the above equation \(P_u\) denotes the partition function, \(k_B\) the Boltzmann constant, and \(T\) the temperature. Using Eq. (6.5), we may write Eq. (6.7) explicitly in the form

\[
\rho_{\mu,\mu'}^{J}(f, i) = \sum_L \sum_{L'} (-i\mu)^h(L) (i\mu')^{h(L')} \mathcal{J}_L \mathcal{J}_{L'} \sum_{m_u,m_l,m_u',m_l'} c_{m_u}^{L} c_{m_l}^{L'} c_{m_u'}^{L} c_{m_l'}^{L'} C(J_l, L, J_u, M, M_u) C(J_{l'}, L', J_{u'}, M, M_{u'})
\]

\[
D_{\mu,\mu'}^{L}(\phi_k, \theta_k, -\phi_k) D_{\mu,\mu'}^{L'}(\phi_k, \theta_k, -\phi_k) .
\]
where the unnormalized profile function

$$F(f, i; x) = \frac{1}{(E_i - E_0 - x\Gamma_{f,i})^2 + \Gamma_{f,i}^2},$$

(6.10)

is written in terms of $x = (\omega - \omega_0)/\Gamma_{f,i}$ where $\omega_0 = E_0 - E_f$ denotes the conventional frequency displacement from the line center expressed in natural width units and $\Gamma_{f,i} = \Gamma_f + \Gamma_i$.

Observing from Eqs. (6.3) and (6.4) that

$$t_{q_u}^{k_u}(i) = [J_u] \sum_{m_u=-J_u}^{J_u} (-1)^{m_u-J_u} C(J_u, J_u, k_u; m_u', -m_u, q_u)c_{m_u}^i c_{m_u'}^{q_u},$$

and

$$t_{q_l}^{k_l}(f) = [J_l] \sum_{m_l=-J_l}^{J_l} (-1)^{m_l-J_l} C(J_l, J_l, k_l; m_l', -m_l, q_l)c_{m_l}^f c_{m_l'}^{q_l},$$

(6.11)

(6.12)

and making use of standard Racah algebra techniques, we may now express $\rho_{\mu,\mu'}(f, i)$ in the elegant form

$$\rho_{\mu,\mu'}(f, i) = \sum_{k_u} \sum_{k_l} \sum_{K=|k_u-k_l|} ((t_{k_u}^{k_u}(i) \otimes t_{k_l}^{k_l}(f))^K \cdot \mathcal{A}^K(\mu, \mu')).$$

(6.13)

In the above equation, the analyzing powers are given by

$$\mathcal{A}^K_{Q}(\mu, \mu') = p_i F(f, i; x) \sum_{L, L'} (-i\mu)^{L}(i\mu')^{L'} J_L J_{L'} \sum_{K=0}^{L} \frac{[J_u][k_l][J_u][K]}{[J_l][L]} \begin{bmatrix} J_u & k_u & J_u \\ k_l & J_l & k_l \\ K & L & L' \end{bmatrix} \begin{bmatrix} C(L', K, L; M', -Q, M) \end{bmatrix} \begin{bmatrix} (L'+K)^{k_u} C(L', K, L; M', -Q, M) \end{bmatrix}$$

(6.14)

$$D_{M,\mu}(\phi_k, \theta_k, -\phi_k)^* D_{M',\mu'}(\phi_k, \theta_k, -\phi_k).$$

where \{\} denotes the Wigner $9-j$ symbol (Varshalovich et al. 1988). The irreducible tensor of rank $K$ constructed out of the two Fano statistical tensors is expressed in
Eq. (6.13) using the shorthand notation

\[
(t^k_u(i) \otimes t^l(f))_Q^K = \sum_{q_l} C(k_u, k_l, K; q_u, q_l, Q) t^k_u(i) t^l_q(f). 
\]  

(6.15)

6.3 Special case \( J_u = 1, J_l = 0 \)

We may show that the above general result readily specializes to give a simple form when \( J_u = 1 \) and \( J_l = 0 \). Defining the Fano statistical tensors through

\[
t^k_u(i) = \sqrt{3} \sum_{m_u = -1}^{1} (-1)^{m_u-1} C(1, 1, k_u; m_u', -m_u, q_u),
\]  

(6.16)

\[
t^l_q(f) = \delta_{k_0} \delta_{q_0},
\]  

(6.17)

we can express Eq. (6.13) in another elegant form

\[
\rho^\gamma_{\mu\mu'} = \sum_{K=0}^{2} (t^K \cdot A^K(\mu, \mu'))
\]

\[
= \sum_{K=0}^{2} \sum_{Q=-K}^{K} (-1)^Q t^K_Q A^K_{-Q}(\mu, \mu'),
\]  

(6.18)

where the \( t^K_Q \equiv t^k_u(i) \) and analyzing powers are given by

\[
A^K_{-Q}(\mu, \mu') = \frac{1}{3} p_i F(f, i; x) |\mathcal{J}|^2 \sum_{M=-1}^{1} (-1)^{1-M} C(1, 1, K; M, -M, -Q)
\]

\[
D^1_{M\mu}(\phi_k, \theta_k, -\phi_k)^* D^1_{M'\mu'}(\phi_k, \theta_k, -\phi_k).
\]  

(6.19)

6.4 Zeeman terms and Cross terms

Using the well-known Clebsch-Gordan theorem for the \( D \) matrices we may eliminate the sum over \( M \) and reduce the product of the two \( D \) matrices in Eq. (6.14) into a
single $D$ matrix element. Explicitly, we can make the replacement

$$
\sum_{M=-L}^{L} C(L, K, L; M, -Q, M) D_{M\mu}^{L}(\phi_k, \theta_k, -\phi_k) D_{M'\mu'}^{L'}(\phi_k, \theta_k, -\phi_k) =
$$

$$
(-1)^{L'+Q-\mu} \frac{[L]}{[K]} C(L', L; K; \mu', -\mu, \nu) D_{Q\nu}^{K}(\phi_k, \theta_k, -\phi_k),
$$

(6.20)
on the right hand side of Eq. (6.14). The Stokes parameters (Chandrasekhar 1950)

$I, Q, U, V$ are now readily expressed, following Section 5.4 by

$$
S_0 = I = \rho_{+1,+1}(f, i) + \rho_{-1,-1}(f, i),
$$

$$
S_1 = Q = \rho_{+1,-1}(f, i) + \rho_{-1,+1}(f, i),
$$

$$
S_2 = U = i(\rho_{+1,-1}(f, i) - \rho_{-1,+1}(f, i)),
$$

$$
S_3 = V = \rho_{+1,+1}(f, i) - \rho_{-1,-1}(f, i),
$$

(6.21)
in terms of the elements given by the Eq. (6.13), which is the central result of this chapter. We note that $\nu = 0$ in Eq. (6.20) for $S_0 = I$, when

$$
D_{Q0}^{K}(\phi_k, \theta_k, -\phi_k) = \sqrt{\frac{4\pi}{2K+1} Y_{KQ}^{*}(\theta_k, \phi_k)},
$$

(6.22)
and hence the intensity can be expressed as a weighted sum of $Y_{KQ}(\theta_k, \phi_k)$ and the sum involves only those $K$ which satisfy $(-1)^{L'+L-K} = 1$. On the other hand, we note $\nu = 0$ for $S_3 = V$ as well so that the relation in Eq. (6.22) can be used. But it is important to note that $S_3 = V$ is a weighted sum of $Y_{KQ}(\theta_k, \phi_k)$ for those $K$ which satisfy $(-1)^{L'+L-K} = -1$. For $S_1 = Q$ and $S_2 = U$, we note that $\nu = \pm 2$ and therefore $K$ has necessarily to be $\geq 2$.

The Stokes parameters $S_\alpha(\alpha = 0, 1, 2, 3)$ defined above, may also be expressed
using the Eq. (6.9) in the form

\[
S_\alpha = \sum_{m_u=-J_u}^{J_u} \sum_{m_l=-J_l}^{J_l} |c_{m_u}^l|^2 |c_{m_l}^l|^2 \mathcal{Z}_\alpha (m_l; m_u) + \\
\sum_{m_u \neq m_u'} \sum_{m_l \neq m_l'} \sum_{m_u'=-J_u}^{J_u} \sum_{m_l'=-J_l}^{J_l} c_{m_u}^l c_{m_u'}^l c_{m_l}^l c_{m_l'}^l \mathcal{Z}_\alpha (m_l; m_u, m_u', m_l', m_l') , \quad (6.23)
\]

where the Stokes parameters \( \mathcal{Z}_\alpha (m_l; m_u) \) for \( \alpha = 0, 1, 2, 3 \) for a pure Zeeman transition from \( |J_u, m_u\rangle \) to \( |J_l, m_l\rangle \) are readily expressible, following Eq. (6.21) as

\[
\mathcal{Z}_0 (m_l; m_u) = \mathcal{Z}_{+1,1} (m_l; m_u) + \mathcal{Z}_{-1,-1} (m_l; m_u) , \\
\mathcal{Z}_1 (m_l; m_u) = \mathcal{Z}_{+1,-1} (m_l; m_u) + \mathcal{Z}_{-1,1} (m_l; m_u) , \\
\mathcal{Z}_2 (m_l; m_u) = i \left[ \mathcal{Z}_{+1,-1} (m_l; m_u) - \mathcal{Z}_{-1,1} (m_l; m_u) \right] , \\
\mathcal{Z}_3 (m_l; m_u) = \mathcal{Z}_{+1,1} (m_l; m_u) - \mathcal{Z}_{-1,-1} (m_l; m_u) , \quad (6.24)
\]

in terms of

\[
\mathcal{Z}_{\mu,\mu'} (m_l; m_u) = p_i F(f, i; x) \sum_L \sum_{L'} (-i\mu)^{h(L)} (i\mu')^{h(L')} J_L J_{L'}^* C(J_l, L, J_u; m_l, M, m_u) C(J_l', L', J_u; m_l, M, m_u) \\
\times D^L_{M,\mu} (\phi_k, \theta_k, -\phi_k)^* D^{L'}_{M,\mu} (\phi_k, \theta_k, -\phi_k) . \quad (6.25)
\]

The first part of right hand side in Eq. (6.23) is thus a statistically weighted sum of pure Zeeman contributions which we may refer to, for brevity, as the Zeeman term.

Likewise, the \( C_\alpha (m_l, m_l'; m_u, m_u') \) can be expressed as

\[
C_0 (m_l, m_l'; m_u, m_u') = C_{+1,1} (m_l, m_l'; m_u, m_u') + C_{-1,-1} (m_l, m_l'; m_u, m_u') , \\
C_1 (m_l, m_l'; m_u, m_u') = C_{+1,-1} (m_l, m_l'; m_u, m_u') + C_{-1,1} (m_l, m_l'; m_u, m_u') , \\
C_2 (m_l, m_l'; m_u, m_u') = i \left[ C_{+1,-1} (m_l, m_l'; m_u, m_u') - C_{-1,1} (m_l, m_l'; m_u, m_u') \right] , \\
C_3 (m_l, m_l'; m_u, m_u') = C_{+1,1} (m_l, m_l'; m_u, m_u') - C_{-1,-1} (m_l, m_l'; m_u, m_u') , \quad (6.26)
\]
in terms of

$$C_{\mu,\mu'}(m_i, m_i'; m_u, m_u') = p_i F(f, i; x) \sum_{L} \sum_{L'} (-i\mu)^{b(L)}(i\mu')^{b(L')} J_L J_{L'}^*$$

$$C(J_i, L, J_u; m_i, M, m_u) C(J_i', L', J_u; m_i', M', m_u') \quad (6.27)$$

$$D_{M,\mu}^L(\phi_k, \theta_k, -\phi_k)^* D_{M',\mu'}^{L'}(\phi_k, \theta_k, -\phi_k).$$

Clearly, the $C_\alpha$ denote the contributions to the Stokes parameters from the cross terms which arise due to quantum interference effects. The second part of Eq. (6.23) may be referred to, for brevity, as cross term contributions.

6.5 Polarization of emitted lines in $J = 1$ to $J = 0$ transition

In this section we investigate the nature of polarized line spectra of an atom making a transition from an upper level with spin-1 to a lower level with spin-0, using the theoretical results obtained in the above. We show the characteristically different behavior of polarization on emitted line spectra arising because of external electric quadrupole and magnetic fields.

6.5.1 Polarization density matrix for radiation

When an atom makes a transition from an upper level with energy $E_u$, spin-1 and parity $\pi_u$ to a lower level with energy $E_l$, spin-0 and parity $\pi_l$, angular momentum and parity are conserved. The transition matrix elements may then be expressed in
the form

\[ \langle 0, 0; k, \mu | T | 1, m \rangle = D^{i}_{m\mu}(\phi_k, \theta_k, -\phi_k)^* J, \]  

(6.28)

where the transition strength

\[ J = -i \sqrt{3} (2\pi)^{1/2} \langle 0 || T || 1 \rangle, \]  

(6.29)

where \( J \) indicates that either a magnetic or an electric dipole type, depending on
the parities of the upper and lower levels. If the lower level is the ground state and
if the transition is from an upper state \( |\Psi_i\rangle \) which is of the form given by Eq. (6.1),
and \( |\Psi_i\rangle \) has a width \( \Gamma_i \), then the transition is described by matrix elements denoted
by

\[ T_{\mu,i} = J f(i) \sum_{m=-1}^{1} c^i_m D^{i}_{m\mu}(\phi_k, \theta_k, -\phi_k)^*, \]  

(6.30)

where

\[ f(i) = \frac{1}{(E_i - E_f - \omega - i\Gamma_i)}, \]  

(6.31)

incorporates the frequency dependence due to the natural width \( \Gamma_i \). The density
matrix \( \rho^\gamma \) describing the state of polarization of radiation can be written as

\[ \rho^\gamma_{\mu\mu'} = \sum_{i=1}^{3} T_{\mu,i} p_i T_{\mu',i}^* = \sum_{i=1}^{3} \rho^\gamma_{\mu\mu'}(i), \]  

(6.32)

where \( \mu, \mu' \) take values \( \pm 1 \), the probabilities \( p_i \) are in general given by Eq. (6.8)
and \( \rho^\gamma(i) \) correspond to the transition from an upper level \( |\Psi_i\rangle \) to the lower level
\( |\Psi_f\rangle = |0, 0\rangle \) representing the ground state. The elements of the density matrices
\( \rho^\gamma(i) \) are the form

\[ \rho^\gamma_{\mu\mu'}(i) = G(i) R^\gamma_{\mu\mu'}(i) \]  

(6.33)
where

\[ R_{\mu \mu'}^{\gamma}(i) = \sum_{m,m'=-1} c_m^* c_{m'} D_{m \mu}^{1\gamma}(\phi_k, \theta_k, -\phi_k)^* D_{m' \mu'}^{1\gamma}(\phi_k, \theta_k, -\phi_k), \]  

(6.34)

governs completely the angular dependence. The \( G(i) \) are given by

\[ G(i) = |J|^2 p_i F(i, x), \]  

(6.35)
in terms of the profile function

\[ F(i, x) = |f(i)|^2 = \frac{1}{(E_i - E_0 - x \Gamma_i)^2 + \Gamma_i^2}, \]  

(6.36)

where

\[ x = \frac{\omega - \omega_0}{\Gamma_i}, \quad \omega_0 = E_0 - E_l, \]  

(6.37)
denotes the conventional frequency displacement from the line center in natural width units. It may also be noted that the frequency dependence of \( \rho^{\gamma}(i) \) as well as the line strengths are completely taken care of by \( G(i) \).

### 6.5.2 Stokes line profiles formed in the presence of a pure magnetic field

The states \(|\Psi_i\rangle\) in this case are readily identified with \(|1, m\rangle\), where \( m = 1, 0, -1 \) defined with respect to the axis of quantization parallel to \( B \). Note that the variation with the frequency in the vicinity of \( E_m - E_l \) for each \( E_m \) is governed by the profile function \( F(m, x) \), which is shown in Fig. (6.2) with the peaks normalized to unity.

The widths \( \Gamma \) have been chosen as \( 2.18 \times 10^8 \text{s}^{-1} \) independent of \( m \). We can now
explicitly write the density matrices $\rho^m(m)$ for $m = 1, 0, -1$ using Eq. (6.33), where we have

\[
R^l_{+1,+1}(m = 1) = \frac{1}{4} (1 + \cos \theta_k)^2 ,
\]
\[
R^l_{+1,-1}(m = 1) = \frac{1}{4} \sin^2 \theta_k e^{-2i\phi_k} ,
\]
\[
R^l_{-1,+1}(m = 1) = \frac{1}{4} \sin^2 \theta_k e^{2i\phi_k} ,
\]
\[
R^l_{-1,-1}(m = 1) = \frac{1}{4} (1 - \cos \theta_k)^2 ,
\] (6.38)

for the transition from the $m = 1$ state. Using Eq. (6.21), the Stokes parameters for this line component are given by

\[
I(m = 1) = \frac{1}{2} G(m = 1) (1 + \cos^2 \theta_k) ,
\]
\[
Q(m = 1) = \frac{1}{2} G(m = 1) \sin^2 \theta_k \cos 2\phi_k ,
\]
\[
U(m = 1) = \frac{1}{2} G(m = 1) \sin^2 \theta_k \sin 2\phi_k ,
\]
\[
V(m = 1) = G(m = 1) \cos \theta_k .
\] (6.39)

Like wise, for the transition from $m = 0$ upper state, we have

\[
R^l_{+1,+1}(m = 0) = \frac{1}{2} \sin^2 \theta_k ,
\]
\[
R^l_{+1,-1}(m = 0) = -\frac{1}{2} \sin^2 \theta_k e^{-2i\phi_k} ,
\]
\[
R^l_{-1,+1}(m = 0) = -\frac{1}{2} \sin^2 \theta_k e^{2i\phi_k} ,
\]
\[
R^l_{-1,-1}(m = 0) = \frac{1}{2} \sin^2 \theta_k ,
\] (6.40)

and the Stokes parameters are

\[
I(m = 0) = G(m = 0) \sin^2 \theta_k ,
\]
\[ Q(m = 0) = -G(m = 0) \sin^2 \theta_k \cos 2\phi_k , \]
\[ U(m = 0) = -G(m = 0) \sin^2 \theta_k \sin 2\phi_k , \]
\[ V(m = 0) = 0 , \]  

(6.41)

whereas for the transition from \( m = -1 \) state, we have

\[ R_{+1,+1}^\gamma(m = -1) = \frac{1}{4} (1 - \cos \theta_k)^2 , \]
\[ R_{+1,-1}^\gamma(m = -1) = \frac{1}{4} \sin^2 \theta_k e^{-2i\phi_k} , \]
\[ R_{-1,+1}^\gamma(m = -1) = \frac{1}{4} \sin^2 \theta_k e^{2i\phi_k} , \]
\[ R_{-1,-1}^\gamma(m = -1) = \frac{1}{4} (1 + \cos \theta_k)^2 , \]  

(6.42)

and the Stokes parameters

\[ I(m = -1) = \frac{1}{2} G(m = -1) (1 + \cos^2 \theta_k) , \]
\[ Q(m = -1) = \frac{1}{2} G(m = -1) \sin^2 \theta_k \cos 2\phi_k , \]
\[ U(m = -1) = \frac{1}{2} G(m = -1) \sin^2 \theta_k \sin 2\phi_k , \]
\[ V(m = -1) = -G(m = -1) \cos \theta_k . \]  

(6.43)

The above results are graphically presented in Fig. (6.3) for some select angles \( \theta_k = 0, \pi/4, \pi/2 \) and \( \phi_k = 0 \). Clearly the Zeeman components with \( m = \pm 1 \) are 100% circularly polarized and the \( m = 0 \) component is absent at \( \theta_k = 0 \) and \( \pi \), whereas at \( \theta_k = \pi/2 \), the circular polarization asymmetry is zero for all the three components each of which is now linearly polarized. It is obvious that there should be no polarization for zero field, when the lines corresponding to the transitions from the \( \Psi_+ \) and \( \Psi_- \) states occur at the same point \( x = 0 \), with the same intensity, so
that \( I = I(m = +1) + I(m = -1) \) increases, whereas \( V = V(m = +1) - V(m = -1) \) becomes zero. If there were no widths, even a slight deviation from \( x = 0 \) on either side should be characterized by \( I = 0 \). On the other hand \( I \) decreases slowly on either side of \( x = 0 \) because of the finite width. The finite width is also responsible for the depolarization in \( V \) as the lines corresponding to \( m = +1 \) and \( m = -1 \) approach \( x = 0 \) from either side, in the weak field case. While \( V \), being the difference in the intensities of \( m = +1 \) and \( m = -1 \), decreases and \( I \), being the sum of \( I = I(m = +1) \) and \( I(m = -1) \) remains sufficiently large so that the depolarization is pronounced when we plot \( V/I \) in Fig. (6.3).

6.5.3 Stokes line profiles formed in the presence of a pure electric quadrupole field

In this case, the relevant eigenstates \( \Psi_i, i = x, y, z \) of the atomic density matrix \( \rho \) are given by Eq. (2.18) and the respective energy level separations are shown in Fig. (2.2). The profile functions for the pure electric quadrupole case are shown in Fig. (6.4a-c) with peaks normalized to unity and with a choice of \( \Gamma = 2.18 \times 10^8 \text{s}^{-1} \) independent of \( i \), for illustrative purposes. It may be emphasized that the profile functions for the pure electric field are characteristically different from those for the pure magnetic field. Moreover they are dependent on the asymmetry parameter \( \eta \). For \( i = x \) the elements of \( \rho^i(x) \) are explicitly given in terms of the photon emission angles \((\theta_k, \phi_k)\) by

\[
R_{1,1,+1}^i(x) = \frac{1}{4} \left[ (1 + \cos^2 \theta_k) - \sin^2 \theta_k \cos 2\phi_k \right],
\]
\[ R_{1,-1}(x) = \frac{1}{4} \left[ \sin^2 \theta_k e^{-2i\phi} - \frac{1}{2}(1 - \cos \theta_k)^2 e^{-4i\phi} - \frac{1}{2}(1 + \cos \theta_k)^2 \right], \]
\[ R_{1,+1}(x) = \frac{1}{4} \left[ \sin^2 \theta_k e^{2i\phi} - \frac{1}{2}(1 - \cos \theta_k)^2 e^{4i\phi} - \frac{1}{2}(1 + \cos \theta_k)^2 \right], \]
\[ R_{-1,-1}(x) = \frac{1}{4} \left[ (1 + \cos^2 \theta_k) - \sin^2 \theta_k \cos 2\phi_k \right]. \]

From the above expressions, we obtain the emitted Stokes parameters
\[
I(x) = \frac{1}{2} G(x) \left[ (1 + \cos^2 \theta_k) - \sin^2 \theta_k \cos 2\phi_k \right], \\
Q(x) = \frac{1}{2} G(x) \left[ \sin^2 \theta_k \cos 2\phi_k - \frac{1}{2}(1 - \cos \theta_k)^2 \cos 4\phi_k - \frac{1}{2}(1 + \cos \theta_k)^2 \right], \\
U(x) = \frac{1}{2} G(x) \left[ \sin^2 \theta_k \sin 2\phi_k - \frac{1}{2}(1 - \cos \theta_k)^2 \sin 4\phi_k \right], \\
V(x) = 0. \tag{6.44}
\]

Similarly, the matrix elements \( R_{\mu\nu'}(y) \) are given by
\[
R_{1,+1}(y) = \frac{1}{4} \left[ (1 + \cos^2 \theta_k) + \sin^2 \theta_k \cos 2\phi_k \right], \\
R_{1,-1}(y) = \frac{1}{4} \left[ \sin^2 \theta_k e^{-2i\phi} + \frac{1}{2}(1 - \cos \theta_k)^2 e^{-4i\phi} + \frac{1}{2}(1 + \cos \theta_k)^2 \right], \\
R_{-1,+1}(y) = \frac{1}{4} \left[ \sin^2 \theta_k e^{2i\phi} + \frac{1}{2}(1 - \cos \theta_k)^2 e^{4i\phi} + \frac{1}{2}(1 + \cos \theta_k)^2 \right], \\
R_{-1,-1}(y) = \frac{1}{4} \left[ (1 + \cos^2 \theta_k) + \sin^2 \theta_k \cos 2\phi_k \right]. \tag{6.46}
\]

The corresponding expressions for the Stokes parameters are
\[
I(y) = \frac{1}{2} G(y) \left[ (1 + \cos^2 \theta_k) + \sin^2 \theta_k \cos 2\phi_k \right], \\
Q(y) = \frac{1}{2} G(y) \left[ \sin^2 \theta_k \cos 2\phi_k + \frac{1}{2}(1 - \cos \theta_k)^2 \cos 4\phi_k + \frac{1}{2}(1 + \cos \theta_k)^2 \right], \\
U(y) = \frac{1}{2} G(y) \left[ \sin^2 \theta_k \sin 2\phi_k + \frac{1}{2}(1 - \cos \theta_k)^2 \sin 4\phi_k \right], \\
V(y) = 0. \tag{6.47}
\]

For \( \rho^7(z) \), we note that the angular dependence given by \( R_{\mu\nu'}(z) \) is identical to that given by Eq. (6.40), since \( \Psi_z = \Psi_{1,0} \). However, the frequency dependence...
is governed by $G(z)$, which is characteristically different from $G(m = 0)$ since the energy eigenvalue $E_z$ is different from $E_0$ and the spectrum of pure electric quadrupole field (dependent on $\eta$ in addition to $A$) is different from the spectrum in the case of Zeeman effect. This results in non-symmetric Stokes profiles, unlike in the case of the Zeeman effect. Thus, the Stokes parameters are given by,

\[
\begin{align*}
I(z) &= G(z) \sin^2 \theta_k \\
Q(z) &= -G(z) \sin^2 \theta_k \cos 2\phi_k \\
U(z) &= -G(z) \sin^2 \theta_k \sin 2\phi_k \\
V(z) &= 0. 
\end{align*}
\] (6.48)

These Stokes parameters are plotted in Figs. (6.5), (6.6) and (6.7) for the values of $\eta = 0, 0.5$ and 1 respectively for the pure electric quadrupole field case. The lack of azimuthal symmetry is evident here when $\eta \neq 0$. Appearance of linear polarization $Q/I$ in the $Z$-direction of the PAF, is a remarkable feature observed in this case. Moreover, the $V$ parameter is absent, since the superposition $\Psi_{1,+1}$ and $\Psi_{1,-1}$ results in producing only linear polarization.

6.5.4 Stokes line profiles formed in the presence of both magnetic and electric quadrupole fields

6.5.4.1 Magnetic field $B$ along the $Z$-axis of PAF

We consider in this section only the simple case of magnetic field $B$ along the $Z$-axis of the PAF. The corresponding eigenstates $\Psi_i, i= 1, 2, 3$ of the atomic density matrix
\( \rho \) are therefore given by Eq. (3.4) in section (3.2.1) of Chapter 3 and the respective energy level separations, which depend on the magnetic field strength and the electric quadrupole field parameters \( A \) and \( \eta \), are shown in Fig. (3.1). In this case, the level separations are unequal, except when \( (A^2\eta^2 + D^2)^{1/2} = A \). The angular dependence in the elements of \( \rho^7(1) \) is the same as in Eq. (6.40), since the state \( \Psi_1 \) is identical to \( \Psi_{1,0} \), but the frequency dependence is characteristically different since it is governed by \( G(1) \). It is important to note that \( G(m = 0), G(z) \) and \( G(1) \) are responsible for the remarkable changes in the Stokes line profiles considered here in the three cases of pure magnetic field, pure electric quadrupole field and the combination of electric and magnetic fields respectively although the factors responsible for angular distribution are identical. The stokes parameters here are given by

\[
\begin{align*}
I(1) &= G(1) \sin^2 \theta_k , \\
Q(1) &= -G(1) \sin^2 \theta_k \cos 2\phi_k , \\
U(1) &= -G(1) \sin^2 \theta_k \sin 2\phi_k , \\
V(1) &= 0 .
\end{align*}
\tag{6.49}
\]

For the transition from the state \( \Psi_2 \), the \( R_{\mu\mu'}^7(2) \) are given by,

\[
\begin{align*}
R_{+1,+1}^7(2) &= \frac{1}{4} \left[ a^2 (1 - \cos \theta_k)^2 + 2ab \sin^2 \theta_k \cos 2\phi_k + b^2 (1 + \cos \theta_k)^2 \right] , \\
R_{+1,-1}^7(2) &= \frac{1}{4} \left[ (a^2 + b^2) \sin^2 \theta_k e^{-2i\phi_k} + ab (1 - \cos \theta_k)^2 e^{-4i\phi_k} + ab (1 + \cos \theta_k)^2 \right] , \\
R_{-1,+1}^7(2) &= \frac{1}{4} \left[ (a^2 + b^2) \sin^2 \theta_k e^{2i\phi_k} + ab (1 - \cos \theta_k)^2 e^{4i\phi_k} + ab (1 + \cos \theta_k)^2 \right] , \\
R_{-1,-1}^7(2) &= \frac{1}{4} \left[ a^2 (1 + \cos \theta_k)^2 + 2ab \sin^2 \theta_k \cos 2\phi_k + b^2 (1 - \cos \theta_k)^2 \right] . \tag{6.50}
\end{align*}
\]
The coefficients \(a\) and \(b\) are real and are given by

\[
\begin{align*}
\alpha & = -\frac{A\eta - D + (A^2 \eta^2 + D^2)^{1/2}}{2[A^2 \eta^2 + D^2 + A\eta(A^2 \eta^2 + D^2)^{1/2}]^{1/2}}, \\
\beta & = -\frac{A\eta + D + (A^2 \eta^2 + D^2)^{1/2}}{2[A^2 \eta^2 + D^2 + A\eta(A^2 \eta^2 + D^2)^{1/2}]^{1/2}}.
\end{align*}
\]  

(6.51)

The corresponding Stokes parameters for this transition are

\[
\begin{align*}
I(2) & = G(2)\left[\frac{1}{2}(a^2 + b^2)(1 + \cos^2 \theta_k) + a \, b \, \sin^2 \theta_k \, \cos 2\phi_k\right], \\
Q(2) & = \frac{1}{2}G(2)\left[(a^2 + b^2) \, \sin^2 \theta_k \, \cos 2\phi_k + a \, b \, (1 - \cos \theta_k)^2 \, \cos 4\phi_k + a \, b \, (1 + \cos \theta_k)^2\right], \\
U(2) & = \frac{1}{2}G(2)\left[(a^2 + b^2) \, \sin^2 \theta_k \, \sin 2\phi_k + a \, b \, (1 - \cos \theta_k)^2 \, \sin 4\phi_k\right], \\
V(2) & = G(2)(b^2 - a^2) \, \cos \theta_k.
\end{align*}
\]  

(6.52)

For the transition from the state \(\Psi_3\), like wise, we have

\[
\begin{align*}
R_{1,+1}^3(3) & = \frac{1}{4}\left[a^2(1 + \cos \theta_k)^2 - 2 \, a \, b \, \sin^2 \theta_k \, \cos 2\phi_k + b^2(1 - \cos \theta_k)^2\right], \\
R_{1,-1}^3(3) & = \frac{1}{4}\left[(a^2 + b^2) \, \sin^2 \theta_k \, e^{-2i\phi_k} - a \, b \, (1 - \cos \theta_k)^2 \, e^{-4i\phi_k} - a \, b \, (1 + \cos \theta_k)^2\right], \\
R_{1,+1}^3(3) & = \frac{1}{4}\left[(a^2 + b^2) \, \sin^2 \theta_k \, e^{2i\phi_k} - a \, b \, (1 - \cos \theta_k)^2 \, e^{4i\phi_k} - a \, b \, (1 + \cos \theta_k)^2\right], \\
R_{1,-1}^3(3) & = \frac{1}{4}\left[a^2(1 - \cos \theta_k)^2 - 2 \, a \, b \, \sin^2 \theta_k \, \cos 2\phi_k + b^2(1 + \cos \theta_k)^2\right].
\end{align*}
\]  

(6.53)

The Stokes parameters for this transition are given by

\[
\begin{align*}
I(3) & = G(3)\left[\frac{1}{2}(a^2 + b^2)(1 + \cos^2 \theta_k) - a \, b \, \sin^2 \theta_k \, \cos 2\phi_k\right], \\
Q(3) & = \frac{1}{2}G(3)\left[(a^2 + b^2) \, \sin^2 \theta_k \, \cos 2\phi_k - a \, b \, (1 - \cos \theta_k)^2 \, \cos 4\phi_k - a \, b \, (1 + \cos \theta_k)^2\right], \\
U(3) & = \frac{1}{2}G(3)\left[(a^2 + b^2) \, \sin^2 \theta_k \, \sin 2\phi_k - a \, b \, (1 - \cos \theta_k)^2 \, \sin 4\phi_k\right], \\
V(3) & = G(3)(a^2 - b^2) \, \cos \theta_k.
\end{align*}
\]  

(6.54)

The above results are presented in Figs. (6.8), (6.9) and (6.10) for different values of the asymmetry parameter \(\eta = 0, 0.5\) and \(1\) respectively for varying ratios of
magnetic field to electric field strengths. This case, i.e. where the radiating atom is in the presence of both an electric quadrupole field and a magnetic field, presents a host of interesting features. As is evident from the figures, we tend toward the Zeeman case when the magnetic field becomes stronger. It is worth drawing one's attention to the $V$ parameter in these cases, a parameter not present in the pure electric quadrupole field case. We notice a modification in the shape of this profile near line center, due to the presence of the electric quadrupole field. Thus, a careful examination of the shape of the $V$ profile near line center, can serve as a diagnostic for the presence electric quadrupole field in addition to the magnetic field.

6.5.4.2 The case of arbitrary orientation of $B$ with respect to the PAF

We present in this section the results for arbitrary orientations of the magnetic field. The Stokes line profiles are presented graphically in Fig. (6.11) for chosen values of $\eta = 1$ and $R = 1$. For convenience of presentation, we choose a direction of magnetic field with $\theta_B = \pi/6$ and $\phi_B = \pi/4$. The azimuth angle of line of sight is chosen to be $\phi_k = \pi/4$. With this choice the angle between the magnetic field direction and line of sight direction becomes $\gamma = (\theta_B - \theta_k)$ where $\theta_k$ represents the orientation of the LOS (see Fig. 6.1).

In Fig. (6.11), we show the Stokes profiles arising due to contributions from the Zeeman term (dashed lines), cross term (solid lines) and the general case of combined terms (dotted lines). We present the actual Stokes line profiles $I, Q, U, V$ instead of the ratios $Q/I, U/I$ and $V/I$ in order to theoretically establish the correspondence between the individual terms of Eq. (6.23) and the emergent Stokes line profiles. In
the $\gamma = 0$ case (panel-a), the $Q$ Stokes parameter arises only due to the presence of electric quadrupole field (note that when $\gamma = 0$, the $Q$ Stokes parameter vanishes in the pure Zeeman case). A similar effect is seen in the Stokes $V$ profile also when $\gamma = \pi/2$ (magnetic field transverse to the LOS). For this chosen value of $\phi_B = \pi/4$, which highlights the contribution from cross terms, one can see that the linear polarization ($Q$ profile) arises only from these terms (pure Zeeman terms vanish for this value of $\phi_B$). However, the Zeeman terms contribute to the linear polarization for other arbitrary choices of $\phi_B$. The cross term contributions in $|\Psi_1\rangle$ and $|\Psi_3\rangle$ lead to strong linear polarization components in the blue and near-red wings of the $Q$ profiles respectively. However, the cross term contributions in $|\Psi_2\rangle$ lead to the appearance of a very weak linear polarization signal in the far red wing. This fact is also readily seen by examining the magnitude of cross term contributions as shown in Table (6.1). We have chosen $\phi_k = \pi/4$, in order to show the sensitivity of the Stokes $U$ profiles to the physical parameters. Contributions arise from both Zeeman and cross terms in the Stokes $U$ profiles. In our earlier calculation Stokes $V$ parameter for the pure Zeeman case revealed only two (left and right) circularly polarized components when $\gamma = 0$ (see Fig. 6.3). In contrast, presently all the three circularly polarized components can be seen. This is a consequence of the superposition of all the concerned basis states (magnetic substates). The appearance of circularly polarized component (Stokes $V$ profile) in the red wing is a consequence of the contribution of cross terms. We notice that the effects of quadrupole electric field and magnetic field are comparable only when $R \sim 1$. For large values of $R$ (say $R > 4$) the Zeeman effect dominates. Similarly, for $R < 1$ (say $R \sim 0.2$) the electric field effect dominates. Thus, we can consider $0.1 < $
R < 3 as the sensitivity regime for quadrupole electric field effects on polarized line profiles.

6.6 Polarization of emitted lines in $J = 3/2$ to $J = 1/2$ transition

In this section we investigate the nature of polarized line spectra of an atom making a transition from an upper level with spin-3/2 to a lower level with spin-1/2 taking only the dominant dipole transition (i.e $L = 1$ contribution). It is interesting to note that the spin-3/2 level is sensitive to the electric quadrupole as well as uniform magnetic fields, whereas the spin-1/2 levels get split only due to the magnetic field. We therefore show the physical significance of the emitted Stokes parameters are discussed as functions of the relevant physical parameters for some typical cases.

6.6.1 Polarization density matrix for radiation

The transition matrix elements of an atom, which makes a transition from an upper spin $J_u = 3/2$ level with energy $E_u$ and parity $\pi_u$ to lower spin $J_l = 1/2$ level with energy $E_l$ and parity $\pi_l$, may be expressed in the form

$$
\langle 1/2, m_l; k, \mu | T | 3/2, m_u \rangle = D_{M\mu}^{1/2}(\phi_k, \theta_k, -\phi_k) \mathcal{J},
$$

(6.55)

where the transition strength $\mathcal{J}$ may be displayed in the form

$$
\mathcal{J} = -i \sqrt{3} (2\pi)^{1/2} \langle 1/2 || T || 3/2 \rangle.
$$

(6.56)
The transition matrix elements are given by
\[ T_{\mu,i} = J f(i) \sum_{m=-3/2}^{3/2} c_m^i D^1_{M\mu}(\phi_k, \theta_k, -\phi_k)^*, \quad (6.57) \]
where the frequency dependence function \( f(i) \) is expressed in Eq. (6.31) of sect. 6.5.

The density matrix \( \rho^\gamma \) showing the state of polarization of emitted radiation can be written as
\[ \rho^\gamma_{\mu\mu'} = \sum_{i=1} T_{\mu,i} p_i T^*_{\mu',i} = \sum_{i=1} \rho^\gamma_{\mu\mu'}(i), \quad (6.58) \]
where the density matrix elements \( \rho^\gamma_{\mu\mu'}(i) \) are in the form
\[ \rho^\gamma_{\mu\mu'}(i) = G(i) R^\gamma_{\mu\mu'}(i), \quad (6.59) \]
where \( G(i) \) are given in Eq. (6.35). The angular dependence part \( R^\gamma_{\mu\mu'}(i) \) can be written by
\[ R^\gamma_{\mu\mu'}(i) = \sum_{m,m' = -3/2}^{3/2} c_m^i c_{m'}^{*i} D^1_{M\mu}(\phi_k, \theta_k, -\phi_k)^* D^1_{M\mu'}(\phi_k, \theta_k, -\phi_k), \quad (6.60) \]

6.6.2 Stokes line profiles formed in the presence of a pure magnetic field

We know that a uniform magnetic field removes the degeneracy (Zeeman effect). Thus the magnetic field splits the upper \( J_u = 3/2 \) level into four magnetic sub-levels with \( m_u = 3/2, 1/2, -1/2, -3/2 \) and the lower \( J_l = 1/2 \) level into two magnetic sub-levels with \( m_l = \pm 1/2 \). Six transitions are possible between these split levels when we consider only the dipole type transition. The total number of transitions is determined by the selection rule \( (\Delta m = 0, \pm 1) \) of the transition, while the state of polarization is
determined by the conservation of angular momentum, namely Zeeman components with $\Delta m = \pm 1$ represent pairs of left and right circularly polarized radiation ($\sigma$ lines) and $\Delta m = 0$ components represent plane polarized radiation ($\pi$ lines). The profile functions $F(m, x)$ for all transitions are shown in Fig. (6.12).

The density matrix elements for each transition and corresponding Stokes parameters are given below explicitly.

(1) $m_u = 3/2 \rightarrow m_l = 1/2$ transition

\[
R^\gamma_{1,1}(m_u = 3/2) = \frac{1}{4} (1 + \cos \theta_k)^2, \\
R^\gamma_{1,-1}(m_u = 3/2) = \frac{1}{4} \sin^2 \theta_k e^{-2i\phi_k}, \\
R^\gamma_{-1,1}(m_u = 3/2) = \frac{1}{4} \sin^2 \theta_k e^{2i\phi_k}, \\
R^\gamma_{-1,-1}(m_u = 3/2) = \frac{1}{4} (1 - \cos \theta_k)^2, \tag{6.61}
\]

and

\[
I(m_u = 3/2) = \frac{1}{2} G(m_u = 3/2 \rightarrow m_l = 1/2) (1 + \cos^2 \theta_k), \\
Q(m_u = 3/2) = \frac{1}{2} G(m = 3/2 \rightarrow m_l = 1/2) \sin^2 \theta_k \cos 2\phi_k, \\
U(m_u = 3/2) = \frac{1}{2} G(m = 3/2 \rightarrow m_l = 1/2) \sin^2 \theta_k \sin 2\phi_k, \\
V(m_u = 3/2) = G(m = 3/2 \rightarrow m_l = 1/2) \cos \theta_k. \tag{6.62}
\]

(2) $m_u = 1/2 \rightarrow m_l = 1/2$ transition

\[
R^\gamma_{1,1}(m_u = 1/2) = \frac{1}{3} \sin^2 \theta_k, \\
R^\gamma_{1,-1}(m_u = 1/2) = -\frac{1}{3} \sin^2 \theta_k e^{-2i\phi_k}, \\
R^\gamma_{-1,1}(m_u = 1/2) = -\frac{1}{3} \sin^2 \theta_k e^{2i\phi_k},
\]
\[ R_{-1,-1}(m_u = \frac{1}{2}) = \frac{1}{3} \sin^2 \theta_k , \]  \hspace{1cm} (6.63) 

and

\[ I(m_u = \frac{1}{2}) = \frac{2}{3} G(m_u = \frac{1}{2} \rightarrow m_l = \frac{1}{2}) \sin^2 \theta_k , \]
\[ Q(m_u = \frac{1}{2}) = -\frac{2}{3} G(m = \frac{1}{2} \rightarrow m_l = \frac{1}{2}) \sin^2 \theta_k \cos 2\phi_k , \]
\[ U(m_u = \frac{1}{2}) = -\frac{2}{3} G(m = \frac{1}{2} \rightarrow m_l = \frac{1}{2}) \sin^2 \theta_k \sin 2\phi_k , \]
\[ V(m_u = \frac{1}{2}) = 0 . \]  \hspace{1cm} (6.64) 

(3) \( m_u = 1/2 \rightarrow m_l = -1/2 \) transition

\[ R_{+1,+1}(m_u = \frac{1}{2}) = \frac{1}{12} (1 + \cos \theta_k)^2 , \]
\[ R_{+1,-1}(m_u = \frac{1}{2}) = \frac{1}{12} \sin^2 \theta_k e^{-2i\phi_k} , \]
\[ R_{-1,+1}(m_u = \frac{1}{2}) = \frac{1}{12} \sin^2 \theta_k e^{2i\phi_k} , \]
\[ R_{-1,-1}(m_u = \frac{1}{2}) = \frac{1}{12} (1 - \cos \theta_k)^2 , \]  \hspace{1cm} (6.65) 

and

\[ I(m_u = \frac{1}{2}) = \frac{1}{6} G(m_u = \frac{1}{2} \rightarrow m_l = -\frac{1}{2}) (1 + \cos \theta_k)^2 , \]
\[ Q(m_u = \frac{1}{2}) = \frac{1}{6} G(m_u = \frac{1}{2} \rightarrow m_l = -\frac{1}{2}) \sin^2 \theta_k \cos 2\phi_k , \]
\[ U(m_u = \frac{1}{2}) = \frac{1}{6} G(m_u = \frac{1}{2} \rightarrow m_l = -\frac{1}{2}) \sin^2 \theta_k \sin 2\phi_k , \]
\[ V(m_u = \frac{1}{2}) = \frac{1}{6} G(m_u = \frac{1}{2} \rightarrow m_l = -\frac{1}{2}) \cos \theta_k . \]  \hspace{1cm} (6.66) 

(4) \( m_u = -1/2 \rightarrow m_l = 1/2 \) transition

\[ R_{+1,+1}(m_u = -\frac{1}{2}) = \frac{1}{12} (1 - \cos \theta_k)^2 , \]
\[ R^\gamma_{+1,-1}(m_u = -\frac{1}{2}) = \frac{1}{12} \sin^2 \theta_k e^{-2i\phi_k} , \]
\[ R^\gamma_{-1,+1}(m_u = -\frac{1}{2}) = \frac{1}{12} \sin^2 \theta_k e^{2i\phi_k} , \]
\[ R^\gamma_{-1,-1}(m_u = -\frac{1}{2}) = \frac{1}{12} (1 + \cos \theta_k)^2 , \]

and

\[ I(m_u = -\frac{1}{2}) = \frac{1}{6} G(m_u = -\frac{1}{2} \rightarrow m_l = \frac{1}{2}) (1 + \cos \theta_k)^2 , \]
\[ Q(m_u = -\frac{1}{2}) = \frac{1}{6} G(m_u = -\frac{1}{2} \rightarrow m_l = \frac{1}{2}) \sin^2 \theta_k \cos 2\phi_k , \]
\[ U(m_u = -\frac{1}{2}) = \frac{1}{6} G(m_u = -\frac{1}{2} \rightarrow m_l = \frac{1}{2}) \sin^2 \theta_k \sin 2\phi_k , \]
\[ V(m_u = -\frac{1}{2}) = -\frac{1}{6} G(m_u = -\frac{1}{2} \rightarrow m_l = \frac{1}{2}) \cos \theta_k . \]

(5) $m_u = -1/2 \rightarrow m_l = -1/2$ transition

\[ R^\gamma_{+1,+1}(m_u = -\frac{1}{2}) = \frac{1}{3} \sin^2 \theta_k , \]
\[ R^\gamma_{+1,-1}(m_u = -\frac{1}{2}) = -\frac{1}{3} \sin^2 \theta_k e^{-2i\phi_k} , \]
\[ R^\gamma_{-1,+1}(m_u = -\frac{1}{2}) = -\frac{1}{3} \sin^2 \theta_k e^{2i\phi_k} , \]
\[ R^\gamma_{-1,-1}(m_u = -\frac{1}{2}) = \frac{1}{3} \sin^2 \theta_k , \]

and

\[ I(m_u = -\frac{1}{2}) = \frac{2}{3} G(m_u = -\frac{1}{2} \rightarrow m_l = -\frac{1}{2}) \sin^2 \theta_k , \]
\[ Q(m_u = -\frac{1}{2}) = -\frac{2}{3} G(m_u = -\frac{1}{2} \rightarrow m_l = -\frac{1}{2}) \sin^2 \theta_k \cos 2\phi_k , \]
\[ U(m_u = -\frac{1}{2}) = -\frac{2}{3} G(m_u = -\frac{1}{2} \rightarrow m_l = -\frac{1}{2}) \sin^2 \theta_k \sin 2\phi_k , \]
\[ V(m_u = -\frac{1}{2}) = 0 . \]
(6) $m_u = -3/2 \rightarrow m_l = -1/2$ transition

\[
R_{1,1+1}(m_u = -\frac{3}{2}) = \frac{1}{4} (1 - \cos \theta_k)^2 ,
\]

\[
R_{1,1-1}(m_u = -\frac{3}{2}) = \frac{1}{4} \sin^2 \theta_k e^{-2i\phi_k} ,
\]

\[
R_{1,1+1}(m_u = -\frac{3}{2}) = \frac{1}{4} \sin^2 \theta_k e^{2i\phi_k} ,
\]

\[
R_{1,1-1}(m_u = -\frac{3}{2}) = \frac{1}{4} (1 + \cos \theta_k)^2 ,
\]

and

\[
I(m_u = -\frac{3}{2}) = \frac{1}{2} G(m = -\frac{3}{2} \rightarrow m_l = -\frac{1}{2}) (1 + \cos^2 \theta_k) ,
\]

\[
Q(m_u = -\frac{3}{2}) = \frac{1}{2} G(m = -\frac{3}{2} \rightarrow m_l = -\frac{1}{2}) \sin^2 \theta_k \cos 2\phi_k ,
\]

\[
U(m_u = -\frac{3}{2}) = \frac{1}{2} G(m = -\frac{3}{2} \rightarrow m_l = -\frac{1}{2}) \sin^2 \theta_k \sin 2\phi_k ,
\]

\[
V(m_u = -\frac{3}{2}) = G(m = -\frac{3}{2} \rightarrow m_l = -\frac{1}{2}) \cos \theta_k .
\]

In this case we present the ratio of emergent Stokes parameters $Q/I, U/I$ and $V/I$ in order to provide a correspondence with the polarization measurements of the solar spectrum. They are presented graphically in Fig. (6.13) for the selected parameters $\theta_k = 0, \pi/4, \pi/2$ and $\phi_k = \pi/6$. We can observe four $\sigma$ lines when the LOS is parallel to the quantization axis (magnetic field direction) and two $\pi$ lines and four $\sigma$ lines when the LOS is perpendicular to the quantization axis. They are symmetric around the line center. In linearly polarized Stokes $Q/I$ and $U/I$ profiles, the $\sigma$ components appear in the blue and red line wings with a positive sign and the $\pi$ components appear with a negative sign. The circularly polarized Stokes $V/I$ profile is an antisymmetric line profile since the $\sigma$ components in blue and red line wings have opposite signs.
6.6.3 Stokes line profiles formed in the presence of a pure
electric quadrupole field

In this case, the electric quadrupole field splits the upper \( J_u = 3/2 \) level into four
energy eigenstates which are given by Eq. (2.21) in Chapter 2. One can see from
Eq. (2.21) that each of two energy eigenstates are degenerate. These degenerate
energy eigenstates are superposition of basis states \(| \frac{3}{2}, \frac{3}{2} \rangle \) and \(| \frac{3}{2}, -\frac{1}{2} \rangle \), or superposition
of \(| \frac{3}{2}, \frac{1}{2} \rangle \) and \(| \frac{3}{2}, -\frac{3}{2} \rangle \). We note that the energy levels corresponding to these energy
eigenstates are equally spaced and symmetric about the line center (see Fig. 2.3).
These level spacings depend on electric field strength \( A \) and asymmetry parameter \( \eta \).
Increase in \( A \) or \( \eta \) or both results in increased spacing between these energy levels.
However the lower \( J_l = 1/2 \) level is unaffected by the electric quadrupole field. In the
present calculation, we assume the lower energy eigenstate is \(| \frac{1}{2}, \frac{1}{2} \rangle \). Thus only two
transition components can be seen. The profile functions \( F(i, x) \) for these transitions
are shown in Fig. (6.14). The density matrix elements of each transition and the
corresponding Stokes parameters are shown below.

(1) From \(| \Psi_1 \rangle \) transition

\[
\begin{align*}
R_{1,+1}^1 (1) &= \frac{1}{4} \left[ a (1 + \cos \theta_k)^2 + b \sin^2 \theta_k e^{2i\phi_k} + b \sin^2 \theta_k e^{-2i\phi_k} + \frac{1}{3} c (1 - \cos \theta_k)^2 \right], \\
R_{1,-1}^1 (1) &= \frac{1}{4} \left[ a \sin^2 \theta_k e^{-2i\phi_k} + b (1 + \cos \theta_k)^2 + b (1 - \cos \theta_k)^2 e^{-4i\phi_k} + \frac{1}{3} c \sin^2 \theta_k e^{-2i\phi_k} \right], \\
R_{-1,+1}^1 (1) &= \frac{1}{4} \left[ a \sin^2 \theta_k e^{2i\phi_k} + b (1 + \cos \theta_k)^2 + b (1 - \cos \theta_k)^2 e^{4i\phi_k} + \frac{1}{3} c \sin^2 \theta_k e^{2i\phi_k} \right],
\end{align*}
\]
\[ R_{1,1}^{1}(1) = \frac{1}{4} \left[ a (1 - \cos \theta_k)^2 + b \sin^2 \theta_k e^{2i\phi_k} + b \sin^2 \theta_k e^{-2i\phi_k} + \frac{1}{3} c (1 + \cos \theta_k)^2 \right], \tag{6.73} \]

where the coefficients \( a, b \) and \( c \) are
\[ 1 = \frac{K + 3A}{2K}, \quad b = \sqrt{\frac{K^2 - 9A^2}{12K^2}}, \quad c = \frac{K - 3A}{2K}. \tag{6.74} \]

\[
I(1) = G(1) \left[ \left( \frac{1}{2} a + \frac{1}{6} c \right) (1 + \cos^2 \theta_k) + b \sin^2 \theta_k \cos 2\phi_k \right],
\]
\[
Q(1) = G(1) \left[ \left( \frac{1}{2} a + \frac{1}{6} c \right) \sin^2 \theta_k \cos 2\phi_k + \frac{1}{2} b (1 + \cos \theta_k)^2 + \frac{1}{2} b (1 - \cos \theta_k)^2 \cos 4\phi_k \right],
\]
\[
U(1) = G(1) \left[ \left( \frac{1}{2} a + \frac{1}{6} c \right) \sin^2 \theta_k \sin 2\phi_k + \frac{1}{2} b (1 - \cos \theta_k)^2 \sin 4\phi_k \right],
\]
\[
V(1) = (a - \frac{1}{3} c) \cos \theta_k. \tag{6.75} \]

(2) From \(|\Psi_2\rangle\) transition
\[
R_{1,1}^{1}(2) = \frac{1}{4} \left[ c (1 + \cos \theta_k)^2 - b \sin^2 \theta_k e^{2i\phi_k} - b \sin^2 \theta_k e^{-2i\phi_k} + \frac{1}{3} a (1 - \cos \theta_k)^2 \right],
\]
\[
R_{1,1}^{1}(1) = \frac{1}{4} \left[ c \sin^2 \theta_k e^{2i\phi_k} - b (1 + \cos \theta_k)^2 - b (1 - \cos \theta_k)^2 e^{-4i\phi_k} + \frac{1}{3} a \sin^2 \theta_k e^{-2i\phi_k} \right],
\]
\[
R_{2,1}^{1}(2) = \frac{1}{4} \left[ c \sin^2 \theta_k e^{2i\phi_k} - b (1 + \cos \theta_k)^2 - b (1 - \cos \theta_k)^2 e^{4i\phi_k} + \frac{1}{3} a \sin^2 \theta_k e^{2i\phi_k} \right],
\]
\[
R_{2,1}^{1}(1) = \frac{1}{4} \left[ c (1 - \cos \theta_k)^2 - b \sin^2 \theta_k e^{2i\phi_k} - b \sin^2 \theta_k e^{-2i\phi_k} + \frac{1}{3} a (1 + \cos \theta_k)^2 \right]. \tag{6.76} \]

and
\[
I(1) = G(2) \left[ \left( \frac{1}{2} c + \frac{1}{6} a \right) (1 + \cos^2 \theta_k) - b \sin^2 \theta_k \cos 2\phi_k \right],
\]
$Q(1) = G(1) \left[ (\frac{1}{2} c + \frac{1}{6} a) \sin^2 \theta_k \cos 2\phi_k - \frac{1}{2} b (1 + \cos \theta_k)^2 - \frac{1}{2} b (1 - \cos \theta_k)^2 \cos 4\phi_k \right],$

$U(1) = G(1) \left[ (\frac{1}{2} c + \frac{1}{6} a) \sin^2 \theta_k \sin 2\phi_k - \frac{1}{2} b (1 - \cos \theta_k)^2 \sin 4\phi_k \right],$

$V(1) = (c - \frac{1}{3} a) \cos \theta_k.$ 

(6.77)

(3) From $|\Psi_3\rangle$ transition

\[
R^\gamma_{1,1}(3) = \frac{1}{3} c \sin^2 \theta_k ,
\]

\[
R^\gamma_{1,-1}(3) = -\frac{1}{3} c \sin^2 \theta_k e^{-2i\phi_k} ,
\]

\[
R^\gamma_{-1,1}(3) = -\frac{1}{3} c \sin^2 \theta_k e^{2i\phi_k} ,
\]

\[
R^\gamma_{-1,-1}(3) = \frac{1}{3} c \sin^2 \theta_k ,
\]

(6.78)

and

\[
I(3) = \frac{2}{3} c \sin^2 \theta_k ,
\]

\[
Q(3) = -\frac{2}{3} c \sin^2 \theta_k \cos 2\phi_k ,
\]

\[
U(3) = -\frac{2}{3} c \sin^2 \theta_k \sin 2\phi_k ,
\]

\[
V(3) = 0 .
\]

(6.79)

(4) From $|\Psi_4\rangle$ transition

\[
R^\gamma_{1,1}(4) = \frac{1}{3} a \sin^2 \theta_k ,
\]

\[
R^\gamma_{1,-1}(4) = -\frac{1}{3} a \sin^2 \theta_k e^{-2i\phi_k} ,
\]

\[
R^\gamma_{-1,1}(4) = -\frac{1}{3} a \sin^2 \theta_k e^{2i\phi_k} ,
\]

\[
R^\gamma_{-1,-1}(4) = \frac{1}{3} a \sin^2 \theta_k ,
\]

(6.80)
\[ I(4) = \frac{2}{3} a \sin^2 \theta_k , \]
\[ Q(4) = -\frac{2}{3} a \sin^2 \theta_k \cos 2\phi_k , \]
\[ U(4) = -\frac{2}{3} a \sin^2 \theta_k \sin 2\phi_k , \]
\[ V(4) = 0 . \quad (6.81) \]

The results are presented graphically in Fig. (6.15) for the selected parameters \( \theta_k = 0, \pi/4, \pi/2 \) and \( \phi_k = \pi/6 \). We present the emergent Stokes profiles \( Q/I, U/I \) and \( V/I \) for a chosen asymmetry parameter \( \eta = 1 \). It is interesting to note that the difference in height of the emergent \( I \) profiles is caused by unequal superposition of basis states. As in the case of \( J_u = 1 \rightarrow J_l = 0 \) transition (see Fig. 6.11 for comparison) the appearance of linear polarization \( Q/I \) profile even when the LOS is parallel to the Z-axis of the PAF is a remarkable feature of the electric quadrupole field effect. In the present calculation (\( J_u = 3/2 \rightarrow J_l = 1/2 \) transition), the \( V/I \) profile is non-zero (see panels-a,b), because of unequal superposition of basis states. In contrast, in the previous calculation in sect. 6.5 (\( J_u = 1 \rightarrow J_l = 0 \) transition), the \( V/I \) profile always vanishes because of equal superposition of basis states (see Fig. 6.5-6.7). We do not show the results for the lower energy eigenstate \( |\frac{1}{2}, -\frac{1}{2} \rangle \) because, in that case \( V/I \) components simply change sign while \( Q/I \) and \( U/I \) profiles remain invariant with respect to the \( |\frac{1}{2}, \frac{1}{2} \rangle \) case discussed above.
6.6.4 Stokes line profiles formed in the presence of combined electric quadrupole field and arbitrary orientation of magnetic field

The theoretical results for this situation are presented in Fig. (6.16) for values of the free parameters given by $\eta = 1$ and $R = 1$. The magnetic field direction is again represented by $\theta_B = \pi/6$ and $\phi_B = \pi/4$. We confine the azimuth angle of line of sight direction to $\phi_k = \pi/4$, so that the angle between the magnetic field direction and line of sight direction is given by the relation $\gamma = \theta_B - \theta_k$ (see Fig. 6.1). The Table (6.2) summarizes the extent of contributions to the wave functions $|\Psi_1\rangle$, $|\Psi_2\rangle$ and $|\Psi_3\rangle$ from the basis states $|1, \pm 1\rangle$ and $|1, 0\rangle$, for chosen values of $\theta_B, \phi_B, \eta$ and $R$. These contributions represented as probabilities $|c_{m_u}^i|^2$ for the basis states under consideration, are displayed under the columns pertaining to the Zeeman terms. The cross terms viz. $2\text{Re}(c_{m_u}^i c_{m_u'}^{i*})$, which are representative of the quantum interference play a significant role in determining the Stokes line profiles.

The combined effect of electric quadrupole field and arbitrarily oriented magnetic field is to split the upper $J_u = 3/2$ level into four energy eigenstates. However, the lower $J_l = 1/2$ level is split into two non-degenerate energy eigenstates due to external magnetic fields itself. Recall that the electric quadrupole field does not affect $J_l = 1/2$ level. Thus, eight transition components can arise between the split levels. In this figure, we present the Stokes $I, Q, U$ and $V$ line profiles arising due to contributions from the Zeeman term, cross term and the sum of these two (combined effects). As in the case of $J_u = 1 \rightarrow J_l = 0$ (see Fig.6.11), $Q$ profiles exclusively arise from the cross terms. For the Stokes $U$ and $V$ parameters, both Zeeman and cross terms contribute.
Table 6.1: Contributions arising from the Zeeman terms and cross terms to the Stokes \( I, Q, U, V \) profiles for a transition \( J_u = 1 \rightarrow J_l = 0 \). The electric and magnetic field parameters employed are \( \eta = 1 \), \( R = B/A = 1 \), \( \theta_B = \pi/6 \) and \( \phi_B = \pi/4 \).

<table>
<thead>
<tr>
<th>Energy eigenstate</th>
<th>multiplying coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Zeeman terms ((m_u = m'_l))</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(</td>
<td>\psi_1\rangle )</td>
</tr>
<tr>
<td>(</td>
<td>\psi_2\rangle )</td>
</tr>
<tr>
<td>(</td>
<td>\psi_3\rangle )</td>
</tr>
</tbody>
</table>

Table 6.2: Contributions arising from the Zeeman terms and cross terms to the Stokes \( I, Q, U, V \) profiles for a transition \( J_u = 3/2 \rightarrow J_l = 1/2 \). The electric and magnetic field parameters employed are \( \eta = 1 \), \( R = B/A = 1 \), \( \theta_B = \pi/6 \) and \( \phi_B = \pi/4 \) with \( m_u = m'_l = \pm 1/2 \).

<table>
<thead>
<tr>
<th>Energy eigenstate</th>
<th>multiplying coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Zeeman terms ((m_u = m'_l))</td>
</tr>
<tr>
<td></td>
<td>((m_u = m'_l = \pm 1/2))</td>
</tr>
<tr>
<td>3/2</td>
<td>1/2</td>
</tr>
<tr>
<td>(</td>
<td>\psi_1\rangle )</td>
</tr>
<tr>
<td>(</td>
<td>\psi_2\rangle )</td>
</tr>
<tr>
<td>(</td>
<td>\psi_3\rangle )</td>
</tr>
<tr>
<td>(</td>
<td>\psi_4\rangle )</td>
</tr>
</tbody>
</table>
Figure 6.1: The magnetic field $\mathbf{B}$ and the direction $\mathbf{k}$ of emission of radiation identified as line of sight (LOS) with reference to the principal axes frame (PAF) coordinate system. $\gamma$ is the angle between the direction of the magnetic field and the line of sight.
Figure 6.2: The profile function $F(m, x)$ showing the pure magnetic field effect. The solid, dotted and dashed lines correspond to the excited states $\Psi_{-1}, \Psi_0, \Psi_{+1}$ respectively. $x$ is the frequency displacement from the line center in natural width units.
Figure 6.3: The effect of pure magnetic field. Panels (a)-(c) represent the emission Stokes line profiles for assumed temperature value $T = 6000K$ and natural width $\Gamma = 2.18 \times 10^8 s^{-1}$. The three different curves (solid, dash-dot-dot and dashed) correspond to the ratio of level splitting to natural width $= 0.4, 4, 8$ respectively. The Stokes $Q$ and $V$ are expressed in units of intensity. $x = \frac{\omega - \omega_0}{\Gamma}$ is the frequency displacement from the line center in natural width units.
Figure 6.4: The profile function $F(i, x)$ showing the pure electric quadrupole field effect. Panels (a)-(c) represent the asymmetry parameter $\eta = 0, 0.5, 1$. The solid, dotted and dashed lines correspond to the excited states $\Psi_x, \Psi_y, \Psi_z$ respectively. $x$ is the frequency displacement from the line center in natural width units.
Figure 6.5: The effect of pure electric quadrupole field ($\eta = 0$). Panels (a)-(c) represent the emission Stokes line profiles for assumed temperature value $T = 6000K$ and natural width $\Gamma = 2.18 \times 10^8 s^{-1}$. The three different curves (solid, dash-dot-dot and dashed) correspond to the ratio of level splitting to natural width $= 0.4, 4, 8$ respectively. The Stokes $Q$ is expressed in unit of intensity. $z = \frac{\omega - \omega_0}{\Gamma}$ is the frequency displacement from the line center in natural width units.
Figure 6.6: The effect of pure electric quadrupole field ($\eta = 0.5$). Panels (a)-(c) represent the emission Stokes line profiles for assumed temperature value $T = 6000K$ and natural width $\Gamma = 2.18 \times 10^8 s^{-1}$. The three different curves (solid, dash-dot-dot and dashed) correspond to the ratio of level splitting to natural width $= 0.4, 4, 8$ respectively. The Stokes $Q$ is expressed in unit of intensity. $x = \omega \frac{a}{\Gamma}$ is the frequency displacement from the line center in natural width units.
Figure 6.7: The effect of pure electric quadrupole field ($\eta = 1$). Panels (a)-(c) represent the emission Stokes line profiles for assumed temperature value $T = 6000K$ and natural width $\Gamma = 2.18 \times 10^8 s^{-1}$. The three different curves (solid, dash-dot-dot and dashed) correspond to the ratio of level splitting to natural width $= 0.4, 4, 8$ respectively. The Stokes $Q$ is expressed in unit of intensity. $x = \frac{\nu - \nu_{\text{lin}}}{\Gamma}$ is the frequency displacement from the line center in natural width units.
Figure 6.8: The effect of combined magnetic and electric quadrupole fields on Stokes line profiles for $\eta = 0$. Panels (a)-(c) represent the emission Stokes line profiles for assumed temperature value $T = 6000K$ and natural width $\Gamma = 2.18 \times 10^5 s^{-1}$. The three different curves (solid, dash-dot-dot and dashed) correspond to the ratio of magnetic to electric fields strengths = 1, 3, 5 respectively. The Stokes $Q$ and $V$ are expressed in units of intensity. $x = \frac{\omega - \omega_0}{\Gamma}$ is the frequency displacement from the line center in natural width units.
Figure 6.9: The effect of combined magnetic and electric quadrupole fields on Stokes line profiles for $\eta = 0.5$. Panels (a)-(c) represent the emission Stokes line profiles for assumed temperature value $T = 6000K$ and natural width $\Gamma = 2.18 \times 10^9s^{-1}$. The three different curves (solid, dash-dot-dot and dashed) correspond to the ratio of magnetic to electric fields strengths $= 1, 3, 5$ respectively. The Stokes $Q$ and $V$ are expressed in units of intensity. $x = \frac{\omega - \omega_0}{\Gamma}$ is the frequency displacement from the line center in natural width units.
Figure 6.10: The effect of combined magnetic and electric quadrupole fields on Stokes line profiles for $\eta = 1$. Panels (a)-(c) represent the emission Stokes line profiles for assumed temperature value $T = 6000K$ and natural width $\Gamma = 2.18 \times 10^8 s^{-1}$. The three different curves (solid, dash-dot-dot and dashed) correspond to the ratio of magnetic to electric fields strengths = 1, 3, 5 respectively. The Stokes $Q$ and $V$ are expressed in units of intensity. $x = \frac{\nu - \nu_0}{\Gamma}$ is the frequency displacement from the line center in natural width units.
Figure 6.11: The effect of combined electric quadrupole field and chosen direction of magnetic field $\theta_B = \pi/6, \phi_B = \pi/4$ and azimuth of the LOS $\phi_k = \pi/4$, on Stokes line profiles for $\eta = 1$ and $R = 1$. Panels (a)-(c) represent the emission Stokes line profiles for different values of $\gamma$ with assumed temperature $T = 6000 \text{ K}$ and natural line width $\Gamma = 2.18 \times 10^8 \text{ s}^{-1}$. The three different curves (dashed, solid and dotted) correspond respectively to the Zeeman term contributions, cross term contributions and the combined effect polarisations. The Stokes parameters $I, Q, U$ and $V$ are expressed in arbitrary units. The quantity $z = (\omega - \omega_0)/\Gamma$ is the frequency displacement from the line center in natural width units. The values $z > 0$ refer to the blue wing and $z < 0$ refer to the red wing of the line profile.
Figure 6.12: The profile function $F(m, x)$ showing the pure magnetic field effect.

The solid, dashed, dotted, dash-dot, dash-dot-dot and dash-dot-dot-dot profiles correspond to the all possible transition presented in section (6.6.2). $x$ is the frequency displacement from the line center in natural width units.
Figure 6.13: The effect of pure magnetic field in the emission Stokes line profiles of transition $J_u = 3/2 \rightarrow J_l = 1/2$ for a given temperature $T = 6000$ K and natural line width $\Gamma = 0.63 \times 10^8 \text{s}^{-1}$. The dashed and solid curves correspond to two values (5,10 respectively) for the ratio of level splitting to natural line width. The percentage of linear and circular polarization ($Q/I$, $U/I$ and $V/I$ profiles) are expressed in units of intensity. The quantity $x = (\omega - \omega_0)/\Gamma$ is the frequency displacement from the line center expressed in natural width units.
Figure 6.14: The profile function $F(i, x)$ showing the pure electric quadrupole field effect. The solid and dashed curves correspond to the transitions from the excited states $\Psi_2, \Psi_4$ and $\Psi_1, \Psi_3$ respectively. $x$ is the frequency displacement from the line center in natural width units.
Figure 6.15: The effect of pure electric quadrupole field ($\eta = 1$ case) in the emission Stokes line profiles for the transition $J_u = 3/2 \rightarrow J_l = 1/2$ when the temperature $T = 6000$ K and natural line width $\Gamma = 0.63 \times 10^8$ s$^{-1}$. The dashed and solid curves correspond to two values (2, 4 respectively) for the ratio of level splitting to natural line width. The percentage of linear and circular polarization ($Q/I, U/I$ and $V/I$ profiles) are expressed in units of intensity. The quantity $x = (\omega - \omega_0)/\Gamma$ is the frequency displacement from the line center expressed in natural width units.
Figure 6.16: The effect of combined electric quadrupole field (with $\eta = 1$) and the magnetic field (with $\theta_B = \pi/6, \phi_B = \pi/4$) on the Stokes line profiles for $\phi_B = \pi/4$ and $R = 1$. Panels (a)-(c) represent the emission Stokes line profiles for the transition $J_u = 3/2 \rightarrow J_l = 1/2$ for different values of $\gamma$. A temperature of $T = 6000$ K and natural width $\Gamma = 0.63 \times 10^6 \text{s}^{-1}$. The dashed, solid and dotted curves correspond respectively to the Zeeman term contributions, cross term contributions and the combined effect polarisations. The Stokes $I, Q, U$ and $V$ parameters are expressed in arbitrary units. The quantity $x = (\omega - \omega_0)/\Gamma$ is the frequency displacement from the line center expressed in natural width units.