Chapter I

INTRODUCTION

is well known that Sun's radiation and solar activity have influence on the Earth's atmosphere, Ionosphere, Magnetsphere and space environment. Although activity is observed in many distant stars and galaxies, Sun is the only star whose activity can be seen in enough detail to guide us toward understanding of its mechanism. Indeed the Sun, the nearest star, is the "nearest astrophysical laboratory", where we can learn about fundamental process in astrophysics. Solar physics is essential to give us the necessary background to understand the working of our Universe (Belvedere & Lanzafame 1997). Because of this broad appeal, solar activity and solar cycle phenomena continue to inspire a vast amount of research worldwide.

It has been generally accepted that interactions of Sun's differential rotation and magnetic field play a basic role in the generation of solar activity and solar cycle, but the exact mechanism of solar activity cycle is not yet known. In this thesis I present studies of solar rotation, solar meridional flows and solar activity which give some interesting and potentially important clues for understanding the physical processes responsible for solar activity and solar cycle phenomena.

The main conclusions of this study are as follows: Legendre Fourier (LF) analysis of magnetic field inferred from sunspot group suggests that the solar activity may be created in the Sun by some kind of 'slow' MHD waves constituting 'global MHD oscillations'. Our study of periodicities in the rotation parameters, the depth dependence of the rotation of sunspot groups (and its temporal variations), and the temporal variations in the meridional motions of sunspot groups suggest existence of couplings between the spatial (radial, latitudinal and longitudinal) and temporal (few days to few decades) variations of Sun's magnetic field, rotation, and meridional flow. This supports the suggestion that the aforementioned oscillations are 'global MHD oscillations'. We find substantial variations in the differential rotation, whose periodicities and parity suggest that these oscillations are torsional in nature.

In the next section I give plan of the thesis. In the remaining sections of this chapter I give a brief description of solar activity, solar rotation and meridional motion and review the results of some of the recent studies.
1.1. Plan of the Thesis

In Chapter II, I present our study of global modes of the Sun’s MHD oscillations by LF analysis of solar magnetic field inferred from sunspot group data during 1874–1976.

In Chapter III, I present our study of periodicities in the solar differential rotation parameters using the 103 year sunspot data and the 26 year Mt. Wilson velocity data (Mt. Wilson values of coefficients A, B, C were kindly provided by Dr. R. F. Howard). The temporal variations in the differential rotation parameters represent Sun’s torsional oscillations of “even-parity” (symmetric about the solar equator). Fast Fourier Transform (FFT) and Maximum Entropy Method (MEM) are used to determine periodicities in the rotation parameters.

In Chapter IV, I present our study of periodicities in the north–south asymmetry of the differential rotation coefficients determined from the spot group data and the Mt. Wilson velocity data. The temporal variations in the north–south asymmetries of the differential rotation parameters represent ‘odd-parity’ torsional oscillations (i.e., they are anti-symmetric about the solar equator). We also comment on the matching of dominant periodicities of solar differential rotation with periods of configurations of dominant planets.

In Chapter V, we determine the dependence of the rotation of sunspot groups on their life span and age, and compare these dependences with the radial variation of the angular velocity of Sun’s rotation. From this we determine the depths of initial anchoring and the rate of radial rise of sunspot magnetic structures. In this chapter, we also study the relation between area and life span of sunspot groups, and the distributions of the number of spot groups (N) with respect to their life span. The importance of the area–life span relation in connection with anchoring depths and rising rates of sunspot magnetic structures is pointed out.

In Chapter VI, using the upgraded Greenwich spot group data during 1879–1975 we determine the following: (i) cycle-to-cycle modulations of the differential rotation parameters and (ii) variations of these parameters during the solar cycle. In the same chapter, we also study the periodicities in the differential rotation by determining it separately from well defined data samples of ’long lived’ young and old sunspot groups.

In Chapter VII, from the data on sunspot groups compiled during 1874–1981 we investigate the following: (i) dependence of the ‘initial’ meridional motion (v_m(t)) of sunspot groups on their life span (τ) in the range 2–12 days, (ii) dependence of the mean meridional motion (v(t)) of sunspot groups of life spans 10–12 days on their age (τ) and (iii) dependence of the mean meridional motion of spot groups, (with and without inclusion of second and subsequent appearances of recurrent spot groups), on the phase of the solar cycle.
In Chapter VIII, first I summarize the conclusions from Chapters II to VII and then provide some overall interpretations of the results in Chapters II-VII. More work will be necessary to acquire further insight into the real mechanism of solar cycle. For this purpose I list, in the end, important investigations I plan to undertake in near future.

1.2. Solar Activity

The term “solar activity” applies to a large variety of structures and phenomena in Sun's atmosphere created by interaction of Sun’s magnetic field with ionized gases on a variety of length scales and time scales. For example, sunspots, flares, active prominences, which occur in low latitudes and spicules, bright chromospheric emission along the supergranule boundaries, ephemeral bipolar regions, X-ray bright points, etc. which occur at any latitudes in Sun’s atmosphere.

Sunspots are the earliest observed solar phenomenon of solar activity. The earliest known records of sunspots observed by naked eye were found in Chinese documents that go back 2,000 years (Wittmann & Xu 1987; Li 1999). Sunspots are essentially small dark areas on the Sun’s visible surface (the photosphere). They appear within 35° solar latitudes on either side of the Sun’s equator. A sunspot may last from a few hours to several weeks depending on the maximum size it attains. The area of a sunspot is measured in a unit called a millionth of solar hemisphere - briefly, mh (≈ 3 x 10^6 km²). The typical sizes of sunspots ranges from ~ 10 mh to 10^3 mh. Those exceeding 25 mh generally have a dark core called an umbra occupying the central 15 to 20 per cent of the spot’s area. This is surrounded by a less dark region called the penumbra which contains hundreds of radially oriented, densely packed long, thin, dark and bright structures called penumbral filaments. Small dark regions resembling small sunspot umbra without a penumbra are also observed and are called pores. These have lifetimes of few hours to few days only.

Sunspots appear dark because they are only 10 to 20 per cent as bright as the surrounding photosphere. The gases in the umbra and the penumbra have temperatures of about 4000°-5000° K respectively while that of the normal photosphere is about 6000° K. It is believed that sunspots are ‘cool’ because of their strong magnetic fields, about 3000 G (with magnetic fluxes in the range of 10^{20}-10^{22} Mx), which are thousands of times stronger than the average field in the normal photosphere. Pores have field strengths of 2000-2500 G, with fluxes in the range of 10^{19}-10^{20} Mx.

Although individual sunspots are frequent, most sunspots occur in groups. Spot groups are often large and complex. Depending on their size and complexity spot groups can be classified into number of classes (see Bray & Loughhead 1964). Spots of each group are spread over a generally elongated area called the active region. Most active
regions are bipolar, with two main spots and either surrounding plages of opposite magnetic polarities or a single plage covering both. The main westward spot is called the leading spot and the main eastward spot is called follower spot. The line segment joining the two spots is tilted with respect to the east–west direction making a small angle with the local latitude (on the Sun) such that the leading spot is nearer to the equator. Careful examination of the sunspot data also reveals that the magnitude of this tilt with respect to the east–west direction increases with heliocentric latitudes, from 4° for sunspot pairs located near the equator to 10° at ~ 35° latitude (see also Gilman & Howard 1986; Wang & Sheely 1989). This result is usually known as Joy’s Law.

Sunspots are associated with a number of other remarkable solar activity phenomena, such as flares, prominences, filaments, plages etc. (detailed descriptions of solar activity phenomena are given in Zirin 1966; Gibson 1973; Priest 1982).

1.2.1. THE ELEVEN-YEAR SUNSPOT CYCLE

During the years 1826–1851 a German amateur astronomer, Heinrich Schwabe, noted that the yearly number of sunspots systematically decreased and increased alternatively during the periods 1826 to 1830, 1831 to 1837, 1838 to 1843, 1844 to 1848 and 1849 to 1851. He thus discovered “the ~ 11-year sunspot cycle” (which is commonly known as solar activity cycle or solar cycle). Soon after, Wolf (1852) defined the “relative sunspot number” and extended knowledge of its value more than a century into the past by assembling old records. Since then a careful tradition at Zurich has maintained the long term uniformity of this solar activity index (Waldmeier 1961). Detailed information on the solar activity cycle is obtained from a variety of sources. Sunspot observations provide many important details and are available for more than 20 sunspot cycles. In recent decades sunspot observations have been augmented with direct observations of the Sun’s magnetic field and we now have nearly continuous observations of the photospheric velocity field as well. These data sets are supplemented by synoptic observations of the Sun’s outer layers at wavelengths ranging from the radio to hard x-rays (Hathaway 1998), which include observation at important optical lines Ca II K and H-alpha. Figure 1.1(a) shows the presence of a cycle with an average period of about 11-yr in yearly sunspot numbers from 1870 to the present. The cycles vary in amplitudes, duration or period (cycle length) and shape. The length of a sunspot cycle varies from 9 to 14 years. Most cycles are asymmetric with a rapid rise in numbers from sunspot minimum and a slower decline in numbers from sunspot maximum. Larger cycles tend to rise faster and reach maximum in a shorter time than the smaller cycles (Waldmeier effect).

Sunspot positions reveal additional details about the activity cycle. Such measurements have been routinely obtained at many observatories including Greenwich, Mt.
Wilson, Kodaikanal, and the USAF/NOAA SOON network (cf., http://wwwssl.msfcc.nasa.gov/ssl/pad/solar/greenwch.htm). Before the sunspot number of any cycle reaching minimum, sunspots of the new cycle appear at latitude 35° on either side of the equator, and then onwards the mean latitude of sunspots activity systematically migrates towards the equator (Figure 1.1(b), 'Butterfly Diagram'). The equatorward migration of the active latitude bands and the overlaps of cycles near minima are both important features of the activity cycle. The solar activity also shows long-term variation, such as a modulation of about 90 years, called 'Gliesberg cycle'. A large number of minor periodicities in sunspot activity have also been reported (Wolf 1976; Carbonell & Ballester 1992). Many of the periodicities in solar activity happens to coincide with periodicities in planetary configurations (e.g., review by Seymure et al. 1992). Existence of a small north–south asymmetry in the activity is also known. Some authors also reported existence of ~ 11-year periodicity and a few other periodicities in the north–south asymmetry (see e.g., Carbonell et al. 1993 and references therein).

Figure 1.1(a). Yearly sunspot numbers from 1870 to the present (cf., http://www.astro.cma.be/SIDC/DATA/yearssn.dat).

Figure 1.1(b). A Butterfly diagram showing the latitudinal distribution of area of sunspots from 1870 to the present (cf., http://wwwssl.msfcc.nasa.gov/ssl/pad/solar/images/bfly.gif).

Periods of suppressed activity like the *Maunder minimum* from 1645 to 1715 and the *Spörer minimum* in the 15th century as well as periods of increased activity such as *current Modern Maximum* and *Medieval Maximum* in the 12th century are known to
have occurred. As deduced combining, where available, sunspot and aurora occurrence reports, paleoclimatic and paleomagnetic data and $^{14}C$ concentration in tree ring, there probably existed ten similar solar activity minima and eight maxima in total since 5300 BC, with significant impact on Earth’s climate and geomagnetic activity (Maunnder 1922; Eddy 1976, 1978; Ding Youji 1978; see also reviews by Rosner & Weiss 1992, Nesme-Ribes et al. 1996; Polygiannakis et al. 1996). However, even during the Maunder Minimum there are indications that the 11-yr cycle was still at work (Ribes & Nesme-Ribes 1994; Merzlyakov 1997 and references therein).

Several different approaches have been made for understanding, (and predicting, if possible), the short and long-term evolution of the sunspot cycle, through spectral, statistical and morphological studies of sunspot activity (see Polygiannakis et al. 1996, Li 1997 and references therein).

There are several other measures that can be used for studying solar cycle: for example the 10.7-cm radio flux, which comes primarily from the higher levels in solar atmosphere; Mount Wilson magnetic plage strength index (MPSI), Kitt Peak magnetic index (KPMI); and UVI, a measurement of the solar UV variability as determined by the Mg II 280 nm cor-to-wing ratio (also visit http://www.sunspotcycle.com).

1.2.2. SOLAR MAGNETIC CYCLE

Photospheric magnetic field measurements reveal the magnetic nature of sunspots and the solar activity cycle. During each sunspot cycle, the magnetic polarity of the leading spots in the north hemisphere is the same as that of the weak field near the Sun’s north pole at the beginning of that cycle. A similar rule holds good in the southern hemisphere, wherein polarities are opposite to those in the northern hemisphere. The weak fields in the polar regions themselves reverse their polarities within one or two years after the year of maximum sunspots. Polarity orientations of spot groups in each hemisphere reverse from one sunspot cycle to the next, which was first demonstrated by Hale and his collaborators (Hale et al. 1919). This is known as Hale’s Polarity Law. Thus, the Sun’s magnetic properties constitute a 22-yr cycle (consisting of two consecutive sunspot cycles) called the ‘solar magnetic cycle’.

1.2.3. THE EXTENDED SOLAR CYCLE

The ‘butterfly diagrams’ show overlap of the old and the new sunspot cycles, near sunspot minima. This has been well known for several years. In view of this and from analyses of coronal green line emission, ephemeral active regions and torsional oscillation signal, Wilson et al. (1988) concluded that sunspot activity is simply the main phase of what they call ‘an extended-cycle’ (also see Altrock 1997). This ‘extended-cycle’ begins at high latitudes before the maximum of a sunspot cycle and progress
towards the equator during the next 18-22 year. However, according to Stenflo (1992) the concept of an extended cycle is superfluous and leading to confusion.

1.2.4. THEORY

There are two main approaches for explaining the mechanism of solar cycle, viz., one is based on a turbulent dynamo operating in or immediately below the solar convective envelope and the other is on large scale oscillation superposed on a fossil magnetic field in the radiative core. The dynamo theory has been very well studied by many authors (Parker 1979a, 1993; Roberts 1972; Cowling 1981; Stix 1981, 1991; Dikpati & Charbonneau 1999; see also reviews by Deluca & Gilman 1991; Rosner & Weiss 1992) and the models based on this theory agree with many observed features of solar magnetic activity. With regard to the hydromagnetic oscillator, clear and complete mechanism explaining the large-scale oscillations of the magnetic field has not been found (Piddington 1976; Layzer et al. 1979; Dicke 1979; also review by Rosner & Weiss 1992).

**Turbulent Dynamo:** Basic idea of the turbulent dynamo theory is that the solar magnetic fields are generated and maintained by complicated nonlinear interactions between the solar plasma and magnetic fields. For a complete solution to this highly nonlinear dynamo problem, it is necessary to solve the full magnetohydrodynamic equations and demonstrate that: (i) there is a velocity field \(v\) which can maintain an oscillating magnetic field \(B\); and (ii) this velocity field is itself maintained by the available forces, such as those exerted by convective turbulence and Lorentz force. These two steps involve solving the induction equation and the equation of motion, respectively, under suitable assumptions. Solving both equations simultaneously is extremely difficult. Most work was restricted to the first step alone, which is referred to as the *kinetic dynamo problem* (see Priest 1982).

With a turbulent kinetic dynamo the global properties of the solar magnetic field can be reasonably well described. Such a dynamo consists of two basic elements: (a) differential rotation, which produces a toroidal field (east-west component) by continuously winding up a poloidal field (\(\omega\) effect), and (b) the ‘\(\alpha\)-effect’ which is induction effect of cyclonic turbulence that regenerates the poloidal (north-south component) field component, and (c) enhancement of diffusion by turbulence. The ‘\(\alpha\)-effect’ effect is crucial in turbulent dynamo models.

Though the models based on turbulent dynamo theory agree with many observed features of the solar cycle, a sufficiently detailed and realistic model of the dynamo process to account for all the different aspects of solar magnetism is not yet available. The available turbulent dynamo models have several difficulties: for example, in such models the role of the differential rotation in the cyclic variation of the solar activity is
not clear (e.g., Gilman 1992). The turbulent dynamo theory requires angular velocity increasing radially inward (e.g., Stix 1981; Parker 1987) whereas helioseismical data show that the angular velocity in the equatorial latitudes is either constant or slightly decreases radially inward throughout the convection zone except in a thin layer at the top (e.g., Brown et al. 1989; Libbrecht 1989; Dziembowski et al. 1989; Goode et al. 1991; Tomezyk et al. 1995; Antia & Chitre 1996; Dalsgaard & Thompson 1999, see also Section 1.3.5). This theory is developed by averaging over turbulent fluctuations. The 'first order smoothing approximation' used in the models of Mean-Field Electrodynamics may not valid on the Sun (for more detail see Priest 1982; Rosner & Weiss 1992).

Oscillator Models: The basic idea of magnetic oscillator models is to consider the observed oscillating large-scale solar poloidal and toroidal field as effects of periodic amplification of primordial fields due to oscillations in the differential rotation rate of the solar interior. The main difficulty in the oscillator models is regarding energetics. No oscillator model offers means of maintaining the oscillations against dissipation of velocity and magnetic fields. There is no observational evidence for the ambient field about which oscillations are claimed to occur or for variations in angular velocity with a 22-year period. Many solar physicists suspect that the Sun is too old and has had too much convection to have fossil field in its radiative core (see review by Rosner & Weiss 1992).

1.3. Sun's Rotation

Study of solar rotation rate and its variation with latitude (the differential rotation) and time is extremely important for understanding the Sun's internal dynamics and solar magnetic activity cycle. The study of solar rotation dates back to the first telescope observation of sunspots and solar rotation is the most studied of solar motion fields. However, the dynamics that produces the differential rotation are still not understood. The well known reviews on theoretical background of solar rotation are written by Gilman (1974, 1976) and Dicke (1970) and discussed in detail the theories of the differential rotation, various fluid dynamical models.

Solar rotation has been investigated by a variety of techniques which fall basically in three categories: (i) Doppler-shift measurements of particular spectral lines, (ii) tracking of tracers (like sunspots, sunspot groups, faculae, small magnetic elements, plages, filaments, etc.), and (iii) helioseismical measurement of the Sun's internal rotation. Methods (i) and (ii) are less accurate than the method (iii). But methods (i) and (ii) enable us to study Sun's rotation in the past.

Doppler and tracers measurements agree within a 5% accuracy level, but they do not at 1% level. At this level of accuracy the measured rotation rate depends not only
pon whether Doppler or tracer methods are applied but also upon the choice of line in Doppler method) and what types of tracers and what characteristic properties (size, ge, etc.) of tracers are used. Obviously, the Doppler measurements yield rotation of surface layers. Because of magnetic origin of all solar features their rotation is more likely to represent rotation of deeper layers where the magnetic fields of the features anchored (e.g., Foukal 1972; Schüssler 1987). However, the reason for the discrepancy in the results derived from tracers and from Doppler measurements, and also in the results obtained from different tracers, is not yet clear (e.g., D'Silva & Howard 1994).

There are several other significant pitfalls in both Doppler and tracer methods (e.g., Schröter 1985).

There are several good reviews in the literature on the topic of solar rotation. (e.g., Jilman 1974, 1980; Howard 1978, 1984, 1996a; Paternó 1978; Schröter & Wöhl 1978; Schröter 1985, Bogart 1987; Howard et al. 1991, Libbrecht & Morrow 1991; Snodgrass 1992) In the following subsections, I present only a very brief review of studies of solar rotation.

1.3.1. PHOTOSPHERIC DOPPLER VELOCITY MEASUREMENTS

Several observatories have synoptic programs, where full disk velocity measurements are performed daily: Mt. Wilson, Stanford, Kitt Peak, Crimea, etc. Synoptic programs provide consistent and high accuracy measurements over an extended period of time. For example, Mt. Wilson rotation measurements cover more than three 11-yr cycles from 1967 to the present. Doppler velocity measurements are now routinely obtained by several instruments including those of GONG and those on the SOHO satellite. These observations provide spatial and temporal coverage of the photospheric flows (Hathaway et al. 1996; Hathaway 1996, 1998). Limitations and sources of systematic errors in Doppler measurements are discussed in detail by Schröter (1985).

Livingston (1969) was the first who published comprehensive measurement of solar rotation taken at the Kitt Peak Observatory, covering the period 1966-1968. He obtained the value 13°.74 day⁻¹ for the equatorial rotation rate. Subsequently, a number of authors published the values of the rotation rate and the differential rotation rate determined from modern Doppler method (see Table I in the review by Schröter (1985)). The practice of making polynomial fits to the full-disk maps of Doppler data in daily Mount Wilson Magnetograms was started by Howard & Harvey (1970). They fitted the Mt. Wilson data from 350 magnetograms (or Dopplergrams) obtained during 1966-1968 to the following form: \[ \omega(\lambda) = A + B \sin^2 \lambda + C \sin^4 \lambda, \]
where \( \omega(\lambda) \) is the solar rotation at latitude \( \lambda \), the parameter \( A \) represents the equatorial rotation rate, \( B \) and \( C \) measure the latitude gradient of the rotation rate with \( B \) representing mainly low latitudes and \( C \) representing largely higher latitudes. They obtained \( A = 13.76, B = -1.74 \) and \( C = -2.19 \). The units are degrees (°) day⁻¹ sidereal.
1.3.2. ROTATION MEASUREMENTS FROM MAGNETIC STRUCTURES AS TRACERS

Determination of solar rotation using magnetic structures as tracers can be done by tracking relatively stable and long-lived individual magnetic structures, such as sunspots, faculae, plages, which are relatively scarce. When the features are sufficiently plentiful, for example, the small magnetic features in a magnetogram, the rotation can be determined by correlation methods (Wilcox & Howard 1970; Wilcox et al. 1970; Stenflo 1974, 1977, 1989; Sheely et al. 1992; Snodgrass 1983, 1991; Komm et al. 1993a). Sunspots are relatively good tracers of rotation and other motions at the surface in the sense that they are small, relatively well defined and unchanging, and often live for several days or more. Sometime sunspots live long enough to cross the central meridian more than once. An important drawback is that sunspots are confined to low latitude belts.

Heliographic coordinates are an obvious choice for determining accurately the Sun’s rotation and meridional motion (north–south motion of solar plasma) by tracer method (Schröter 1985). The accuracy in determination of heliographic coordinates of mass center of a sunspot group is \( \sim 0.5^\circ \) and therefore error in the calculations of daily velocity values is about \( 1^\circ \text{ day}^{-1} \) \( (1.4 \times 10^4 \text{ cm s}^{-1}) \) (Balthasar & Wöhl 1980; Zappalá & Zuccarello 1991; Paternó et al. 1991, Zuccarello 1993). However, error in determination of the mean velocity is inversely proportional to the square root of the number of observations (N) used, it is \( 0.07^\circ \text{ day}^{-1} \) \( (9.9 \times 10^2 \text{ cm s}^{-1}) \) for \( N = 200 \).

Carrington (1863) showed, using sunspots as tracers for the first time, that sun has a differential rotation, i.e., the rotation is fastest at the equator and decreasing gradually towards the pole, in both the northern and southern hemispheres. Of all the tracers, sunspots are the most extensively used in the studies of the solar rotation and the solar meridional motion.

Greenwich Photoheliographic Results (GPR) compiled during 1874–1976 give a long data base for the studies of solar activity and the solar cycle, the solar rotation and other properties of motions in solar convection zone. This data has been extensively used by several authors for a long time to determine the solar rotation and the differential rotation (e.g., Newton & Nunn 1951; Ward 1965a; 1966; Godoli & Mazzucconi 1979, Balthasar & Wöhl 1980; Arévalo et al. 1982; Lustig & Dvorak 1984; Balthasar et al. 1986; Tuominen & Virtanen 1987). The other large data sets are Mount Wilson sunspot data set covering the time period 1917–1985 (Howard et al. 1984) and Kodaikanal data set covering the time period 1906–1987 (Sivaraman et al. 1993; Gupta 1994. Howard et al. 1999). Measurements of these data were made using the same technique. Some other data bases are: sunspot drawings of Kanzelhöhe Observatory, data of digitized Ca II K spectroheliograms recorded at Meudon Observatory (Ribes et al. 1985), Solar patrol data of the Catania Observatory (e.g., Ternullo et al. 1981) and sunspot draw-
ings obtained at the National Astronomical Observatory of Japan during 1954–1976 (Kambry & Nishikawa 1990; Yoshimura & Kambry 1993). A distinct advantage of the Mount Wilson and Kodaikanal data sets is that besides positions of spot groups, individual sunspot positions and areas have also been measured. Hence, the rotation and other motions of individual spots, besides spot groups, are also studied from the Mount Wilson and Kodaikanal data sets. The studies based on Meudon Observatory refer to rotation of sunspots. Studies based on Greenwich data generally refer to rotation of sunspot groups.

Newton & Nunn (1951) using the data on long-lived and recurrent sunspots during the years 1978–1944 derived a historic law of the differential rotation,

$$\omega(\lambda) = A + B \sin^2 \lambda,$$

where the equatorial rotation rate $A = 14.368 \pm 0.004$ and the latitude gradient of the rotation rate $B = -2.69 \pm 0.04$ (see Schröter 1985). The units are degrees (°) day$^{-1}$ sidereal and $\lambda$ is heliographic latitude.

In general, most of the previous authors in their statistical studies had only considered recurrent phenomena, whereas Ward (1964, 1965a, b, 1966) also included the young and short-lived non-recurrent sunspots. He found a slight higher equatorial rotation rate and somewhat steeper $\omega(\lambda)$–profile than those found by previous authors. From his studies it also emerged, for the first time, that the solar rotation rate determined from sunspots depends on certain characteristics of the sunspots used. Since then, several attempts have been made by a number of authors to derive the rotation rate accurately using large data sets and to study the rate of rotation of sunspots by classifying the spots according to their classifications, such as, single, bipolar, follower, leader, complex structures, and characteristics, such as, area, life span, age (young and old spots or spot groups), angles of inclinations, etc (see Howard 1984, 1996a; Schröter 1985). These studies yielded several vital clues for the understanding of the dynamics of flows in the solar convection zone and the cyclic activity of the solar magnetic field.

Figure 1.2 (adopted from Snodgrass 1992), shows time-averaged rotation profiles determined using various photospheric indicators that are available. The rotation rate from Doppler measurements is significantly lower than the magnetic and sunspot rates. The slowest rate is coming from the photospheric plages (see also, Snodgrass 1992; Howard 1984; Schröter 1985). It is difficult to establish rotation as a function of height or depth, since it is difficult to establish accurately the height or depth of tracers (Schröter 1985).
1.3.3. NORTH–SOUTH ASYMMETRY IN THE ROTATION OF OUTER LAYERS

Existence of north–south asymmetry in solar activity is shown by several statistical studies for most of the activity phenomena (see Carbonell et al. 1993 and references therein). Small hemispheric difference in the solar rotation is also known (Howard & Harvey 1970; Schröter et al. 1978; Godoli & Mazzucconi 1979; Arévalo et al. 1982; Lustig 1983; Howard et al. 1984; Howard & Gilman 1986; Balthasar et al. 1986; Hathaway & Wilson 1990). Antonucci et al. (1990) performed a Fourier analysis of large-scale photospheric features on magnetograms, in different latitude bands, and detected a strong north–south asymmetry. Meunier et al. (1997) analyzed photospheric faculae data over cycle 19 and found existence of a strong north–south asymmetry in the mean rotation rate of faculae. Recently, existence of slight north–south asymmetry in the differential rotation rate is shown also by time-distance helioseismic measurements (Duvall et al. 1998; Giles & Duvall 1998). Hathaway & Wilson (1990) analyzed rotation rate of sunspots from Mt. Wilson sunspot data for individual sunspots during 1923–1975 and found that the southern hemisphere, having fewer spots, rotates faster (by $0.016 \pm 0.004^\circ \text{ day}^{-1}$) than the northern hemisphere.
1.3.4. TIME DEPENDENCE OF THE ROTATION OF OUTER LAYERS

Since differential rotation plays an important role in producing solar activity, the study of temporal variations in solar rotation might provide vital clues for the mechanism of solar cycle. Many attempts to study the cycle dependence of solar rotation have been made (see Howard 1984 and Schröter 1985 for reviews). The most extensive studies are based upon long time series of sunspots and sunspot groups covering several solar cycles. An 11-year pattern of changes in rotation related to the activity cycle is shown by a number of authors using sunspot data. An increase in the equatorial rotation rate is found at solar minimum (Tuominen & Kyrolaninen 1982; Arvalo et al. 1982; Lustig 1983; Gilman & Howard 1984; Balthasar et al. 1986; Kambry & Nishikawa 1990). The discovery of the so-called 'torsional oscillations' by Howard & LaBonte (1980) and LaBonte & Howard (1982a) from the analysis of Mt. Wilson Doppler measurements during 1967-1982, stimulated further study of the spatial structure and temporal variation of the differential rotation. The observed 'torsional oscillations' consist of alternating bands of rotation faster (or slower) than average, and moving in each hemisphere, from high latitudes towards the equator in ~ 22 years (see Figure 1.3). In a given latitude the velocity of torsional oscillation changes its direction from east to west and vice versa during 11 years with amplitude of about 3 m s⁻¹. It has also been suggested that the Sun's rotation may be different during the extended minimum periods, such as Maunder minimum (Ribes et al. 1987). However, there is, at present time, no uncontroversial evidence for long-term, or secular, changes in the solar rotation rate (Howard et al. 1991).

Howard & LaBonte (1980) and LaBonte & Howard (1982a) argued in favor of rotation combined with other convective phenomena driving the dynamo and that the observed torsional oscillations pattern is a direct manifestation of this causal connection. Theoretical models attempt to explain the torsional oscillations pattern by linking it to the equatorward propagation of dynamo waves associated with the activity cycle. Yoshimura (1981) and Schüssler (1981) calculated a pattern by considering the Lorentz force back-reaction of bands of toroidal magnetic field on the surface plasma. Kliorin & Ruzmaikin (1984) show that the Lorentz force from a subsurface dynamo wave could produce the pattern and refute arguments against this model made by LaBonte & Howard (1982a). Rüdiger & Kichatinov (1990) consider the influence of large-scale magnetic fields on Reynolds and turbulent stresses, and find that Lorentz stress is dominated by the microscale feedback due to Reynolds stresses. In this model, the torsional oscillation is a real oscillation associated with the changing viscosity tensor in the presence of the azimuthal magnetic field of the dynamo wave. The authors do not discuss how the pattern produced in this fashion matches the observed torsional pattern. Wilson (1987) and Snodgrass & Wilson (1987) suggest, based on observations, that the torsional pattern is the surface signature of the link between the polar fields.
Figure 1.3. Contour plot of the zonal-excess rotation velocity. The zonal excess is the zonal rotation velocity minus a smooth curve fitted to each of 34 independent zones of equal width in sine latitude. The daily values have been averaged over independent intervals of four Carrington rotations (109.1012 days) to reduce the random solar noise level. Contour levels are 1.5, 3, and 6 m s\(^{-1}\). Solid contours represent regions of faster rotation (westward flow): dashed contours, slower rotation (eastward motion). Regions of faster rotation have been shaded to render the latitude drift of the velocity bands obvious. The heavy dashed lines mark the latitudes of the maxima of the magnetic flux distribution, observed simultaneously with the velocity data (adopted from Howard & LaBonte 1980).
and the active-region fields. In this model, the zone of enhanced shear is produced by the Coriolis force due to a downflow either from an azimuthal convective pattern or from convergence of meridional flows. A more detailed discussion of this model is presented in Snodgrass (1992). Gilman (1992) proposes that the torsional oscillations are a secondary flow arising from the thermal and mechanical disturbance caused by solar activity and the subsurface toroidal field at certain latitudes.

1.3.5. HELIOSISMIC MEASUREMENT OF THE SUN'S INTERNAL ROTATION

Heliosismology is a branch of solar physics, in which one uses the knowledge of oscillations of the Sun for probing the Sun's internal structure. Since last two decades this powerful technique has been used to study radial gradient of plasma rotation and other internal properties of the Sun through the study of the solar acoustic modes. The observed frequencies of the solar acoustic (p) modes are used for determining the internal rotation. The observed oscillations are called five-minute oscillations because they have periods in the vicinity of five minutes. The modes of the observed oscillations are distinguished not only by their different frequencies, but also by their different patterns on the surface of the Sun. The p-modes are identified by the radial order \( n \) and the spherical harmonic degree \( l \) and azimuthal order \( m \), with \( 2l + 1 \) possible \( m \) values, from \(-l\) to \(+l\). \( |m| \) indicates the number of node circles crossing a line of latitude, while \( l - |m| \) gives the number of node circles crossing a line of longitude. Modes of the same \( l \) and \( m \) but different \( n \) have different frequencies, with the spacing between modes of adjacent \( n \) being related to the inverse of sound speed. In the absence of rotation the frequencies of the modes of the same \( n \) and \( l \) are independent of \( m \), owing to absence of any preferred axis. However, rotation breaks this symmetry and remove the degeneracy of the frequencies. Thus the splitting due to the rotation as given by observations is

\[
\delta \nu_{nml} = \nu_{nml} - \nu_{nl} ,
\]

where \( \nu_{nml} \) is the frequency of an individual mode and \( \nu_{nl} \) is the central frequency of the multiplet. For the sake of convenience, the angular frequency \( \omega_{nml} = 2\pi \nu_{nml} \) can be used instead of \( \nu_{nml} \). The quantity \( L = \sqrt{l(l + 1)} \) is also commonly used, and \( \nu_{nml}/L \) is monotonically related to the lower turning point radius of the \((n, l, m)\) mode. The observed splittings related to the internal rotation in the following way (e.g., Di Mauro et al. 1998)

\[
\delta \omega_{nml} = \int_0^{R_\odot} \int_0^{\pi} K_{nml}(r, \theta) \Omega(r, \theta) rdrd\theta ,
\]

where \( r \) is the radial distance from the center and \( \theta \) is the colatitude. \( K_{nml}(r, \theta) \) are the kernels functions which depend on the equilibrium model quantities and oscillation eigen functions. Thus by knowing observationally determined rotational frequency
splittings and theoretical eigen functions, the internal rotation can be determined. The dependence of splittings on angular velocity can be used in a 2-dimensional inverse problem to probe the Sun’s internal differential rotation.

Figure 1.4. A contour diagram of the solar rotation rate as obtained by the 1.5D inversion technique using GONG months 4–14 data. Due to the symmetry of the inversion results, the rotation rate has been shown for just one quadrant only. Contours are drawn at intervals of 5 nHz. The highest level near the equator around $r = 0.9 - 0.95$ is 465 nHz. The red lines are at interval of 5 nHz and blue ones at 20 nHz. The $x$-axis represents the solar equator while the $y$-axis represents the rotation axis (courtesy H. M. Antia).

A typical internal rotation profile inferred from heliosismological data is given in Figure 1.4 (kindly provided by H. M. Antia). Note that, the differential rotation exist down to base of the convection zone. The rotation is not constant on cylinders aligned with the rotation axis, as was predicted by simulations in the early 1980s. There is a transition to latitudinally independent rotation near the base of the convection zone, in a layer which has become known as the tachocline (see Kosovichev 1996, Hill 1998). This layer is of interest as it is now thought that the dynamo is located there. Recently, F. Hill reviewed solar cycle dependence of all areas of global helioseismology which
oscillations that provide longitudinally-averaged information (Hill 1998).

Some inversions of the odd splitting coefficients appear to show an equatorial rotation rate that does evolve as the cycle phase changes (e.g. Goode & Dziembowski 1991; Woodard & Libbrecht 1993b). Other analyses either conclude that there is no change (e.g. Woodard & Libbrecht 1993a, Schou 1990; Antia et al. 1996) or that the evidence is weak (Gough & Stark 1993b). Recently, using GONG and SOHO data over 4.5 yr time span, Howe et al. (2000a) have detected changes in the rotation near the base of the convective envelope, including a prominent variation with a period of 1.3 yr at low latitudes. Using splitting measurements of the f mode from SOI data, Kosovichev & Schou (1997) and Schou (1998, 1999) have inferred the latitudinal variation of angular velocity. They find some agreement between the features seen in the f-mode inversion and those seen in a measurement of the surface torsional oscillation pattern from GONG data (Hathaway et al. 1996). Using the GONG and SOHO data, Howe et al. (2000b) have shown that the torsional oscillation pattern extend downward at least 60 Mm.

Local helioseismology, in which sections of the solar surface are studied, now provides information on the cellular patterns below the surface (e.g. Hill 1990, Duvall et al. 1997). While supergranulation is readily observed, giant cell convective motion still remain unresolved with these techniques (Hathaway 1998).

1.4. Meridional Circulation

Models of solar activity cycle and solar differential rotation suggest that there might exist solar meridional motion and might play an important role in generation of solar activity cycle and in the maintenance of solar differential rotation (see Schröter 1985). Some of the observed properties of solar activity also suggest existence of meridional motion: for example, the equatorward shift of average activity latitude bin and the observed poleward migration of unipolar magnetic field regions and polar filaments bands, over the solar cycle (Howard et al. 1991). Tracking of tracers (like sunspots, sunspot groups, faculae, plages, prominences, small magnetic features) and direct Doppler measurements are the two principle methods used to measure the meridional flow. Because the magnitude of the meridional flow is at least two order smaller than the rotation speed, it is relatively more difficult to measure (Howard 1996a). Though the existence of meridional flow is confirmed observationally, its magnitude and direction is not clear. Meridional motions derived by using sunspots as tracers generally lead to smaller amplitudes of a few ms$^{-1}$ (see, for example, Balthasar et al. 1986; Howard & Gilman 1986; Lustig & Wöhl 1991), whereas a majority of Doppler measurements suggest a poleward flow of 10-20 ms$^{-1}$ (for example, Duvall 1979; LaBonte & Howard 1982b; Ulrich et al. 1988; Hathaway 1996).