Chapter 3
Modeling Stiles Crawford Effect of the first Kind as Pupil Apodization
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Light entering the periphery of the pupil appears dimmer than light entering axially. This is known as the Stiles-Crawford effect of the first kind (SCE I) [21,34,261]. Recent tests of visual optics using coherent light with annular and half-annular apertures have revealed a surprising absence of the SCE I [117], and there is no reason to suppose that a full pupil admitting a coherent beam would behave differently [37]. Here we demonstrate for sine, rectangular, square, triangular, and saw-tooth wave gratings that this behavior is in fact consistent with the standard model of SCE I as a pupil apodization. Thus, for coherent light such as for the integrated SCE can cancel the wavefront and thereby the SCE and for interference gratings this still holds true and the challenge still lies in how to translate this knowledge into the case of partially coherent or incoherent light where this does not occur.

3.1 Introduction

A beam of light stimulates the retina weakly when its entry to the pupil is gradually shifted from the centre toward the edge. This is manifested as a reduction in visibility, also known as Stiles-Crawford effect of the first kind (SCE I) [34]. The same kind of reduction in brightness in an image is also observed when the pupil allowing the light to enter is covered with a filter which becomes less and less transparent from the centre toward the edge. This is known as apodization [36]. Due to this outward resemblance between the retinal Stiles-Crawford effect and pupil apodization, the SCE I has been modeled as a pupil apodization in studying retinal light distributions in many imaging situations [38-40,44,48,89-91]. And this approach of treating SCE I as pupil apodization is justified as light is assumed to enter into the receptor through a single accepting aperture [17]. Here we have modeled the SCE I as a pupil apodization in the diffraction calculations of the retinal light distributions for periodic targets of different transmission profiles like sine, square, rectangular, triangular, and saw-tooth in coherent illumination and incoherent illumination.
Though a sinusoidal grating producing sinusoidal variations of brightness in the image is best suited for a test target [109,113,130,262-63], the difficulty in obtaining such a grating of unit contrast over a wide frequency range has led to the use of objects with square, rectangular, triangular and saw-tooth waveforms as test targets [46,49,92,264-272]. Similarly the choice of illumination was inclined more towards the use of the completely incoherent type, both uniform and non uniform, i.e. the extensive literature available for modulation evaluation studies has made use of various test targets under incoherent illumination [46, 49, 92, 264-272]. But no work appears to have been done which simultaneously combine all the three parameters of i. treating the SCE I as a pupil apodization, ii. choosing test target of different transmission profiles and iii. using a spatially coherent entering beam to evaluate modulation [93,94,97,98]. This has the significance of knowing about the changes in retinal light spatial distribution as a result of pupil apodization due to a grating structure of different transmission profiles and the novelty of promising manipulation of the coherence in the entering beam as an optical tool to obtain retinal response of a human eye in the presence of Stiles-Crawford effect of the first kind.

Although the above apodization model accounts for the SCE I, the phenomenon has a retinal basis in the optical behavior of photoreceptor cones [77]. The retina of a human eye uses the image formed on it of a physical object as a stimulus in causing photoisomerization of the photopigment molecules in the photoreceptors of a human eye. This is in effect the spatial distribution of photon absorption in the receptors [17] or retinal response. The total number of photoisomerizations depends on the number of absorbed photons, on the quantum efficiency of the photopigment for the wavelength distribution of the incoming light, and on the amplitude of light integrated over the wavefront passing through the pupil as conditioned by the receptor’s acceptance lobe. Hence, the spatial distribution of excitement of receptors otherwise called the retinal response will differ depending on their Stiles-Crawford effect [39,44].

The reduction in visibility can be expressed mathematically either as \( \eta = 10^{-\rho_{10}r^2} \)

or \( \eta = e^{-\rho_{e}r^2} \), where \( r \) is the distance in the entrance pupil from the origin of the function and \( \eta \) is the visibility [34]. The coefficients (of directionality that measures the width of the pupil apodization) \( \rho_e \) and \( \rho_{10} \) are related by \( \rho_e = \ln 10 \rho_{10} = 2.3 \rho_{10} \). The means and standard deviations [43] for \( \rho_{10} \) are: horizontal meridian, \( 0.048 \pm 0.013 mm^{-2} \); vertical
meridian, $0.053 \pm 0.012 \text{mm}^{-2}$. In the absence of aberrations, both the approaches, of modeling the Stiles-Crawford effect of the first kind as pupil apodization and the waveguiding of light in the photoreceptors give identical predictions for effective retinal light distributions [61,62,196]

Moreover unlike incoherent illumination, the non-linearity in intensity for coherent imaging does not allow the formulation of any general statements about the retinal image [109]. Thus the particular choice of test target and pupil opening (that too, in the absence of aberrations and defocus) will have its own kind of imaging characteristics in coherent illumination. What is then the series of transformations in case of coherent imaging ultimately leading to the fundamental imaging property of a human eye which is contained either in the modulation transfer function in the Fourier domain or in the image intensity distribution of a point object in the space domain? The amplitude of light in the pupil plane is the Fourier transform of the amplitude in the plane of the rectangular grating. This distribution represents the spatial frequency content of the amplitude displayed at distances from the geometrical image of the source proportional to the spatial frequencies. So the amplitude distribution of light in the plane of the retina is subjected to a non-linear transformation, that is, squaring, and the resulting intensity distribution constitutes the retinal image [109]. In apodization a change in the point spread function (PSF) occurs due to a gradual shading of the aperture toward its edge [39,44], because it reduces the height of the surrounding rings in the Airy disk.

3.2 Theory and Methods

The technique of coherent imaging involves the following stages: first, the amplitude of light coming from a specific periodic target is spatially Fourier analyzed; next, it is filtered, and resynthesized as a distribution of amplitude of light in the retinal plane. At the end, the amplitude is squared to get retinal light distributions [264-266]. That is, an object with a complex amplitude distribution is Fourier transformed to produce its corresponding spatial frequency spectrum. The apodizer, here SCE I modifies the object spectrum and the modified object spectrum is then inverse Fourier transformed to get the image complex amplitude distribution. Finally, the squared modulus of this amplitude gives the intensity distribution in the image from which the modulation is computed to ascertain the changes in retinal light spatial distribution in coherent illumination. The reasonable assumption of light
entering into a cone through a single transverse aperture allows us to incorporate the SCE I into the computation of retinal light distributions in the manner of an apodizing effect [17]. In the absence of aberrations this gives the identical prediction for effective retinal diffraction images with the photoreceptor waveguiding provided we are careful in multiplying the amplitude of the wavefront at each pupil location with the square root of the SCE I at that pupil location (square root because the SCE I has been measured for intensity and not amplitude) prior to the transformation leading to the retinal image light distribution [109]. This we have done by evaluating the modulation, an effective parameter for image quality assessment.

### 3.2.1 Images of Periodic Sine-Wave Targets

The object amplitude transmission in a sinusoidal grating object of period $p$ can be mathematically expressed as [95]

$$A(x,y) = a_1 + a_2 \cos(\omega x)$$

Where $a_1$ is the average amplitude in the object and $a_2$ is the modulation in the object amplitude, $\omega$ is the angular spatial frequency in reduced units, that is, $\omega = \frac{2\pi}{p}$ and $x, y$ are the horizontal and vertical coordinates of space.

The Fourier transform of Eq. 3.1 leads to

$$a(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [a_1 + a_2 \cos(\omega x)] e^{-i(ux+vy)} dxdy$$

(3.2)

Use of the two dimensional Dirac delta function defined as

$$\delta(p,r) = \iint_{-\infty}^{\infty} e^{-i(pq+rs)} dqs$$

reduces Eq.3.2 to

$$= a_1 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(ux+vy)} dxdy + a_2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cos(\omega x) e^{-i(ux+vy)} dxdy$$

$$= a_1 \delta(u,v) + \frac{a_2}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ e^{-i[(u-\omega)x+vy]} + e^{-i[(u+\omega)x+vy]} \right] dxdy$$
\[ a_1 \delta(u,v) + \frac{a_2}{2} [\delta(u - \omega, v) + \delta(u + \omega, v)] \]

Given that the Fourier transform of the input to the pupil is \( a(u,v) \), pupil exit function will be \( f(u,v)a(u,v) = a'(u,v) \) for \( f \) being the amplitude transfer function which we approximate as follows [51]:

\[
f(u,v) = \begin{cases} 
\frac{e^{-\frac{(u^2+v^2)}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} & \text{if } u^2 + v^2 < K, \\
0 & \text{otherwise}
\end{cases}
\]

where \( K = 1 \) for diffraction-limited coherent imaging systems, and \( K = 2 \) for diffraction-limited incoherent imaging systems [38, 51,112] for apodization parameter \( \sigma = 3.086 \) [43]

\[
a'(u,v) = e^{-\frac{(u^2+v^2)}{2\sigma^2}} \left[ a_1 \delta(u,v) + \frac{a_2}{2} [\delta(u - \omega, v) + \delta(u + \omega, v)] \right]
\]  

(3.3)

The image amplitude distribution at the exit pupil is obtained by the inverse Fourier transform of the spectrum \( a'(u,v) \). Thus

\[
A'(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} a'(u,v)e^{i(ux+vy)}dudv
\]

\[
A'(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(u,v) \left[ a_1 \delta(u,v) + \frac{a_2}{2} [\delta(u - \omega, v) + \delta(u + \omega, v)] \right] e^{i(ux+vy)}dudv
\]

\[
= a_1 + a_2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{[\delta(u - \omega, v) + \delta(u + \omega, v)]}{2} f(u,v)e^{i(ux+vy)}dudv
\]

\[
= a_1 + a_2 f(\omega,0) \cos(\omega x)
\]

(3.4)

Finally, the image energy distribution will be given by the squared modulus of above as

\[
|A'(x,y)|^2 = [a_1 + a_2 f(\omega,0) \cos(\omega x)]^2
\]  

(3.5)
\( \omega x \) can vary from 0 to \( \pi \) to compute maximum and minimum image energy distribution.

The modulation in the image (for unit contrast in the object) can be defined as

\[
M = \frac{|A(x,y)|^2_{max} - |A(x,y)|^2_{min}}{|A(x,y)|^2_{max} + |A(x,y)|^2_{min}}
\]

\[
|A(x,y)|^2_{max} = \left[ a_1 + a_2 e^{-\frac{\omega^2}{\sigma^2}} \right]^2
\]

\[
= a_1^2 + a_2^2 e^{\frac{2\omega^2}{\sigma^2}} + 2a_1a_2 e^{-\frac{\omega^2}{\sigma^2}}
\]

\[
|A(x,y)|^2_{min} = \left[ a_1 - a_2 e^{-\frac{\omega^2}{\sigma^2}} \right]^2
\]

\[
= a_1^2 + a_2^2 e^{-\frac{2\omega^2}{\sigma^2}} - 2a_1a_2 e^{-\frac{\omega^2}{\sigma^2}}
\]

Thus, taking \( a_1 = a_2 = \frac{1}{2} \) we get

\[
M = \frac{4a_1a_2 e^{\frac{\omega^2}{\sigma^2}} + 2e^{\frac{2\omega^2}{\sigma^2}}}{1 + e^{\frac{2\omega^2}{\sigma^2}}} = \frac{2e^{-\frac{\omega^2}{\sigma^2}}}{e^{\omega^2/(\sigma^2 + e^{\omega^2/\sigma^2})}} = \text{sech} \frac{\omega^2}{\sigma^2}
\]

\[
M_{coh} = \text{sech} \frac{\omega^2}{\sigma^2}
\]

(3.7)

As the incoherent imaging is linear in image energy distribution, rather than amplitude the image energy distribution and the corresponding modulation equations for incoherent illumination will be given by

\[
I(x,y)_{incoh} = a + be^{-\frac{\omega^2}{\sigma^2}} \cos (\omega x)
\]

(3.8)

where \( a \) is average intensity and \( b \) is modulation.

\[
M_{inc} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}
\]

\[
I_{max} = a + be^{-\frac{\omega^2}{\sigma^2}}
\]

\[
I_{min} = a - be^{-\frac{\omega^2}{\sigma^2}}
\]

Taking average irradiance (or intensity) and the modulation in the object energy distribution as equal, we get the contrast in the image as

\[
M_{inc} = e^{-\frac{\omega^2}{\sigma^2}}
\]

(3.9)
3.2.2 Images of Periodic Square-Wave Targets

A vertical rectangular wave grating with average amplitude $a$, modulation $b$, period $p$ and duty cycle $\alpha$ can be expressed as follows [264,268,270,273]:

$$A(x,y) = (a - b + 2ab) + \frac{4b}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\alpha\pi)}{n} \cos(n\omega x)$$  \hspace{1cm} (3.10)

where $\omega = \frac{2\pi}{p}$ and $x,y$ are the horizontal and vertical coordinates of space.

The Fourier transform of Eq.3.10 as usual leads to

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[(a - b + 2ab) + \frac{4b}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\alpha\pi)}{n} \cos(n\omega x)\right] e^{-i(ux + vy)} dxdy$$  \hspace{1cm} (3.11)

Using two dimensional Dirac delta function defined above and assuming

$$(a - b + 2ab) = C$$ and $K_n = \frac{4b \sin(n\pi\alpha)}{n\pi}$ for simplicity of mathematical steps

Reduces Eq. 3.11 to

$$C \delta(u,v) + \frac{1}{2} \sum_{n=1}^{\infty} K_n[\delta(u - n\omega,v) + \delta(u + n\omega,v)]$$

Following the steps discussed in 4.2.1, the above equation becomes

$$a'(u,v) = e^{-\frac{(u^2 + v^2)}{\sigma^2}} \left[ C \delta(u,v) + \frac{1}{2} \sum_{n=1}^{\infty} K_n[\delta(u - n\omega,v) + \delta(u + n\omega,v)] \right]$$  \hspace{1cm} (3.12)

The inverse Fourier transform of the spectrum $a'(u,v)$ leads to
\[ C + \sum_{n=0}^{\infty} K_n f(n\omega,0)\cos(n\omega x) \]

Substituting back in for \( C \) and \( K_n \) yields
\[ A'(x,y) = a - b + 2ab + \frac{4b}{\pi} \sum_{n=1}^{\infty} \frac{\sin(na\pi)}{n} f(n\omega,0)\cos(n\omega x) \]  \hspace{1cm} (3.13)

As \( f(n\omega,0) = 0 \) for \( n\omega \geq 1 \) the upper limit \( n' \) of \( n \) is such that \( n\omega < 1 \). So, Eq. 3.13 will be
\[ A'(x,y) = a - b + 2ab + \frac{4b}{\pi} \sum_{n=1}^{n'} \frac{\sin(na\pi)}{n} f(n\omega,0)\cos(n\omega x) \]  \hspace{1cm} (3.14)

Finally, the image energy distribution will be given by the squared modulus of above as
\[ |A'(x,y)|^2 = \left[ (a - b + 2ab + \frac{4b}{\pi} \sum_{n=1}^{n'} \frac{\sin(na\pi)}{n} f(n\omega,0)\cos(n\omega x) \right]^2 \]  \hspace{1cm} (3.15)

For a square wave grating of unit amplitude, as \( a = b = \frac{1}{2}, \alpha = \frac{1}{2} \Rightarrow (a - b + 2ab) = \frac{1}{2} \), Eq. 3.15 reduces to
\[ \left[ \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{n'} \frac{\sin(na\pi)}{n} f(n\omega,0)\cos(n\omega x) \right]^2 \]  \hspace{1cm} (3.16)

\( \omega x \) can vary from 0 to \( \pi \) to compute maximum and minimum image energy distribution.

The modulation in the image (for unit contrast in the object) can be computed by using the expression given below
\[ M = \frac{|A'(x,y)|^2_{\max} - |A'(x,y)|^2_{\min}}{|A'(x,y)|^2_{\max} + |A'(x,y)|^2_{\min}} \]  \hspace{1cm} (3.17)

For incoherent illumination
\[ I(x,y)_{\text{incoherent}} = a - b + 2ab + \frac{4b}{\pi} \sum_{n=1}^{n'} \frac{\sin(na\pi)}{n} f(n\omega,0)\cos(n\omega x) \]  \hspace{1cm} (3.18)

where \( a \) is average intensity and \( b \) is modulation.

As \( f(n\omega,0) = 0 \) for \( n\omega \geq 2 \) the upper limit \( n' \) of \( n \) is such that \( n\omega < 2 \) for incoherent illumination [51].
\[ M_{\text{inc}} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \]  \hspace{1cm} (3.19)
3.2.3 Images of Periodic Triangular Wave Targets

The amplitude transmittance of a vertical triangular wave grating, modeled as a transmitting structure in the object space with average amplitude \( a \), modulation \( b \), period \( p \) and duty cycle \( \alpha \) can be expressed as follows [94,268-270]:

\[
A(\xi, \eta) = (a - b + 2ab) + \frac{4b}{\pi^2 \alpha} \sum_{n=1}^{\infty} \frac{\sin^2(n\alpha \pi)}{n^2} \cos(n\omega \xi)
\]  

(3.20)

where \( \omega = \frac{2\pi}{p} \) and \( \xi, \eta \) are the horizontal and vertical coordinates of space respectively.

Along the other axis of the grating (parallel to the bars), the amplitude is fixed; thus,

\[
A(\xi, \eta) = A(\xi), \text{ for all } \eta.
\]

The Fourier transform of Eq.3.20 leads to

\[
a(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(\xi, \eta) e^{-i(\xi u + \eta v)} d\xi d\eta = \\
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ (a - b + 2ab) + \frac{4b}{\pi^2 \alpha} \sum_{n=1}^{\infty} \frac{\sin^2(n\alpha \pi)}{n^2} \cos(n\omega \xi) \right] e^{-i(\xi u + \eta v)} d\xi d\eta
\]

(3.21)

With use of 2-D Dirac delta function and the simplification of mathematical steps discussed above this reduces to

\[
C \delta(u, v) + \frac{1}{2} \sum_{n} \left[ \delta(u - n\omega, v) + \delta(u + n\omega, v) \right] a(u, v) \\
= (a - b + 2ab) \delta(u, v) \\
+ \frac{2b}{\pi^2 \alpha} \sum_{n=1}^{\infty} \frac{\sin^2(n\alpha \pi)}{n^2} \left[ \delta(u - n\omega, v) + \delta(u + n\omega, v) \right]
\]

Following similar steps outlined in 3.2.1 & 3.2.2 this becomes

\[
a'(u, v) = e^{-\frac{|u^2 + v^2|}{\sigma^2}} \left[ C \delta(u, v) + \frac{1}{2} \sum_{n} \left\{ \delta(u - n\omega, v) + \delta(u + n\omega, v) \right\} \right]
\]

Then inverse Fourier transform of \( a'(u, v) \) gives rise to

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{|u^2 + v^2|}{\sigma^2}} \left[ C \delta(u, v) + \frac{1}{2} \sum_{n} \left\{ \delta(u - n\omega, v) + \delta(u + n\omega, v) \right\} \right] dudv
\]

Utilizing the properties of the Dirac delta function and replacing the notations the above equation becomes
Now, $f(0,0)$ can be normalized to unity and for rotationally symmetric system we can write

$$f(n\omega,0) = f(-n\omega,0)$$

As $f(n\omega,0) = e^{-\frac{n^2\omega^2}{\sigma^2}}$. Hence,

$$A'(x,y) = C + \frac{1}{2} \sum_{n} f(n\omega,0) \left[ e^{in\omega x} + e^{-in\omega x} \right] = C + \sum_{n} f(n\omega,0) \cos(n\omega x).$$

As $C$ and

$$\sum_{n} = \frac{4b}{\pi^2 \alpha} \sum_{n=1}^{\infty} \frac{\sin^2(n\alpha \pi)}{n^2}$$

$$A'(x,y) = (a - b + 2ab) + \frac{4b}{\pi^2 \alpha} \sum_{n=1}^{\infty} \frac{\sin^2(n\alpha \pi)}{n^2} f(n\omega,0) \cos(n\omega x)$$

$f(0,0) = 1$ for rotationally symmetric systems and $f(n\omega,0) = 0$ for $n\omega \geq 1$. Thus the upper limit $n'$ of $n$ is such that $n\omega < 1$. Finally, the image intensity distribution will be given by the squared modulus of the above equation as

$$B'(x,y) = |A'(x,y)|^2 = \left[ (a - b + 2ab) + \frac{4b}{\pi^2 \alpha} \sum_{n=1}^{\infty} \frac{\sin^2(n\alpha \pi)}{n^2} f(n\omega,0) \cos(n\omega x) \right]^2$$

For a triangular wave grating of unit amplitude, $a = b = \frac{1}{2}$ and $\alpha = \frac{1}{2}$. Hence,

$$B'(x,y) = \left[ \frac{1}{2} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{\sin^2(n\alpha \pi)}{n^2} f(n\omega,0) \cos(n\omega x) \right]^2 \quad (3.22)$$

$\omega x$ can vary from 0 to $\pi$ to compute maximum and minimum intensity. The modulation in the image (for unit contrast in the object) can be defined as $M = \frac{B_{\text{max}} - B_{\text{min}}}{B_{\text{max}} + B_{\text{min}}} \quad (3.23)$

For the limiting case of $\alpha \to 0$, the triangular wave object reduces to an object with infinitely thin lines. In this case, for transmission of a finite amount of light, $b$ should tend to infinity. Considering normalized intensity in the line as unity, that is, $2ab \to 1$, we find the object function for this case (and for $a = b$) as

$$A(\xi,\eta) \to 1 + 2 \sum_{n=1}^{\infty} \cos(n\omega \xi).$$

Hence the intensity distribution in the image is given by

$$B'(x,y) = \left[ 1 + 2 \sum_{n=1}^{\infty} f(n\omega,0) \cos(n\omega x) \right]^2 \quad (3.24).$$
For $n=1$, $f(n\omega,0)$ reduces to the sine wave response of the system.

### 3.2.4 Images of Periodic Saw-Tooth Wave Targets

The transmission function of a periodic saw-tooth wave pattern can be mathematically defined as follows [93]

$$ A(x) = h + m(x + b) \text{ for } 0 < x < a - b $$

$$ = h + m(x + b - 2a) \text{ for } a - b < x < 2a. $$

Where $h$ is the mean level of irradiance in the object, $m$ is the slope of the pattern and $2a$ is the period for which the corresponding spatial frequency is $\omega = \frac{\pi}{a}$.

The Fourier series representation of the above object transmission function is given by: [112,114]

$$ A(x,y) = h - \sum_{n=1}^{\infty} \frac{2am}{n\pi} \cos(n\pi) \sin(n\omega x) $$

(3.25)

where $\omega = \frac{2\pi}{2a} = \frac{\pi}{a}$ and $x,y$ are the horizontal and vertical coordinates of space.

The Fourier transform of Eq. 3.25 leads to

$$ a(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(x,y) e^{-i(ux+vy)} dx dy $$

$$ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ h - \sum_{n=1}^{\infty} \frac{2am}{n\pi} \cos(n\pi) \sin(n\omega x) \right] e^{-i(ux+vy)} dx dy $$

The evaluation of the above integral calls for the use of the two dimensional Dirac delta function defined as $\delta(p,r) = \int_{-\infty}^{\infty} e^{-i(pq+rs)} dq ds$

Letting Let $K_n = \frac{2am}{n\pi} \cos(n\pi)$ reduces the above equation to

$$ a(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ h - \sum_{n=1}^{\infty} K_n \sin(n\omega x) \right] e^{-i(ux+vy)} dx dy $$

$$ = h \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(ux+vy)} dx dy + \sum_{n=1}^{\infty} K_n \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sin(n\omega x) e^{-i(ux+vy)} dx dy $$
\[
= h \, \delta(u, v) - \sum_{n=1}^{\infty} K_n \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ e^{-i[(u-n\omega)x+vy]} - e^{-i[(u+n\omega)x+vy]} \right] dx dy
\]

\[
= h \, \delta(u, v) - \frac{1}{2i} \sum_{n=1}^{\infty} K_n [\delta(u - n\omega, v) - \delta(u + n\omega, v)]
\]

Given that the Fourier transform of the input to the pupil is \( a(u, v) \), pupil exit function will be \( f(u, v) a(u, v) = a'(u, v) \) for \( f \) being the amplitude transfer function which we approximate as follows [51]:

\[
f(u, v) = \begin{cases} 
e^{-\frac{(u^2+v^2)}{\sigma^2}} & \text{if } u^2+v^2 < K, \\
0 & \text{otherwise}
\end{cases}
\]

where \( K = 1 \) for diffraction-limited coherent imaging systems, and \( K = 2 \) for diffraction-limited incoherent imaging systems [51, 274-76] for apodization parameter \( \sigma = 3.086 \) [43]

\[
a' (u, v) = e^{-\frac{(u^2+v^2)}{\sigma^2}} \left[ h \, \delta(u, v) - \frac{1}{2i} \sum_{n=1}^{\infty} K_n \{\delta(u - n\omega, v) - \delta(u + n\omega, v)\} \right]
\]

(3.26)

The image amplitude distribution at the exit pupil is obtained by the inverse Fourier transform of the spectrum \( a'(u, v) \). Thus

\[
A'(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ a'(u, v) e^{i(ux + vy)} \right] du dv
\]

\[
A'(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(u, v) \left[ h\delta(u, v) - \frac{1}{2i} \sum_{n=1}^{\infty} K_n \{\delta(u - n\omega, v) - \delta(u + n\omega, v)\} \right] e^{i(ux + vy)} du dv
\]

\[
= h - \sum_{n=1}^{\infty} K_n \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\{\delta(u - n\omega, v) - \delta(u + n\omega, v)\}}{2i} f(u, v) \ e^{i(ux + vy)} du dv
\]

\[
= h - \sum_{n=1}^{\infty} K_n f(n\omega, 0) \sin(n\omega x)
\]

Substituting back in for \( K_n \) yields
\[ A'(x, y) = h - \sum_{n=1}^{\infty} \frac{2am}{n\pi} \cos(n\pi)f(n\omega, \theta) \sin(n\omega x) \quad (3.27) \]

As \( f(n\omega, \theta) = 0 \) for \( n\omega \geq 1 \) the upper limit \( n' \) of \( n \) is such that \( n\omega < 1 \). So, Eq. 3.27 will be

\[ A'(x, y) = h - \sum_{n=1}^{n'} \frac{2am}{n\pi} \cos(n\pi)f(n\omega, \theta) \sin(n\omega x) \quad (3.28) \]

Finally, the image energy distribution will be given by the squared modulus of above as

\[ |A'(x, y)|^2 = \left[ h - \sum_{n=1}^{n'} \frac{2am}{n\pi} \cos(n\pi) e^{-0.105n^2\omega^2} \sin(n\omega x) \right]^2 \quad (3.29) \]

\( \omega x \) can vary from 0 to \( \pi \) to compute maximum and minimum image energy distribution.

The modulation in the image (for unit contrast in the object) can be defined as

\[ M = \frac{|A'(x, y)|_{max}^2 - |A'(x, y)|_{min}^2}{|A'(x, y)|_{max}^2 + |A'(x, y)|_{min}^2} \quad (3.30) \]

For incoherent illumination

\[ I(x, y)_{incoherent} = h - \sum_{n=1}^{n'} \frac{2am}{n\pi} \cos(n\pi) e^{-0.105n^2\omega^2} \sin(n\omega x) \quad (3.31) \]

As \( f(n\omega, \theta) = 0 \) for \( n\omega \geq 2 \) the upper limit \( n' \) of \( n \) is such that \( n\omega < 2 \) for incoherent illumination [51,93].

\[ M_{inc.} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \quad (3.32) \]

3.3. Results and Discussion

3.3.1 Sine Wave Response

The retinal light distributions in the images of periodic sine wave targets, formed by a human eye apodized with SCE-I under coherent and incoherent illumination was computed using Eq.(3.6) and Eq (3.8) respectively. Eq. (3.7) is used to find out the modulations in the images of sinusoidal grating targets for coherent illumination. Similarly
modulation is evaluated for the incoherent illumination using Eq. (3.9). The values are plotted from Fig.3.1 to Fig. 3.7. The cut-off frequency is the highest spatial frequency that a normal human eye could resolve if it had perfect optics. And it is 75 cycles/deg for incoherent light and 37.5 cycles/deg [51,112,274-76] for the coherent one.

1. From Fig.3.1, it is evident that for an apodization parameter of \(\sigma = 1.0\), the modulation decreases monotonically with the increase of the spatial frequency in case of incoherent illumination. But for coherent illumination the variation is not pronounced.

2. By changing the apodization parameter to \(\sigma = 2.0\), we see from Fig.3.2 that the modification in the modulation for incoherent illumination continues unhindered while that for coherent light it almost remains unmodified.

3. \(\sigma = 3.086\) is the value of the apodization parameter for a human eye with Stiles Crawford effect of the first kind. This value is important for our study and we see from Fig. 3.3 that the modulation registers a change of 35 \% in the modulation over the entire range of spatial frequencies for incoherent light. But the modulation does not get modified for coherent light except for a minor variation of about 0.5\%.

4. By increasing the apodization parameter to \(\sigma = 5.0\) (Fig. 3.4), even the changes in modulation decreases in case of incoherent illumination also.

5. And with a large apodization of \(\sigma = 30.0\) (Fig. 3.5) the Gaussian character is lost and the modulation even for incoherent illumination hovers around the constant value of unity.

6. Fig. 3.6 depicts the overall response of modulation to spatial frequencies for different apodization parameters when the entering light is coherent. Again the modulation remains unmodified except for \(\sigma = 1.0\) in case of incoherent illumination.

7. And Fig. 3.7 depicts the overall response of modulation to spatial frequencies for different apodization parameters when the entering light is incoherent. There are various degrees of modification in modulation for all apodization parameters except \(\sigma = 30.0\).
There is a departure of almost 70-fold while moving from completely coherent to total incoherent illumination. And the retina responds to this departure by accordingly modifying the modulation as revealed in case of incoherent light.

Figure 3.1: Variation of modulation with spatial frequency for both coherent and incoherent illumination for an apodization parameter $\sigma = 1.0$

Figure 3.2: Variation of modulation with spatial frequency for both coherent and incoherent illumination for an apodization parameter $\sigma = 2.0$

Figure 3.3: Variation of modulation with spatial frequency for both coherent and incoherent illumination for an apodization parameter $\sigma = 3.086$

Figure 3.4: Variation of modulation with spatial frequency for both coherent and incoherent illumination for an apodization parameter $\sigma = 5.0$
3.3.2 Square Wave Response

The image energy distribution in the images of periodic square wave targets, formed by a human eye apodized with SCE-I under coherent illumination was computed to find out the modulations in the images of periodic square wave targets using Eq. (3.16) and Eq. (3.17), respectively. Modulation is plotted graphically in fig. 3.8. Similarly modulation is evaluated for the incoherent illumination using Eq. (3.19) after obtaining intensity from Eq. (3.18) and plotted in fig. 3.9. The Nyquist frequency or the cut-off frequency determined by the geometry of the cone array is the highest spatial frequency that a normal human eye
could resolve if it had perfect optics. So, the fineness of the retinal sampling array imposes a 75 Hz cut-off on square waves that can be delivered to the retina using incoherent light. And for coherent light this will be lower, i.e., 37.5 cycles/deg [51,274-280]. [Appendix I]

From Fig. 3.8, it is evident that though the modulation, first, decreases with the increase of the spatial frequency, attaining next a minimum contrast for a particular spatial frequency of 18 cycles/deg and finally increasing sharply with \( \omega \), the contrast changes only by 0.2 % throughout for coherent entering light. But from Fig. 3.9, it is seen that the modulation registers a variation of as high as 20 % with respect to the spatial frequency for incoherent illumination which is 100 times more compared to that for the coherent case. Again in Fig. 3.10, the modulations for both coherent and incoherent illumination are compared.
And we have found that for coherent illumination the retinal image energy distribution somehow remains insensitive to the SCE I apodization.

Strikingly, with coherent illumination, the Stiles-Crawford effect is absent [37,54,117]. But the Stiles-Crawford effect operates in response to incoherent illumination to accentuate low vs. high spatial frequencies.

In Fig.3.11, the modulation is plotted against spatial frequency for various values of the apodization parameter. It is noticed again that the contrast changes only by 0.4 % over the entire spatial frequency range for coherent illumination, even for different apodization parameters emphasizing the importance of nature of illumination in deciding retinal response.

The modulation’s inability in showing the expected modification in coherent illumination for a human eye where the SCE I is modeled as a pupil apodization points to a more deeper fact in working, that is, the retina prefers to respond to the more immediate stimulus of an interference pattern than to the pupil entry points of the entering beam hitherto governed traditionally by Stiles-Crawford effect of the first kind only. Recent experiments [37, 54,117] have also validated such line of arguments and findings.
In a proceeding paper to be published it is shown that the coherence in the entering beam can be used as an optical tool to regulate the spatial distribution of photon absorption in the photoreceptors of a human eye [56].

3.3.3 Triangular Wave Response

1. For the values of \( \alpha = 0.1, 0.25, 0.50, 0.90 \) and \( a = b = 0.50 \), the intensity distribution in the images has been calculated by use of Eq. 3.22. In each case, \( \omega \) has been given the values of 0.1, 0.25, 0.5, 0.7 and 1.0. The dependence of intensity on \( \omega x \) is illustrated from Figs. 3.12 to 3.15.

![Figure: 3.12. Variation of intensity with reduced distance (\( \omega x \)) for different normalized spatial frequency (\( \omega \)) for \( \alpha = 0.1 \)](image1)

![Figure: 3.13 Variation of intensity with reduced distance (\( \omega x \)) for different normalized spatial frequency (\( \omega \)) for \( \alpha = 0.25 \)](image2)

![Figure: 3.14 Variation of intensity with reduced distance (\( \omega x \)) for different normalized spatial frequency (\( \omega \)) for \( \alpha = 0.5 \)](image3)

![Figure: 3.15 Variation of intensity with reduced distance for different normalized spatial frequency (\( \omega \)) for \( \alpha = 0.9 \).](image4)

It is evident from the figures that the intensity decreases with increasing spatial frequency for all \( \alpha \). But at \( \alpha = 0.5 \), all the spatial frequencies behave alike registering a smooth variation with \( \omega x \).
2. Using Eq. 3.23 we have computed the modulation both for the SCE I apodization and other apodization parameters. Here again the modulation decreases with frequency but the variation is within 0.4 % indicating a non-impact of apodization on visual performance of a human eye in coherent illumination. (Figs. 3.16 & 3.17).

3. Using Eq. 3.24 and Eq. 3.25 we have discussed the interesting case of a periodic object with infinite lines ($\alpha \to 0$), a line object. Here also we have found the intensity to decrease with $\omega x$ for various $\omega$. And the modulation, though appears to decrease with $\omega$, the variation is again restricted to within 0.4 %. (Figures. 3.18 & 3.19).
4. Figure 3.20 compare the normalized intensity distributions in the images for uniform illumination ($\sigma = 0$) and SCE I apodized eye ($\sigma = 3.086$). This shows the effect of an apodization on the intensity distribution in the image. For an eye apodized with the SCE I ($\sigma = 3.086$), at high ($\omega=1.0$) spatial frequencies, the width of the diffraction pattern almost remains unaltered again indicating a non improvement in visual response of the eye under coherent illumination.

5. Figures 3.21 and 3.22 show the equivalent sine wave response of a triangular grating in coherent illumination. The modulation is calculated for various values of $\alpha$. The first thing to note is that the modulation for the triangular wave is always greater than that of the corresponding sine wave for $\alpha < 0.5$ for all values of $\omega$. When $\alpha=0.5$, the modulation of both sine and triangular match for values of $\omega \geq 0.5$, but the sine wave response is smaller than the triangular response for values of $\omega < 0.5$. When $\alpha > 0.5$, the sine wave response gradually closes the gap between it and the corresponding triangular response for $\omega < 0.5$, and maintains the lead over the triangular response for $\omega < 0.5$. If the lower harmonics in the object are eliminated, then the triangular response will mimic the sine wave response for the
value of \( \alpha = 0.5 \) for a SCE I apodized human eye in coherent illumination. Or we can say that as in a triangular wave object the lower harmonics grow weak, the triangular wave response can very well approximate to the sine wave response after elimination of the lower harmonics from the object.

6. Finally, in Figure 3.23 we have compared the modulation of a triangular object both in a SCE I apodized (\( \sigma = 3.086 \)) eye and in a non-apodized eye (\( \sigma = 0 \)) and concluded that SCE I apodized human eye the apodization is acting like a low band pass filter blocking the high frequency components.

3.3.4 Saw-Tooth Wave Response

The retinal light distributions in the images of periodic saw-tooth wave targets, formed by a human eye with SCE-I modeled as pupil apodization under coherent and incoherent illumination was computed using Eq. (3.29) and Eq (3.31) respectively. Eq. (3.30) is used to find out the modulations in the images of saw-tooth wave grating targets for coherent illumination and Eq. (3.32) for incoherent illumination. The values are plotted from Fig.3.24 to Fig. 3.26. As usual the cut off frequency for incoherent light is twice that of the coherent one, i.e., 37.5 cycles/deg.
Table 3.1: Computation of Modulation for different apodization parameters over the entire range of spatial frequencies ($\omega$) under coherent incident illumination

<table>
<thead>
<tr>
<th>$\omega$ in c/deg</th>
<th>$m=0.5$</th>
<th>$m=0.75$</th>
<th>$m=1.0$</th>
<th>$m=1.25$</th>
<th>$m=1.5$</th>
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<tr>
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<td>0.99927</td>
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</tr>
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<td>0.99703</td>
<td>0.98686</td>
<td>0.99606</td>
</tr>
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</table>

1. From Fig. 3.24 and table 3.2, it is seen that to a minor extent the slope governs the modulation, like for different slope the maximum and minimum modulation occur at different spatial frequencies, ex., for $m=0.5$, maximum is at 30 c/deg and minimum at 22.5 c/deg. Similarly for $m=1.0$, maximum is at 18.75 c/deg and minimum is at 26.25 c/deg.

2. But on an average the modification in the modulation registers a change limited to a mere 0.5 % over the entire spatial frequency range for all possible values of the slope.

3. This striking behavior of near constancy of the modulation over the entire spatial frequency range for coherent entering light regardless of the values of the slope shows that coherence of the entering beam has also a role to play in governing retinal light distributions in addition to the pupil entry point of the beam in traditional Stiles Crawford effect. This is experimentally validated by the absence of an integrated Stiles Crawford function for coherent light [54,117].
Table 3.2: Computation of Modulation for different apodization parameters over the entire range of spatial frequencies (ω) under incoherent incident illumination

<table>
<thead>
<tr>
<th>ω in c/deg</th>
<th>m=0.5</th>
<th>m=0.75</th>
<th>m=1.0</th>
<th>m=1.25</th>
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</tr>
<tr>
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<td>1.3232</td>
<td>1.3232</td>
<td>1.3232</td>
<td>1.3232</td>
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</table>
$\sigma = 3.086$ is the value of the apodization parameter for a human eye with Stiles Crawford effect of the first kind [43]. This value is important for our study and we see from Fig. 3.25 that when we incorporate this value in computing the modulation, the modulation falls gradually over the entire range of spatial frequencies in

4.  

Fig. 3.24: Modulation of a saw-tooth wave target for various slopes under coherent illumination. To a minor extent the slope governs the coherent response.

Fig. 3.25: Modulation of a saw-tooth wave target for various slopes under incoherent illumination. The modulation behaves same regardless of the value of the slope.

Fig. 3.26: Modulation of a saw-tooth wave target for various slopes under both coherent and incoherent illumination. The straight line response is for coherent illumination.
incoherent illumination [92]. That is, the incoherent response is only governed by Stiles Crawford effect. This variation is quite appreciable reaching a whopping 98%.

5. The modulation under incoherent illumination keeps its behavior of decreasing with the increase in the value of the spatial frequency intact regardless of the value of the slope as ascertained from Table 3.3. That is, incoherent illumination responds favorably to spatial frequency and is insensitive to change of slope [92]. So, the incoherent response mimics the traditional Stiles Crawford effect.

6. Fig. 3.26 depicts the overall response of modulation to spatial frequencies for apodization parameter 3.086 (a human eye with Stiles Crawford effect modeled as pupil apodization) both for coherent and incoherent light taken together. This figure clearly indicates that coherence in the entering beam (as modulation is quite different for incoherent illumination compared to the coherent one) has the potential to be used as an optical parameter in controlling retinal response [56]. While moving from coherent to incoherent again we encounter a departure of 196-fold forcing the retina to go for a compensative response by decreasing the modulation accordingly.

7. Pupil apodization can be used as an optical technique to obtain retina light distributions in presence of Stiles-Crawford effect of the first kind regardless of the transmission profile of the test targets chosen [94,97].

8. The importance of Stiles-Crawford effect lies in the determination of the effective pupil apodization function for eye and vision modeling [281].

9. The modification the coherence in the entering beam causes to the Stiles Crawford visibility function can be employed as a bio-marker in predicting the early onset of ailments affecting the photoreceptor cones.

### 3.4. Conclusion

Pupil apodization technique can be used to incorporate directional effects of the retina [40,94,196]. And the analysis of this directionality is needed to have a better understanding of the connection between the retinal image and the neural response since the retina plays a key role as the last optical element in the eye. The modulation’s inability in showing the expected modification in coherent illumination for a human eye where the SCE I is modeled as a pupil apodization points to a more deeper fact in working, that is, the retina
prefers to respond to the more immediate stimulus of an interference pattern than to the pupil entry points of the entering beam hitherto governed traditionally by Stiles-Crawford effect of the first kind only. Recent experiments [37,54, 117] have also validated such line of arguments and findings. Moreover, apodization as an optical technique can be successfully used to compute retinal response regardless of the profile of the test objects employed. The significance of the work lies in the potentiality of coherence being used as an optical parameter in regulating retinal stimulation and understanding the spatial distribution of photon absorption in the photoreceptors of a human eye.

**Sine Wave Response**

Though for apodization parameters of $\sigma = 1.0, 2.0$ the modulation decrease monotonically with the increase of the spatial frequency in case of incoherent illumination there is almost no variation for coherent illumination.

But for a human eye with Stiles Crawford effect of the first kind a pupil apodization of $\sigma = 3.086$ a change of 35 % in the modulation is registered over the entire range of spatial frequencies for this value in incoherent illumination and 0.5% for coherent light. And with a large apodization of $\sigma = 30.0$ the Gaussian character is lost and the modulation even for incoherent illumination hovers around the constant value of unity.

**Square Wave Response**

Though the modulation, first, decreases with the increase of the spatial frequency, attaining a minimum contrast at a spatial frequency ($\omega$) of 18 cycles/deg and finally increasing sharply with $\omega$, the contrast changes only by 0.2 % throughout for coherent entering light. For incoherent light it is as high as 20 %. So the departure is 100-fold. Strikingly, with coherent illumination, the Stiles-Crawford effect is absent. But the Stiles-Crawford effect operates in response to incoherent illumination to accentuate low vs. high spatial frequencies.

The modulation’s inability in showing the expected modification in coherent illumination for a human eye where the SCE I is modeled as a pupil apodization points to a more deeper fact in working, that is, the retina prefers to respond to the more immediate stimulus of an interference pattern than to the pupil entry points of the entering beam hitherto governed
traditionally by Stiles-Crawford effect of the first kind only. The 2011 Vohsen Rativa’s experiment has also validated such line of arguments and findings.

**Triangular Wave Response**

Unlike a square wave object, a triangular periodic object’s Fourier series representation is free from Gibbs’ phenomenon, i.e., existence of discontinuities at the edges [282], we have reached some important results regarding the visual performance of a human eye. Moreover, the pupil as an optical system is not linear in intensity (for coherent illumination), hence frequencies cannot remain unaltered. So, the modulation of a triangular object cannot always be greater than that of a sine wave. As the higher spatial frequencies in a triangular test target are associated with gradually decreasing contrast, an attribute of a sine wave target we can always find out the suitable $\alpha = 0.5$ for which the triangular wave response mimics a sine wave response provided the lower harmonics are eliminated from the object.

Unlike other values of $\alpha$, for $\alpha = 0.5$, the variation of intensity is not abrupt but regular. With or without apodization there is no appreciable variation in the modulation for all values of $\alpha$, including $\alpha = 0$. Rather the modulation becoming reluctant in registering a large change unlike incoherent illumination points to a more deeper fact in working, that is, Stiles-Crawford effect of the first kind is absent in coherent illumination in line with recent experiments [37,54,117]. In future, coherence in the entering beam can be used as an optical tool to understand how the spatial distribution of photon absorption in the photoreceptors of a human eye actually works.

**Saw-Tooth Wave Response**

This striking behavior of near constancy of the modulation over the entire spatial frequency range for coherent entering light regardless of the values of the slope for a saw-tooth wave target shows that coherence of the entering beam has also a role to play in governing retinal light distributions in addition to the pupil entry point of the beam in traditional Stiles Crawford effect.

In toto, we can summarise the conclusion of this chapter as: For sine, square and saw-tooth wave gratings the modulation change by 0.2%, 0.5%, and 0.2% for coherent and 35%, 20% and 98% for incoherent over the entire range of spatial frequencies registering departures of 70-fold, 100-fold and 196-fold respectively. Thus with departure from perfect coherence the
retina’s response is compensative in nature as seen by a decreasing modulation in incoherent light for all gratings. This prepares the next round of investigation of looking into the effect of contrast on retinal response in the next chapter.