Chapter 2

Reflection effect in close binaries

2.1 Introduction

Most stars are found in groups that are gravitationally bound to each other. The majority of these stars are found in binary systems which are systems of two stars in orbit around a common center of mass. Binaries are useful systems for astronomers because two stars in orbit obey well understood laws of motion. From their orbital velocities and periods, it's possible to calculate the combined and individual masses of the stars in a system. When the stars are plotted by their brightness and their spectral type on Hertzsprung-Russell diagram, most stars falls on to a narrow band called main sequence. But the stars that compose close binary systems are not the same luminosity as star of similar type, so they fall slightly off the main sequence-the primary stars tend to appear below it, while the secondary above it.

Kopal (1959) first classified all close binaries into three groups - detached, semi-detached, and contact systems. In detached systems both star remain well within their respective Roche lobes (β -Aurigae). In semi-detached systems however, one component fills its Roche lobes (Algol, β Persi). Contact binaries are still exotic - both components fill their Roche lobes and continually interact (44-Bootis B).
One can think of a quite a number of binary star configurations in which relatively simple numerical treatment of reflection will be adequate. Most obvious is that of slowly rotating stars, which are well detached from their limiting lobes and are therefore, not far from spherical. Systems of main sequence stars with radii of the order of 10%-15% of their separation fall into this category and or reasonably common (for stars which are smaller than this there is, in most cases, hardly any reflection effect).

In a close binary system each component will receive the radiation from its companion. If a star is to remain in radiative equilibrium, all the energy received from outside must be re-emitted, without altering the rate of escape of radiation from the deep interior. This phenomenon, known as the "reflection effect" is inevitable in close binaries. To explain this effect properly further study of the problem is necessary to answer - or at least to give some clues to - several fundamental parameters such as mass and radii. In radiative transfer theory, we solve the equation of radiative transfer by assuming certain geometrical configuration such as plane parallel or spherical symmetric stratification of the media. These geometrical configurations assume symmetric boundary conditions and whenever we have asymmetric incident radiation, the solutions developed in the context of symmetrical geometries as mentioned above, will have to be modified. Such problems are encountered in the evaluation of radiation from the irradiated component of the binary system. If we treat one of the component as a point source in a binary system then the problem of incident radiation from such source is equivalent to the searchlight problem. Chandrasekhar (1958) has calculated the diffuse scattering function in a plane parallel medium when a pencil of beam of radiation from a point source is incident. There have been several
attempts for calculating the diffuse radiation field in such simple geometries. However, the calculation of the radiation field during the eclipses in close binaries is of different complexity. There are two important aspects one should take into account: (1) the physical processes that take place inside the atmosphere and (2) the geometrical shape of the illuminated surface which reflects the light. Generally, if the atmosphere of the component under consideration is extended or fills its Roche lobe, then the problem of determining the emergent radiation from such surfaces become very difficult. The process of estimating the radiation field from such surfaces become complicated when the various competing physical processes are taken into account. Geometrical considerations alone would complicate the calculations because of the deformed shape due to tidal effects from the neighbour and due to self radiation. The resultant shape would be an ellipsoid and the problem requires special treatment. The solution of radiative transfer equation either in plane parallel symmetry or in spherical symmetry or in cylindrical symmetry cannot accurately describe the radiation field emanating from such surfaces.

Now in this chapter we will describe a method for estimating the reflected radiation from a primary component in a binary system.

2.2 Distribution of radiation incident from a point source

We shall assume that the components are spherical (the method can be extended for non spherical shapes). We have assumed the albedo of single scattering to be unity, i.e. purely scattering medium. In figure 2.1, let O is the centre of the primary component whose atmosphere is divided in spherical shells. Let S be a point source outside the star (secondary
component) on axis OX and the OY is perpendicular to OX. We shall assume that the radiation is coming from point source S is incident on primary component at points such as P₁, P₂, etc. These rays travel through the medium intersecting the shells at given radial points. We choose a radius vector corresponding to a given \( \theta \) (the colatitude) and calculate the source functions where this radius vector meets the shell boundaries at points such as \( Q₁, Q₂, Q₃, \ldots \), etc. The calculation of source function is done by employing the 'Rod' model of one-dimensional radiative transfer (Sobolev 1963, Wing 1962) along the ray path inside the medium. This means we calculate the source functions at points \( Q₁, Q₂, Q₃, \ldots \), etc.

### 2.2.1 Description of rod model

We shall assume a steady state, monochromatic ray with or without internal sources (see figure 2.2). The optical depth is calculated using the relation

\[
\tau = \tau(\xi) = \int_0^l \sigma(\xi') \, d\tau, \quad \tau(1) = T. \tag{2.1}
\]

The transfer of radiation is assumed to take place along the ray paths \( P₁Q₁, P₂Q₂, P₃Q₃, \ldots \) (in figure 2.1) or along \( 0(\) (in figure 2.2) with isotropic scattering \( \mu = \pm 1 \) and \( p(\tau) = \) phase matrix elements). The source function that includes diffuse radiation can be written as (see Peraiah 1982).

\[
S^+_d(\tau) = S^+(\tau) + \omega(\tau) \left[ p(\tau) I₁e^{-\tau} + (1 - p(\tau)) I₂e^{-(T-\tau)} \right]. \tag{2.2}
\]

and

\[
S^-_d(\tau) = S^-(\tau) + \omega(\tau) \left[ (1 - p(\tau)) I₁e^{-\tau} + p(\tau) I₂e^{-(T-\tau)} \right]. \tag{2.3}
\]
Figure 2.1: Schematic model diagram showing how the radiation field is calculated in the atmosphere of the irradiated component illuminated by a point source

\[ T \quad \longleftrightarrow \quad I^\tau (\tau) \]

\[ 0 \quad \longleftrightarrow \quad I (\tau) \]

\[ T = \text{Total optical depth} \]
\[ 1 = \text{Total geometrical depth} \]

Figure 2.2: Schematic diagram of rod model
where
\[ S^+(\tau) = \omega(\tau) \left[ p(\tau) I^+(\tau) + (1 - p(\tau)) I^- (\tau) \right], \] \[ S^- (\tau) = \omega(\tau) \left[ (1 - p(\tau)) I^+(\tau) + p(\tau) I^- (\tau) \right], \]
where \( \omega(\tau) \) is the albedo for single scattering which is equal to unity in a pure scattering medium, \( p(\tau) \) is the phase matrix (here it is equal to \( \frac{1}{2} \)) and the specific intensities (see figure 2.2) \( I^+(\tau) \) and \( I^- (\tau) \) are given by the differential equations.
\[
\frac{dI^+}{d\tau} + I^+ = S^+, \quad (2.6)
\]
\[
\frac{dI^-}{d\tau} + I^- = S^- . \quad (2.7)
\]
The boundary condition at \( \tau = 0 \) and \( \tau = T \) are given by
\[ I^+(0) = I_1, \quad (2.8) \]
and
\[ I^-(T) = I_2. \quad (2.9) \]
We shall specify \( I_1 \) later and set \( I_2 = 0 \). From equations (2.6) and (2.7) the solution can be obtained and is given by,
\[ I^+(\tau) = I_1 \frac{1 + (T - \tau)(1 - p)}{1 + T(1 - p)}, \quad (2.10) \]
and
\[ I^- (\tau) = I_1 \frac{(T - \tau)(1 - p)}{1 + T(1 - p)}. \quad (2.11) \]
From equations (2.10) and (2.11) we obtain,
\[ I^+(\tau = T) = I_1 \frac{1}{1 + T(1 - p)}, \quad (2.12) \]
\[ I^-(\tau = 0) = I_1 \frac{T(1 - p)}{1 + T(1 - p)} . \quad (2.13) \]
Moreover,
\[ r(T) = \frac{T(1 - p)}{1 + T(1 - p)} \to 1 \quad \text{as} \quad T \to \infty , \quad (2.14) \]
and
\[ t(T) = \frac{1}{1 + T(1 - p)} \to 0, \quad \text{as} \quad T \to \infty \quad (2.15) \]

where \( r(T) \) and \( t(T) \) are the reflection and transmission coefficients respectively. From (2.14) and (2.15) we find
\[ r(T) + t(T) = 1, \quad (2.16) \]

which is the expression for conservation of energy.

Using the results of above analysis we can calculate the source functions according to one-dimensional rod model at points where the radii corresponding to each \( \theta \) meet the shell boundaries. Our aim is to obtain the source functions described in equations (2.2) and (2.3). Here we calculate the optical depth along the ray path e.g., \( P_1Q_1, P_2Q_2, P_3Q_3 \), etc. and employ this optical depth to estimate the specific intensities and source functions at these points.

### 2.2.2 Calculation of self radiation of the primary component

In addition to the incident radiation from the secondary component, we have the radiation of the primary component itself. This self radiation of the primary component can be calculated easily by employing the radiative transfer equation in a spherically-symmetric approximation.

The equation of transfer is given in the form
\[
\frac{\mu}{r^2} \frac{\partial}{\partial r} \left[ r^2 I(r, \mu) \right] + \frac{1}{r} \frac{\partial}{\partial \mu} \left[ (1 - \mu^2) I(r, \mu) \right] + \sigma(r) I(r, \mu) \\
= \sigma(r) \left[ (1 - \omega(r)) b(r) + \frac{1}{2} \omega(r) \int_{-1}^{1} p(r, \mu, \mu') I(r, \mu') d\mu' \right], \quad (2.17)
\]

where \( \omega(r) \) is the albedo for single scattering, \( I(r, \mu) \) is the subject to the conditions in discrete space theory as described in Chapter 1 section 1.2.1).
Therefore, if \( S(r, \theta) \) is the total radiation, \( S_I(r, \theta) \) is the source function due to irradiation from external source and \( S_S(r) \) is that due to self-radiation of the star itself, then

\[
S(r, \theta) = S_I(r, \theta) + S_S(r). \tag{2.18}
\]

Next step is to calculate the distribution of intensities at the internal points like \( Q_1, Q_2 \ldots \) etc., along the radius and find out the distribution of the emergent radiation field at points like \( P_1, P_2, \ldots \) etc. The radiation field is given by the formal solution of radiative transfer equation in plane parallel approximation (Chandrasekhar 1960, equations 64 and 65 on p.12., and figure 2.3 here).

\[
I(\tau, \mu) = I(\tau_1, \mu) \exp\left[-(\tau_1 - \tau)/\mu\right] + \int_\tau^{\tau_1} S(t) \exp\left[-(t - \tau)/\mu\right] \frac{dt}{\mu}, \tag{2.19}
\]

for outward intensities and

\[
I(\tau, -\mu) = I(0, -\mu) \exp\left[-(\tau / \mu)\right] + \int_0^\tau S(t) \exp\left[-(\tau - t)/\mu\right] \frac{dt}{\mu}, \tag{2.20}
\]

for inward intensities. For the explanation of various terms see figure 2.3.

The optical depths are always measured from A to B, and \( 1 > \mu > 0 \) where \( \cos^{-1} \mu \) is the angle made by the ray with AB. The equations (2.19) and (2.20) describe the radiation field emerging from the irradiated surface which receives the incident radiation coming from the point source S.
2.2.3 Brief description of the computational procedure

We have considered a component whose outer radius is $R_{\text{out}} = 10^{12}\text{cm}$ and atmosphere thickness is $\simeq 0.5 R_{\text{out}}$. Let us assume that incident radiation is coming from a point S at a distance of $4.5 \times R_{\text{out}} \text{cm}$ away from the centre of the primary component with its centre at O. We divided the atmosphere of primary component into 25 spherical shells (in terms of shell number where shell numbers are counted outside to inside ie, $n=1$ at $R_{\text{out}}$ and $n=25$ at $R_{\text{in}}$). We calculate the source functions at points $Q_1, Q_2, Q_3, \ldots$, etc., on radius vector co-latitude $\theta$. This is done by employing the rod model described in equations (2.2), (2.3), (2.4) and (2.5). We calculate the segments $P_1Q_1, P_2Q_2, \ldots$, etc., and estimate the optical depths according to the law that the density varies as $\frac{1}{r}$. The electron scattering has been assumed in the medium with $N_0$ (at $R_{\text{out}} = 10^{12}\text{cm}$) = $10^{12}\text{cm}^{-3}$ to $10^{15}\text{cm}^{-3}$. We have presented the results for $N_0 = 10^{14}\text{cm}^{-3}$ and with this density (which is varying as $\frac{1}{r}$), the radial optical depth becomes 1.1. Here while calculating the source functions due to self radiation, we have used the spherically symmetric approximations and calculated the source functions.

The results are given in the form of distribution of radiation at different points along the radii such as $Q_1, Q_2, Q_3, \ldots$, etc., by solving the equations (2.19) and (2.20). Let $I_Q$ be the intensity of radiation incident spherically symmetrically on the inner boundary of the atmosphere of the star and the intensity coming from point S be $I_S$. The incident radiation at the point P will be $I_S \mu \cos \gamma$. We have considered the following cases.

Case 1: $I_Q/I_S = 0.1$
Case 2: $I_Q/I_S = 1$
Case 3: $I_Q/I_S = 10$
The figures 2.4, to 2.9 shows the radiation field $I(r, \mu, \theta)$ are plotted against $\mu$ for a specified $\theta$ and $r$ representing cases 1, 2 and 3 respectively. In these figures the continuous ($I$) curves denote the distribution of radiation due to the incident radiation from the point source and the dotted curves ($I_s$) denote the resultant radiation field due to external and self radiation fields.

2.2.4 Results and discussion

These results represent the radiation on the outermost layers of the reflected surface. In the figures 2.4, 2.5, 2.6, we have plotted $I(n = 1, \mu)$ for case 1, 2 and 3 respectively. From figure 2.4, one can see that the intensities at $\theta = 0^\circ$ are quite small and increase considerably at $\theta = 90^\circ$. In case 2, we have again the same phenomenon (figure 2.5) although at $\theta = 90^\circ$ (along OS see figure 2.1) more radiation goes into the star than the outcoming radiation. $I(n = 1, \mu)$ for case 3 figure 2.6 shows again the same features of figure 2.5, with much less radiation going into the star both at $\theta = 0^\circ$ and $\theta = 90^\circ$.

In the figures 2.7, 2.8, 2.9 we have plotted $I(r, \mu)$ at $\theta = 0^\circ$ for case 1, case 2, and case 3. The continuous curves denote the external radiation and the dotted curve indicate the resultant radiation field due to both external and self radiation. Here the co-latitude $\theta = 0^\circ$, corresponds to the radiation along $OY$ (see figure 2.1). At the bottom of the atmosphere $n = 25$ or $r=R_{in}$) the effect of external radiation is not much in figure 2.7 (ie case 1) but increases when the self radiation is added to it. At the shell $n = 15$, we see that the combined radiation field is maximum while at the outermost layer ($n = 1$) it is not as large as that ($n = 15$). This is not difficult to understand on physical grounds. We have diluted
Figure 2.4: Distribution of the emergent radiation field at $\theta = 0^\circ, 60^\circ, 90^\circ$ for case 1

Figure 2.5: Distribution of the emergent radiation field at $\theta = 0^\circ, 60^\circ, 90^\circ$ for case 2
Distribution of the emergent radiation field at $\theta = 0^\circ, 60^\circ, 90^\circ$ for case 3 self radiation and at $n = 25$ the external radiation becomes weak. In the middle we have the combined radiation field of both, although partly diluted. The similar kind of trend can be seen in figures 2.8, and 2.9.

In the figures 2.10, 2.11, 2.12 we have plotted $I(r, \mu)$ at $\theta = 60^\circ$ for case 1, case 2, and case 3. We also observe similar kind of features as in figures 2.7, 2.8, and 2.9. The main difference between these set of figures is more radiation comes out in all cases when $\theta = 60^\circ$ when compared to that at $\theta = 0^\circ$. 

*Figure 2.6: Distribution of the emergent radiation field at $\theta = 0^\circ, 60^\circ, 90^\circ$ for case 3 self radiation and at $n = 25$ the external radiation becomes weak. In the middle we have the combined radiation field of both, although partly diluted. The similar kind of trend can be seen in figures 2.8, and 2.9.*
Distribution of the emergent radiation field at $\theta = 0^\circ$ for case 1, for the shell numbers shown in the figure. I stands for irradiation, and IS stands for irradiation plus self radiation.

Figure 2.7: Distribution of the emergent radiation field at $\theta = 0^\circ$ for case 2.
Figure 2.9: Distribution of the emergent radiation field at $\theta = 0^\circ$ for case 3

Figure 2.10: Distribution of the emergent radiation field at $\theta = 60^\circ$ for case
Figure 2.11: Distribution of the emergent radiation field at $\theta = 60^\circ$ for case 2

Figure 2.12: Distribution of the emergent radiation field at $\theta = 60^\circ$ for case 3
2.3 Distribution of emergent radiation along the line of sight when incident radiation is from point source

In the previous section we have presented an approach to obtain the reflected radiation from the component of a binary system. We have considered the incident radiation coming from an external point source and estimated how this radiation field has changed the total radiation coming from the irradiated surface. We have noticed several important changes in the radiation field emergent from the irradiated part of the component. The intermediate regions of the irradiated part of the atmosphere become brighter than the extreme regions. This results suggests that the law of limb darkening that is generally used in the light curve analysis (Kopal 1959) should be replaced by the accurate calculations of the distribution of radiation from centre to limb. We shall employ the above procedure to calculate the distribution of radiation from centre to limb as is received at infinity.

2.3.1 Method to calculate radiation field from centre to limb

In figure 2.13, we have shown the portion of the star illuminated by radiation from a point source X. The atmosphere is divided into several shells of equal radial thickness. The radiation from this irradiated part of the atmosphere is received by the observer at infinity. We have chosen a set of parallel rays tangential to the shell boundaries at a point on the axis OX where O is the centre of the component. Let one of these rays meet the shell boundaries at $Q_1, Q_2, Q_3, \ldots$ etc. The intensity along the parallel rays is calculated first by obtaining combined source functions (self+irradiated) at points $Q_1, Q_2, \ldots$ etc.
Figure 2.13: Schematic diagram showing the irradiation of the component. X is the point source of radiation. O is the centre of the component. The specific intensities are calculated along the line of sight. (Q₇, Q₈, etc., R₃, R₂ etc.,)

For this purpose we have (1) the source function at these points due to self-radiation $S_s$ and (2) source function due to irradiation $S_I$. The latter is calculated by using rod model (described in section 2.2). The radiation field is calculated along the lines $Q₁P₁, Q₂P₂, Q₃P₃$ etc. We then add the two source functions to obtain the total source function $S_T$.

$$S_T = S_s + S_I.$$  \hspace{1cm} (2.21)

From figure 2.13 The element $QP$ is given by (see Peraiah 1983a)

$$QP = \left\{a^2 + b^2 + 2ab \cos(\overline{OQP} + \overline{OPQ})\right\}^{\frac{1}{2}},$$  \hspace{1cm} (2.22)

where

$$a = OP \ (OP₁, OP₂ \ etc),$$
\[ b = OQ \ (OQ_1, OQ_2 \ etc), \]
\[ OX = R, \quad OQ_1 = h, \]

(h is measured along \(OX\), and
\[ \sin \overline{OQP} = \frac{R}{b} \left( \frac{b^2 - h^2}{b^2 + R^2 - 2hR} \right)^{\frac{1}{2}}, \]
and
\[ \sin \overline{OPQ} = \frac{R}{a} \left( \frac{b^2 - h^2}{b^2 + R^2 - 2hR} \right)^{\frac{1}{2}}. \]

We have obtained the source terms \(S_I\) at \(Q's\) by calculating the optical depths along \(QP's\). This is done by assuming an electron scattering and the electron density \(\rho\) varying as \(\frac{1}{r^2}\) and \(\frac{1}{r^3}\). The source function due to self radiation \(S_s\) is obtained from the relation
\[ S_s(r) = \frac{1}{2} \int_{-1}^{+1} I(r, \mu) d\mu, \quad (2.23) \]
as we are assuming radiative equilibrium. The specific intensity \(I(r, \mu)\) is obtained by solving the equation of radiative transfer in spherical symmetry solved in Chapter 1 (equations 1.23 and 1.24). The boundary conditions are assumed as follows
\[ I_s/I_I = I, \quad (2.24) \]
where \(I_s\) is the intensity of radiation incident at the inner boundary and \(I_I\) is the intensity of radiation incident from the point source. The radiation incident on the surface at points \(P_1, P_2\ etc.\) is taken to be \(I_I \cos \overline{OPQ}\) and we have set \(I_S = 1\). We have considered both plane parallel and spherically symmetric media for the sake of the comparison and set \(\frac{B}{A} = 1\) and 1.5 respectively where \(B\) and \(A\) are the outer and inner radii of the atmosphere. The purpose of this is to see how the spherical term
\[ \pm \frac{1}{r} \frac{\partial}{\partial \mu} [(1 - \mu^2)U(r, \pm \mu)], \quad (2.25) \]
would change the results.
2.3.2 Results and discussion

We have taken the same data as given in the previous section for OX. \( N_0 \) etc. In figure 2.14-2.16, we have presented the variation of specific intensities from centre to limb in plane parallel case. The curves labeled \( I \) correspond to irradiation and those labeled \( I+S \) correspond to irradiation with self-radiation. The quantity \( h \) is the perpendicular distance to the ray from the centre \( O \) (see figure 2.13). The contribution from irradiation to the intensities \( I(h) \) is several times smaller than the total contribution from both self and irradiation. Figures 2.14 and 2.15 show that the limb is much darker than the centre. Figure 2.16 shows that an increase in electron density increases the brightness at the limb, but when combined with self-radiation, the limb appears dark. In figures 2.17-2.18 we have shown the specific intensities in the spherically symmetric case. We notice that the limb darken and also the intensities fall sharply compared to those in plane parallel case. When the electron density is increased the character of the intensities change, which is shown in figure 2.20. The intensities due to irradiation always show a brightening tendency towards the limb where as the total radiation field falls towards limb but at the limb it shows brightening. This is due to the fact that there are more electrons in the region which scatter more light than when \( N_o = 10^{12} \) or \( 10^{13} \text{cm}^{-3} \).
Figure 2.14: The specific intensities $I(h)$ ($h=0Q$) are plotted with respect to $h$. The curves labeled $I$ correspond to only irradiation and those $I+S$ correspond to irradiation plus self radiation. $N_0$ is the electron density at $A \frac{B}{A} = 1$. $N_0 = 10^{12}\text{cm}^{-3}$ (B and A are the outer and inner radii of the atmosphere. Here $\frac{B}{A} = 1$ means plane parallel atmospheres)

Figure 2.15: $I(h)$ Versus $h$ for $\frac{B}{A} = 1, \quad N_0 = 10^{13}\text{ cm}^{-3}$
Figure 2.16: I(h) Versus h for $\frac{B}{A} = 1$  \( N_0 = 10^{14} \text{ cm}^{-3} \)
Figure 2.17: $I(h)$ Versus $h$ for $\frac{B}{A} = 1.5$, $N_0 = 10^{13}$ cm$^{-3}$, $\rho \approx \frac{1}{r^2}$

Figure 2.18: $I(h)$ Versus $h$ for $\frac{B}{A} = 1.5$, $N_0 = 10^{12}$ cm$^{-3}$, $\rho \approx \frac{1}{r^2}$
Figure 2.19: $I(h)$ Versus $h$ for $\frac{B}{A} = 1.5$, $N_0 = 10^{12} \text{ cm}^{-3}$, $\rho \approx \frac{1}{r^3}$

Figure 2.20: $I(h)$ Versus $h$ for $\frac{B}{A} = 1.5$, $N_0 = 10^{14} \text{ cm}^{-3}$, $\rho \approx \frac{1}{r^3}$
2.4 Temperature changes due to reflection

In this section we investigate how the temperature is redistributed due to incidence of radiation from the point source (figure 2.1). We have assumed radiative equilibrium in scattering medium and therefore, it is easy to calculate the effective temperature, which is proportional to the $S_T^4$ where $S_T$ is the total source function in the scattering medium.

Procedure is described the section (2.1). Here we have considered radius $R_{out}$ and an atmosphere whose thickness is three times the radius. The point source kept at a distance five times (i.e, OS = $5 \times R_{out}$) the outer radius in one case, in ten times (i.e, OS = $10 \times R_{out}$) the outer radius in another case of the component from the centre O. We estimated the changes in temperature along the radii vectors OP corresponding to an angle $\theta$ made with OX. Here position of the point source S is on OX.

2.4.1 Results and discussion

In the regions where $\theta \geq 90^\circ$ the increase in temperature is in the outer layers where as in the regions for $\theta \leq 90^\circ$, the temperature is affected throughout the region. Here figure 2.21 for the density variation for $\rho \sim \frac{1}{r}$. Figure 2.22 for the density variation $\rho \sim \frac{1}{r^2}$. The results in both the figures show similar characteristics.

In figure 2.21 we have plotted the ratio of $\frac{T_\text{new}}{T_\text{old}}$ where $T_\text{new}$ is the new temperature and $T_\text{old}$ is the original temperature along the radius vector OP (see figure 2.1), for various angles. It is very interesting to note that the temperature increases by as much as 40% in the intermediate regions $\theta = 30^\circ$. The figures 2.23 and 2.24 are similar as 2.21 and 2.22 respectively except for the parameter OS (see figure 2.1).
Figure 2.21: Temperature distribution $\frac{T_r}{T_s}$ along the radius vector for each $\theta$ density variation is $\frac{1}{r}$

Figure 2.22: Same as figure 2.21, but the density variation is $\frac{1}{r^{1.2}}$
Figure 2.23: Same as figure 2.21, but $OS = 10 \times R_{out}$

Figure 2.24: Same as figure 2.22, but $OS = 10 \times R_{out}$