Chapter 3

Stokes Polarimeter at the KTT

3.1 Summary

The Stokes polarimeter developed for the KTT is explained in this chapter. Two optical methods were used to test different components of the polarimeter individually. The whole polarimeter was tested in the laboratory and in the field. The details of the experimental setup and the theory needed to extract the misalignments of the optical components are explained. An interferometric method was used to find out the retardance of the quarter-wave plate.

3.2 Introduction

A Stokes polarimeter measures the polarisation state of an input beam by measuring the Stokes parameters which is represented as a column vector $[I, Q, U, V]^T$, where the superscript 'T' represents the transpose operation. A generalised polarimeter is shown in Figure 3.1.

Consider a beam of partially polarised light, $[I, Q, U, V]^T$, which passes through
a retarder of retardance ‘Δ’ with its optic axis kept at an angle ‘θ₁’ with respect to a reference axis. In Figure 3.1, y-axis is taken as the reference axis. The angle is measured positive in the clock wise direction when the observer is looking at the source. The beam of light then passes through a polariser with its transmission axis kept at an angle ‘θ₂’ from the y-axis. Then the output emerging light from the polariser is represented by,

\[ [I_p, Q_p, U_p, V_p]^T = [M_2][M_1][I, Q, U, V]^T \] (3.1)

where \([I_p, Q_p, U_p, V_p]^T\) is the output Stokes vector and \([M_1] & [M_2]\) are the 4×4 Mueller matrices for the retarder and the polariser respectively. The Mueller matrices \([M_1]\) and \([M_2]\) are defined as,

\[
[M_1] = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & c_{21}^2 + s_{21}^2 \beta & c_{21} \cdot s_{21} \cdot (1 - \beta) & -s_{21} \cdot \mu \\
0 & c_{21} \cdot s_{21} \cdot (1 - \beta) & s_{21}^2 + c_{21}^2 \beta & c_{21} \cdot \mu \\
0 & s_{21} \cdot \mu & -c_{21} \cdot \mu & \beta
\end{bmatrix}
\]

where,

\[ c_{21} = \cos(2\theta_1) \quad \& \quad s_{21} = \sin(2\theta_1) \]
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\[ \beta = \cos(\Delta) \quad \& \quad \mu = \sin(\Delta) \]

and

\[ [M^2] = \frac{1}{2} \begin{pmatrix} 1 & c_{22} & s_{22} & 0 \\ c_{22} & c_{22}^2 & c_{22} \cdot s_{22} & 0 \\ s_{22} & c_{22} \cdot s_{22} & s_{22}^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \]

where,

\[ c_{22} = \cos(2\theta_2) \quad \& \quad s_{22} = \sin(2\theta_2) \]

Note that the [M1] and [M2] defined above is general. The Mueller matrix [M1] is applicable to any ideal retarder with retardance \( \Delta \), with its optic axis kept at an angle \( \theta_1 \) from the reference axis. The Mueller matrix [M2] is applicable to any polariser with its transmission axis kept at an angle \( \theta_2 \) from the reference axis. However, [M1] and [M2] does not include the depolarisation effects and other defects which are possible during the manufacturing process.

By substituting [M1] and [M2] in the Equation 3.1, the output intensity in terms of the input Stokes parameters for the system shown in Figure 3.1 can be calculated and is given by,

\[ I_p = \frac{1}{2} \{ I + Q \cdot [c_{22} \cdot (c_{21}^2 + s_{21}^2 \cdot \beta) + s_{22} \cdot c_{21} \cdot s_{21} \cdot (1 - \beta)] + U \cdot [c_{22} \cdot c_{21} \cdot s_{21} \cdot (1 - \beta) + s_{22} \cdot (s_{21}^2 + c_{21}^2 \cdot \beta)] + 1 \cdot [-c_{22} \cdot s_{21} \cdot \mu + s_{22} \cdot c_{21} \cdot \mu] \} \]  \tag{3.2}

Equation 3.2 shows that there are several ways of combining different optical components to calculate all the four Stokes parameters by measuring the output intensity, \( I_p \) (Shurcliff, 1962; Clarke and Grainger, 1971; Stenflo, 1984). Table 3.1 lists the different combinations of the retardance \( \Delta \), the angle \( \theta_1 \) \& \( \theta_2 \) and the associated quantity measured.
Table 3.1: Broad classification of polarimetric methods

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Method</th>
<th>Conditions on the variable</th>
<th>Quantity Measurable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Polariser at different orientation + No retarder</td>
<td>( \theta_2 ) variable, ( \theta_1 ) constant and zero, ( \Delta ) constant and zero</td>
<td>I, Q and U</td>
</tr>
<tr>
<td>2.</td>
<td>Retarder at a fixed orientation and fixed retardance + Polariser at different orientation</td>
<td>( \theta_2 ) variable, ( \theta_1 ) constant, ( \Delta ) constant</td>
<td>I, Q, U and V</td>
</tr>
<tr>
<td>3.</td>
<td>Retarder at different orientation and fixed retardance + Polariser at fixed orientation</td>
<td>( \theta_2 ) constant, ( \theta_1 ) variable, ( \Delta ) constant</td>
<td>I, Q, U and V</td>
</tr>
<tr>
<td>4.</td>
<td>Retarder + Polariser combination at different orientation with their relative orientation fixed and fixed retardance</td>
<td>( \theta_2 ) variable, ( \theta_1 ) variable, ( (\theta_2 - \theta_1) ) constant, ( \Delta ) constant</td>
<td>I, Q, U and V</td>
</tr>
<tr>
<td>5.</td>
<td>Variable retarder with fixed orientation + Polariser at fixed orientation</td>
<td>( \theta_2 ) constant, ( \theta_1 ) constant and ( \Delta ) variable</td>
<td>I, Q, U and V</td>
</tr>
</tbody>
</table>

Each method, listed in Table 3.1, has its own advantages and disadvantages (Clarke and Grainger, 1971). The polarimeter which is developed at the KTT is a rotating prism polaroid with an insertable quarter waveplate (QWP) and hence is the combination of method 1 and 2. The linear polarisation (Q and U) is measured without any retarder (hence method 1) and the circular polarisation is measured by inserting a fixed retarder (quarter wave retarder) at a fixed orientation (hence method 2). The chief consideration for using a rotating prism polaroid rather than a rotating half waveplate (HWP) is that the cross-talk between the Stokes param-
ters arising from the retardance error in the waveplate can be avoided completely. Otherwise, the error in the retardance produces a cross-talk from V (circular) into Q and U (linear) component and vice versa. This can be severe in the case of sunspot measurement where the amount of V is large compared to Q and U, at the umbral region of the sunspot.

For example, if δ is the error in the retardance of a HWP, then the error in the measurement of U can be calculated as,

\[ \frac{\delta U}{U} = \frac{1}{2} (\cos \delta - 1) - \frac{V}{2U} \sin \delta. \]

For very small errors in the retardance, the first term in the above equation can be neglected and hence the residual error in U, i.e., \( \delta U \) is \( \frac{V}{2} \delta \). An error of 0.1° in the retardance (i.e., \( \delta = 0.1° \)) and for a value of 20% in V and 1% in U (typical observational value at the umbra of a sunspot which is very close to the disk center), the error in U (i.e., \( \delta U \)) is about 0.017%. Similarly, the residual error in Q can be calculated as, \( \delta Q = \frac{V}{\sqrt{2}} \delta \) and hence an error of about 0.024% for the same observational parameter.

In this chapter, the design of a polarimeter for modulating the input polarised signal and the demodulation scheme using a CCD chip is discussed. Also, the testing of the polarimeter, both in the laboratory and with the KTT is described.

### 3.3 Polarimeter and the Spectrograph

The Stokes polarimeter designed for the KTT is used in the converging beam of the telescope since the f-ratio of the telescope is high (f-90). The polarimeter consists of a manually insertable QWP, a rotating Glan-Thompson prism polaroid (GTP) as analyser and an insertable polaroid (called as compensating polaroid) for the Stokes U-measurement (Figure 3.2).

An IR sensor with the associated electronics is used to sense the reference position
Figure 3.2: Block diagram of the Stokes polarimeter

which is set at 45° from the slit direction in this case. The transmission axis of the GTP is matched with the sensor position. The compensating polaroid for the Stokes U-measurement is used to compensate the spurious polarisation produced because of the differential grating response to the two orthogonal polarisation, one along the grating groove direction and the other perpendicular to it. The details of the grating response to different input polarisation state and the function of the compensating polaroid will be discussed later in this chapter (refer section 3.5). The whole polarimeter slides on to the slit of the spectrograph in order to minimise vignetting since the polarimeter is used in the converging beam of the telescope. The image plane of the telescope coincides with the slit of the spectrograph. The objective of the telescope can be moved to adjust the focal plane. Since the focal ratio of the objective is f-90, it has a very large depth of focus. Any minor defocusing due to the polarimetric optics can be readjusted by repositioning the imaging objective of the telescope. The acceptance angle constraint posed by the polariser and the QWP is well within the focal ratio of the telescope.
3.3.1 Spectrograph

The spectrograph is a Littrow mount spectrograph whose entrance slit is located at the image plane of the KTT. The spectrograph is housed in a long cylindrical tube running along the length of the tunnel in order to reduce the scattered light from the surrounding. The light from the slit of the spectrograph falls on a 18.3 m focal length, two-element Hilger achromat, in conjunction with a 600 lines/mm Babcock grating, ruled over an area of 200×135 mm and blazed in the fifth order at λλ5000Å. The theoretical resolving power of the spectrograph is 600,000. The advantage of the Littrow mount spectrograph is that the Hilger achromat itself acts as a camera lens to image the spectrum on to a CCD which is kept below the slit. A one-to-one correspondence exists between every point along the length of the slit and the length of the spectrum in a direction perpendicular to the dispersion direction.

3.4 Detector

The data discussed in this thesis are obtained with two types of charge-coupled device (CCD). The first detector is an EEV P8603 CCD 1. Let us call this as Det-1. The other CCD is a Photometrics CCD 2. Let us call this as Det-2.

3.4.1 Detector-1 (Det-1)

Det-1 is an analogue integrated circuit with 576×385 picture elements (pixels) that are light sensitive. It is cooled to temperatures of between 120K to 180K to reduce the thermal dark current generated within the device. At the end of an exposure the

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2 Electronic imaging, 53, Hamilton Road, Cambridge CB4 1BP, Telephone (0223)-63737

device is read out one pixel at a time under the control of the host computer that has
generate signals to cause the device to transfer charge in the two directions and
to return the data value to the computer. This CCD is mounted in a liquid nitrogen
cooled dewar and connected to the CCD 2000 driver electronics rack by a single
flat cable. As a minimum the system must include bias, clock modules, the double-
correlated sampling (DCS) module. The modules communicate with a tri-state bus
on the driver electronics rack to enable the system to be operated at a distance from
the host computer. A bidirectional very high speed (8 Megabaud) serial data link is
used that reduces the interface to a single co-axial cable, transformer isolated at each
end. There is an optional facility to measure various analogue and power supply levels
both with a built in digital voltmeter and remotely under computer control with the
analogue to digital converter on the DCS module. This allows both local and remote
checking very rapidly of the system set-up parameters. Provision is also made for
measuring and (optionally) controlling the operating temperature of the CCD within
the dewar.

The CCD 2000 imaging system consist of three major assemblies. These are,

1. The computer interface unit which transmits control data to and receives data
from the CCD driver electronics rack.

2. The CCD driver electronics rack: This generates the necessary DC supplies
(bias module) and clock waveforms (clocks module) to drive the CCD in the dewar.
The output signal is processed in the double correlated sampling module where the
detected signal is digitised for return to the host computer. A transmitter/receiver
module handles communications with the computer and a multiplexer module decodes
the received control words and routes them to the appropriate module. In order to
facilitate system checkout and alignment a run/display module is included which will
drive the CCD camera without a host computer and generate three analogue signals
that may be used to give an intensity modulated XY display on a suitable monitor
oscilloscope. The temperature of the CCD and of the electronics rack may be sensed
by the monitor/thermal module which can also control the temperature of the CCD mount with the dewar. This module also permits remote monitoring of most of the critical potentials in the system electronics rack by the host computer.

3. Dewar with CCD: Det-1 is mounted inside a dewar cooled with liquid nitrogen.

The sensor array is divided into three regions using an aluminium mask of 1 mm thickness, 12.7 mm length and 8.3 mm breadth. It is held on to the surface of the CCD mount. This mask allows the central (197 pixels; 4.3 mm) region to be exposed to light and the upper and lower regions shielded from the light beam (Refer to Figure 3.3). This demodulation scheme is very similar to the one used by Stockman (1982). The sensor is kept in a liquid nitrogen cooled dewar. The controller is from Astromed, UK. The controller has the necessary clock generation, bias voltages, signal processing, data conversion and data transmission electronics built into it. It operates in slow scan mode with double correlated sampling (DCS) with full 15-bit data conversion. A serial cable link connect the parallel port of the PC to the controller through an external parallel to serial converter. A three meter cable connects the dewar to the controller.

The software has been developed under Linux (a UNIX variant) environment. The application program comprises Graphical User Interface (GUI), image display, a driver software for image data acquisition based on X-windows system and polaroid movements. The software features include simple diagnostic routines for testing the CCD controller and electronics, file handling routines to store and restore image files in ‘Flexible Image Transfer Software (FITS)’ format, image data acquisition routines for acquiring bias, dark or object and simple quick look functions for image data analysis. The hardware and software part of this detector is developed by our engineers Mr. G. Srinivasulu and Mr. A. V. Ananth.
### 3.5 Function of the Polarimeter

#### 3.4.2 Detector-2 (Det-2)

Det-2 is a Photometrics AT200 CCD camera system. It includes three hardware components, (i) An AT200 camera controller, (ii) A CE200A camera electronics unit and (iii) A liquid cooled CH250 camera head. A liquid circulation unit is used for the cooling of the camera head. These components are linked by custom cables and controlled by a host computer.

**AT200 camera controller:** The AT200 camera controller manages communications between a host computer and a CE200A camera electronics unit. A digital signal processor sends control signals to the CE200A via the camera controller cable. CCD data are received through the same cable. The AT200 has no memory of its own, so incoming data must be promptly stored in host RAM. Once collected into host memory, the data can be manipulated by software on the host computer. The AT200 can be synchronised to external equipment or to a manual trigger with the User I/O connector. This feature is used to control the Stokes polarimeter, like sensing the reference position and rotating the stepper motor.

**CE200A camera electronics unit:** The CE200A camera electronics unit contains signal processing, camera control, and temperature control systems. It produces CCD clocking signals for the CCD camera head and manages the transfer of raw CCD data to the AT200.

**CCD camera heads:** Photometrics CCD camera head is cooled to reduce dark current, the spontaneous charge generated by heat and other non-photon sources. Cooling is achieved by thermoelectric (peltier) cooling. The camera head is composed of a sealed CCD enclosure, a shutter assembly. The head contains electronics that are directly associated with CCD operation. The camera head cable transmits voltages and signals to and from the CE200A. A preamplifier raises the CCD output signal to a high level for digitisation.
3.5 Function of the Polarimeter

The function of the polarimeter with Det-1 is as follows, the GTP is brought to the reference position. The Stokes Q measurement corresponds to this position. Spectra from this polarisation state, I+Q is made to fall on the window of the CCD sensor (Det-1) and integrated for 100 msec and then the shutter is closed. Subsequently the GTP is moved to the orthogonal position, I-Q in 60 msec. During this period the exposed region is moved to one of the unexposed region i.e., the masked region. In this position, shutter is opened again and charge integration for 100 msec is performed. After the charge integration, the shutter is closed and this completes one full charge shifting operation. After completing one full operation, the GTP is moved again to the orthogonal position, I+Q and while the charges are shifted in the opposite direction so that the previously exposed charges corresponding to I+Q will now be in the window. The shutter is opened at this position for next 100 msec exposure and the new charges will get added with the old one. The back and forth movement of the charges in the imaging area is achieved by forward/reverse parallel shift in the CCD sensor. These operations, depicted in Figure 3.3, continue until the necessary signal to noise ratio is achieved. The number of full charge shifting operations is nothing but the ratio of the total integration time in msec to 100 msec which is the exposure time for a single spectra in one polarisation state. The number of full charge shifting operations has to be integer multiple in order to get equal total exposure time for both the orthogonal polarisation (I±Q).

Once, the I±Q measurement is over, the GTP is rotated to an angle of 45° from the reference position. The compensating polaroid with its transmission axis at 45° from the slit direction is now inserted at the back (before the slit) of the GTP. Now, the polarimeter is ready for the measurement of I±U. Similar charge shifting operations were carried out as was done for I±Q. However, longer exposure times (i.e., more number of charge shifting operations) are needed because of the light loss due to the
extra polaroid inserted. In practice, the same signal to noise ratio was never achieved. The signal to noise ratio in I±U measurement is lower by a factor of two to that of the I±Q measurement. For the circular polarisation measurement (V), a QWP with its optics axis at 90° to the slit direction is manually inserted in front of the GTP and the compensating polaroid inserted at the back of the GTP for U measurement is removed. Charge shifting is done with the GTP position similar to that of the I±Q measurement.

The operation of the polarimeter with Det-2 is very similar to the one described above except that there is no charge shifting within this CCD. The two orthogonal polarisation states (say, I±Q) are exposed independently and stored in the buffer. However, in order to save the writing time to the computer, a three dimensional data cube is created in the buffer to store all the six measurements (I±Q, I±U and I±V). At first the polarimeter senses the reference position using the IR-sensor and the GTP is positioned for the I + Q measurement. The spectra corresponding to I + Q is exposed. The exposure time used for our observations varies from 100 msec to
3.5: Function of the Polarimeter

500 msec depending on the sky transparency to achieve a good signal to noise ratio. The spectra exposed is transferred to the buffer and the polarimeter rotates to position the GTP in the orthogonal polarisation position for the I - Q measurement. Instead of coming back to the reference position again for the U-measurement, the polarimeter is rotated in the same direction for $45^\circ$ and then $90^\circ$ more to obtain the I±U spectra. The compensating polaroid is inserted behind (in front of the slit) the GTP before the U-exposure takes place. The I±V measurement is carried out by rotating the polarimeter to $45^\circ$ and then $90^\circ$ more from the end position of U-measurement. The compensating polaroid is removed and a QWP is inserted in front of the GTP for the V-measurement. All these six images were stored in the buffer. At the end of the V-measurement, this three dimensional data cube is saved into the computer memory in ‘FITS’ format. The control software is written using the macros available with the Photometrics software. This includes the sensing of the initial position using the IR sensor, movement of the stepper motor to rotate the GTP and controlling the mechanical shutter in front of the CCD to expose the spectra at the correct moment. With the availability of a pentium processor, these measurements are done quickly. All the six measurements for a slit position is done within about 1 minute. This way of measurement will introduce spurious polarisation signal if the sky condition is not good. However, this CCD has the advantage that it is larger in size and hence a larger field of view can be obtained with few more Zeeman sensitive spectral lines in the spectral direction apart from the lines Fe I 6301.5Å and Fe I 6302.5Å.

The polarimeter was tested in the laboratory and in the field with the KTT before the data for the sunspots were obtained. The retardance error in the QWP is tested using a polarisation interferometric technique. Also, the positional alignment error of the QWP, the GTP and the compensating polaroid is tested in the laboratory. Appendix A summarises the algorithm used to find out the Mueller matrix of the polarimeter when errors are present in the alignment of the optic axis of the QWP, retardance of the QWP and the alignment of the GTP, the compensating polaroid.
This algorithm is used to remove the cross-talks between the Stokes parameters introduced by the polarimeter alone.

### 3.6 Testing of Waveplate

Waveplates or retarders are essential in any polarimetric system which measures all the four Stokes parameters. It is necessary to know the characteristics of the waveplate before using it in any scientific instrument. The errors in the waveplate can limit the accuracy of the polarimetric system (West and Smith, 1995). Hence, the accurate measurement of the retardance and the direction of the optic axis is needed before using the waveplate for polarisation analysis. The optical properties of the waveplate, made from a birefringent material, has been studied using different techniques in order to measure the phase retardation precisely (Walter, 1978; Nakadate, 1990; Chidester, Harvey and Hubbard, 1991; Shyu, Chen and Su, 1993). Different methods use different techniques and in general a precision ellipsometric system is needed to measure any arbitrary retardance. Interferometric ellipsometry can be used for the measurement of retardance quite accurately.

#### 3.6.1 Method

Babinet Compensator (BC) can be used as an ellipsometer to measure the ellipsometric angles of any retarding system (Azzam and Bashara, 1977; Born and Wolf, 1984; Sankarasubramanian and Venkatakrishnan, 1996). The principle of the method is the following. When a collimated linearly polarised beam is analysed after passing through the BC, fringes are formed (as explained in Chapter 2; Section 2.4). The contrast and the position of the fringes depend on the input state of polarisation. Hence the fringes formed at the BC gets shifted and the contrast reduces, whenever a waveplate is introduced in front of the BC. From the measurement of the fringe shift
the phase difference which has been introduced by the waveplate can be calculated. However, the fringe shift produced by the waveplate not only depends on the phase difference introduced at the waveplate, but also depends on the angle between the optic axis of the waveplate and the direction of vibration of the input linearly polarised light. The measurement of the fringe shift for different orientation of the optic axis has been taken in this experiment to find out the retardance with better accuracy.

There are several methods to measure the retardance using a BC with high accuracies (Jerrard, 1948). The accuracies involved in the double-pass method used by Hariharan and Sen (1960) is found to be better than the retardance calculated using fringe shift measurement. But the double-pass method needs better photometric accuracies and a stable source to achieve an accuracy of 0.5° in the retardance. The initial knowledge about the orientation of the optic axis of the waveplate is essential in all the methods. The optic axis of the crystal has to be kept at 45° to the direction of vibration of the input linearly polarised light. Again finding out the optic axis of a waveplate requires a crossed polaroid arrangement and the accuracy in finding out the direction of the optic axis depends on the sensitivity of the setup to intensity variations. The inaccuracies in the optic axis determination can give a wrong estimation of the retardance. The determination of the retardance using the heterodyne interferometric techniques developed recently is much superior in terms of the accuracies achieved (Lin, Chou and Chang, 1990; Chou, Huang and Chang, 1997) but it has not yet been applied to the mapping of the retardance errors on the waveplate. An optical setup is developed which measures the retardance of birefringent waveplates without requiring the knowledge about the optic axis. In fact a proper analysis gives the optic axis orientation also with better accuracy.

The measurement of the retardance using BC has been discussed long back (Hariharan and Sen, 1960). In all the cases, the BCs used were variable compensators and the fringe shift was measured using a telescope and a micrometer screw arrangement with manual adjustment. The waveplates were kept with its optic axis at 45° to the
input linearly polarised light. The accuracy with such a system was shown to be of the order of few degrees (Hariharan and Sen, 1960). Using a two dimensional detector (CCD) in place of the micrometer screw-telescope arrangement to record the fringes and analysing the recorded fringes further to measure the fringe shift with a fraction of a pixel accuracy, lead to an accuracy in the retardance measurement of about 2°. By finding the fringe shift for different orientations of the optic axis of the waveplate and fitting a model for the observed fringe shift will increase the accuracy in the retardance calculation further.

3.6.2 Experiment

The experimental setup is given in Figure 3.4. Light from a monochromator is passed through a BC, which is kept in between a crossed polariser (P1 and P2 in Figure 3.4). An EEV-CCD is used to record the fringes formed at the BC with the help of a frame grabber DT-IRIS 2861. The whole setup is aligned and adjusted such that the fringes are along the CCD row and of good contrast in the full field of view. The waveplate to be tested is mounted on a rotating system (R1), which rotates with the help of a stepper motor with a step correspond to an angle of 4.5°, and kept in the collimated beam between the input polariser and the BC to enable the measurement of the fringe shift for different orientation of the optic axis of the waveplate. This rotating system is a part of the polarimeter developed for the polarisation observation of sun at the KTT (Ananth et al, 1994). Fifteen frames for fifteen orientations of the optic axis of the waveplate are taken in this experiment to calculate the retardance.

The stepper motor, CCD and the DT-IRIS 2861 are interfaced to a 286 computer and signals for the integration time (to get a good signal to noise ratio), number of frames to be taken for different orientation of the optic axis of the waveplate and the steps between each frame (minimum of 4.5°) can be given by the user with the help of a FORTRAN program. The reference position marked in the rotating system
Figure 3.4: Block diagram of the experimental setup. M1 - Monochromator, P1 - Prism polariser, L1 - Collimating lens of 20 cm focal length, R1 - Stepper motor controlled rotating stage, S1 - Stepper motor controller, BC - Babinet compensator, P2 - Analyser crossed with P1, CCD - Charge coupled detector, PC - 286 PC to control the stepper motor and the CCD.

can be sensed by an IR sensor and used as a reference for finding out the optic axis of the waveplate from the fringe shift calculation. The axis of the waveplate can be kept at an arbitrary angle to this reference position. A maximum of fifteen frames corresponding to the fifteen orientations of the waveplate optic axis can be frame grabbed online with the frame grabber card DT-IRIS 2861. To cover the fringe shift in both direction of the reference fringe a rotation of 180° in the waveplate orientation is required. A rotation of 13.5°, which corresponds to three steps in the stepper motor used in this experiment, is chosen so that a rotation of 202.5° of the waveplate can be covered with fifteen frames.

The procedure of the experiment is the following: At first the reference position is sensed by the IR sensor and the stepper motor stops at that position. Then the exposure time (t), number of positions of the optic axis of the waveplate (nop) required and the number of steps (n) between each position is fed by the user. The
first frame, which contains the fringes formed by the waveplate, BC and polaroid assembly, is grabbed for this reference position. Once, the frame is grabbed for this position as signaled by the DT-IRIS 2861 card, the stepper motor steps ‘n’ number of steps and stays at that position for the next frame to be grabbed. This repeats for ‘nop’ positions and the grabbed frames which are temporarily stored in the frame grabber buffer are then transferred to the hard disk of the 286 computer. The further analysis are done on a sun workstation to get better accuracies using Interactive data language (IDL) software. All these procedures can be easily implemented as an online process by using a 486/pentium processor with a FORTRAN compiler.

3.6.3 Theory

Each of the optical components in the experimental setup given in Figure 3.4 can be represented by a Mueller matrix (only for polarisation analysis) and the final Mueller matrix can be calculated by just multiplying the individual Mueller matrices in order (Shurcliff, 1962). Assuming \( \theta \) as the angle of the optic axis of the waveplate with respect to the axis of the BC and \( \Delta \) as the retardance of the waveplate, it can be shown that the output intensity of the above described setup is,

\[
I(x, y) = \frac{1}{4}[1 - \sin^2 2\theta \cos \delta(y) - \cos^2 2\theta \cos \delta(y) \cos \Delta + \cos 2\theta \sin \Delta \sin \delta(y)]
\]  

(3.3)

where, \( \delta(y) = \frac{4\pi}{\lambda}(n_e-n_o) y \tan(A) \)

\( A = \) Angle of the wedge of BC

\( \lambda = \) wavelength of the light used.

\( y = \) The spatial position in the y-direction

Equation 3.3 gives the intensity in the detector plane which is a function of the spatial direction \( x \) and \( y \). However, since the parameter \( \delta \), which is the retardance introduced by the BC, is only a function of \( y \), the output intensity is independent of \( x \). The fringes are aligned with the CCD row (x direction), so that the shift in the
ringes will be only along the y-direction (CCD column). Differentiating Equation 3.3 with respect to y, the position of the intensity minima are given as,

\[ y_{min} = \frac{1}{C} \tan^{-1}\left[ \frac{\cos 2\theta \sin \Delta}{(1 - \cos \Delta) \cos^2 2\theta - 1} + m\pi \right], m = 0, 1, \ldots \quad (3.4) \]

where \( C \) is a constant for a particular wavelength which converts the retardance in radians to the fringe shift in number of pixels.

\[ C = \frac{4\pi(n_e - n_o)\tan(A)}{\lambda} \text{pixsiz} \quad (3.5) \]

where pixsiz is the size of a pixel which here is 22\( \mu \).

It can be seen from the Equation 3.5 that when the waveplate is introduced with its optic axis at 0° to that of the BC (i.e., \( \theta = 0° \)) the fringe shift will be proportional to the retardance \( \Delta \) introduced by the waveplate. Assuming \( \theta_1 \) as the angle between the optic axis of the waveplate and a reference position in the rotating system (\( \theta_1 \) can be called as an initial off-set angle), \( n \) as the number for each frame which is captured and \( \delta \theta \) as the angle between each steps then, the position of intensity minima \( y_{min}^n \) for \( m=0 \) can be written as,

\[ y_{min}^n = \frac{1}{C} \tan^{-1}\left[ \frac{\cos 2(\theta_1 + n\delta\theta) \sin \Delta}{(1 - \cos \Delta) \cos^2 2(\theta_1 + n\delta\theta) - 1} \right] \quad (3.6) \]

The above equation is consistent for the optical system described above and used for further analysis of the fringe shifts.

Once the fifteen frames are taken for different orientations of the optic axis of the waveplate, the fringe shift for each frame are calculated using the phase shift matching technique (Wang, Bryanston-cross and Whitehouse, 1996a). The first frame (i.e., \( n=0 \) in Equation 3.6) is taken as the reference frame and the other frames (i.e., \( n=1 \) to 14) are compared with this. The phase shift matching technique, used for calculating the phase difference between two interferograms, is used to calculate the fringe shift. The complete algorithm with simulation and experimental verification is given by Wang, Bryanston-cross and Whitehouse (1996a) and Wang et al (1996b) which can be referred for the details. The principle of the method is to reduce the rms residue
of the difference of a region of the two interferograms by shifting one with respect to the other. The shift which gives the minimum rms residue will be the true shift. A phase curve which is the plot between the shift in pixels to the rms residue is obtained. The minimum in this phase curve will give the value of the fringe shift in pixels. An interpolation scheme is used around this minimum of the phase curve in order to increase the determination of the minimum to sub-pixel accuracy. The simulation without noise shows an achievable accuracy of one hundredth of a pixel in the fringe shift calculation. For the real case with noise the accuracy in the fringe shift calculation is one tenth of a pixel. The advantage of this method over other methods is that it gives reasonably correct fringe shifts even in the presence of noise since the phase shifts are modulated in a particular frequency bin and extracted back by picking up only that frequency bin whereas the white noise will exist equally in all the frequency bins.

3.6.4 Mica Waveplates

Zero-order quarter waveplate for $\lambda=6122\text{Å}$, which is a magnetically sensitive line for Zeeman polarisation measurement of magnetic regions on the sun (Rust and O'byrne, 1990), was made from mica sheets. This waveplate was mounted on the rotating assembly and tested for its retardance. Figure 3.5 shows the 300th column cut of the fringes for two different orientation of the waveplate. The fringe shifts are calculated for different orientations of the optic axis of the waveplate by using the first frame as the reference frame. The same region of size $200 \times 150$ pixels is used to calculate the fringe shift for all the frames. A parametric search algorithm, which minimises the following function by varying $\theta_1$ and $\Delta$ over a range, was used to find out the best fit with the observed fringe shift.

$$\sigma^2(\theta_1, \Delta) = \sum_{i=1}^{15} (y_i - y_i^{\text{the}})^2 \quad (3.7)$$
Figure 3.5: The 300th column cut of two different frames taken with two different orientation of the optic axis.

where $y_i$ are the calculated fringe shifts from the phase matching algorithm and $y^{the}_i$ are the theoretical fringe shifts calculated from Equation 3.8.

$$y^{the}_i = y^{n}_{min} - y^0_{min} \quad (3.8)$$

Figure 3.6 shows the calculated fringe shifts from the phase matching algorithm as asterisks and the fitted theoretical curve as solid line for a particular parameter $\theta_1$ and $\Delta$ which minimises the function $\sigma^2$.

Commercial high-order quarter waveplate at $\lambda = 6302\,\text{Å}$ made of quartz is also tested with this technique. Figure 3.7 shows the calculated fringe shift for different position of the optic axis as asterisks and the best fitted curve as solid line for the high-order
Figure 3.6: The calculated fringe shifts for different positions of the optic axis of the zero-order mica quarter waveplate using the phase matching algorithm and the best fitted model. The calculated data points are shown as asterisks and the best fitted curve is shown as solid line. The parameters for the best fit are $\theta_1 = 85.7^{\circ} \pm 0.3^{\circ}$ and $\Delta = 89.1^{\circ} \pm 0.5^{\circ}$.

It is clearly seen that the best fit in this case has much more deviation than the zero-order case. Table 3.2 lists the experimental results for the zero-order and higher order waveplates along with the accuracies.

The variation of the fringe width with wavelength is used to find out the accuracy in the fringe shift calculation and the accuracies achieved are listed in the second column of the Table 3.2. For a BC, the fringe width is given in terms of number of
Figure 3.7: The calculated fringe shifts for different positions of the optic axis of the higher order quarter waveplate and the fitted curve. The parameters for the best fit are $\theta_1 = 59.3\pm0.3^\circ$ and $\Delta = 87.8\pm0.9^\circ$.

pixels (with 22$\mu$ as one pixel) as,

\[ Fringe Width (FW) = \frac{\lambda}{4.(n_o - n_e) \tan(A) \cdot \text{pixsiz}} \]  \hspace{1cm} (3.9)

The theoretical variation of the fringe width with wavelength (as given in the above equation) is compared with the measured fringe width and the root mean square deviation averaged over wavelength is taken as the experimental accuracy of the fringe shift calculation. The variation of $(n_e - n_o)$ with $\lambda$ is taken from Hariharan (1996). Figure 3.8 shows the measured fringe width for different wavelengths of the input light beam. The solid line is the fitted line using the Equation 3.9.

The accuracy in the retardance calculation from the parametric search are found
Table 3.2: Measured retardance of the waveplates

<table>
<thead>
<tr>
<th>Waveplate</th>
<th>Accuracy in fringe shift calculation (in pixels)</th>
<th>Accuracy in Retardance calculation (in degrees)</th>
<th>Accuracy in optic axis orientation (in degrees)</th>
<th>Calculated Retardance (in degrees)</th>
<th>Orientation of the optic axis* (in degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero-order</td>
<td>0.1</td>
<td>0.5</td>
<td>0.3</td>
<td>89.1</td>
<td>85.7</td>
</tr>
<tr>
<td>Higher order</td>
<td>0.1</td>
<td>0.9</td>
<td>0.3</td>
<td>87.8</td>
<td>59.3</td>
</tr>
</tbody>
</table>

* The orientation of the optic axis is calculated with respect to a reference position marked in the rotating system.

The orientation of the optic axis is calculated with respect to a reference position marked in the rotating system.

Out and listed in the third column of the Table 3.2. The least square fitting used is a non-linear fit and hence there is no analytical formula for the calculation of errors in the fitting parameters (Bevington, 1969). In an approximate way the uncertainty in the fitting parameter can be defined as that change in the parameter value which gives a change in the value of the residuals $\chi^2$ of unity. i.e.,

$$\chi^2(a + \epsilon) = \chi^2(a) + 1$$  \hspace{1cm} (3.10)

where 'a' is the fitting parameter, 'ε' is the uncertainty in 'a' and $\chi^2$ is related to the $\sigma^2$ defined in Equation 3.7 by,

$$\chi^2 = \frac{\sigma^2}{\sigma_{\text{ins}}^2}$$  \hspace{1cm} (3.11)

Equation 3.10 is valid for an experimental setup which has an uniform uncertainty
Figure 3.8: The measured fringe width for different wavelengths of the input light beam is shown as asterisks. The solid line is the theoretical model for the fringe width as given by Equation 3.9.

$\sigma_{ms}^2$ for all the measured data points. Here, the uncertainty (of 0.1 pixel) in the fringe shift measurement is taken as the $\sigma_{ms}$.

3.6.5 Testing of the QWP

A similar set up as described above is used to test the QWP used in the polarimeter of the KTT. A manual rotation of the waveplate is performed in order to get finer sampled data. The fringe shift is calculated for every 10° rotation of the QWP. Figure 3.9 shows a plot of the fringe shift for different rotation angle of the QWP.
The best fit shows that the retardance of the QWP is $94.7^\circ \pm 0.3^\circ$ and the initial offset of the QWP axis is $82.3^\circ \pm 0.2^\circ$.

![Graph showing measured fringe shift in degree for different rotation angles of the QWP. The solid line is the best-fit model.](image)

Figure 3.9: Measured fringe shift in degree for different rotation angle of the QWP are shown as data points. The solid line is the best-fit model.

### 3.6.6 Discussions and Improvements

In this section, a technique to calculate the retardance of any birefringent material using BC with CCD as a detector is discussed. Although the accuracy achievable in the retardance is $2^\circ$ with a single measurement, a series of fringe shift measurements are taken by keeping the optic axis of the waveplate at different orientation and a theoretical model for the experimental setup is fitted to get an accuracy of $0.5^\circ$ in the retardance for zero-order waveplate and $0.9^\circ$ for high-order waveplates. The accuracy achieved for zero-order waveplate are more than for the high-order waveplate.

By taking each single column cut, the variation of the fringe shift over a region is also calculated and shown in Figure 3.10. This shows the one dimensional surface...
variation of the retardance of the waveplate. The rms variation of the fringe shift variation in Figure 3.10 is 0.06 pixel over a region of 0.44 cm. This sigma is of the order of the uncertainty in the fringe shift calculation. Hence, the surface variation of the retardance of the waveplate is less than 0.5° for the zero-order waveplate which has been made in the laboratory using mica.

Figure 3.10: Variation of the fringe shift over a region of about 0.44 cm for the zero-order mica quarter waveplate. The root mean square deviation is 0.06 pixel which is almost equal to the experimental accuracy.

The inherent advantage of this technique is that it does not require any knowledge about the axis of the waveplate. Since it works on the basis of fringe shift calculation, the stability of the input source does not affect the retardance and the optic axis
direction calculation. The surface variation of the retardance of the waveplate can also be studied with the same setup. But it requires an extensive calculation before finding out the retardance and the optic axis direction.

The wedge angle 'A' used for this experiment is 5°. A BC with a wedge angle of 0.5° will give a fringe shift and fringe width of about one order more than the present case. This will definitely improve the accuracy in the retardance measurement. The same experiment can be repeated for several wavelengths and a model can be fitted to find out the retardance value with better accuracy. With the same setup and increasing the number of measurements by reducing the angle between each measurement (currently 13.5°), the least square fitting can be improved which will definitely reduce the uncertainty in the fitting parameters \( \theta_1 \) and \( \Delta \). In conclusion, a retardance measurement of less than 0.1° seems possible with the above improvements.

### 3.7 Testing of Polarisation Optics

A laboratory test has been carried out in order to look for the mis-alignments of the polarisation optics of the polarimeter. The optical setup used is given in Figure 3.11. A monochromator is used to send \( \lambda \lambda 6302 \text{Å} \) light into a polarising Rochon prism. A 20 cm focal length convex lens is used to collimate the beam and sent through the polarimeter, CCD assembly. An aperture is used in front of the polarimeter in order to limit the size of the beam falling on the masked CCD arrangement. Stokes Q, U and V are measured by varying the transmission axis of the input polarising prism. Data are taken for every 5° rotation of the input polarising prism. The exposure time used is between 200 to 600 msec.

Figure 3.12 shows the measured Q/I, U/I and V/I as data points. Since the input is 100% linearly polarised light (polarising prism), the modulation in V/I will give the mis-alignment in the QWP and the error in the retardance. The mis-alignment in the GTP can be identified from the Q/I and U/I modulation. A simple model
Combined performance with the Spectrograph and the KTT

The response of the grating to the input polarised light is measured by sending the sunlight through the polarimeter without introducing the QWP. Since the polarimeter is a rotating analyser, the output light will be polarised and the azimuth will be changing depending on the position of the analyser. The rotating analyser is rotated...
Figure 3.12: The measured percentage degree of polarisation for Q, U and V are plotted for different values of the azimuthal angle of the input prism polaroid. The data plotted with asterisk symbol is for Q and the diamond symbol is for U whereas the triangle symbol is for V. The solid line is the total polarised light and the variation in the total polarisation is less than 0.4% for every 1.8° and the continuum intensity is measured for each position. A simple model for the grating response has been developed in order to estimate the response. The output intensity is related to the grating response coefficient as,

\[ I(\theta) = G_{11} + G_{12} \cos(2\theta) + G_{13} \sin(2\theta) \] (3.12)

where \( G_{11}, \ G_{12} \) and \( G_{13} \) are the response coefficient for the grating. Figure 3.14 shows the measured data and the best fit of this simple model. The derived response coefficients are \( G_{11} = 1.0, \ G_{12} = 0.85, \) and \( G_{13} = 0.0 \).
3.9 Conclusions

In this chapter the modulator and demodulator for the polarimeter developed for KTT was discussed. A CCD based polarisation interferometric technique was developed in order to characterise the QWP used for the polarimeter. An optical setup was used to find out the mis-alignments of the polarimetric optics. The combined performance of the polarimeter with the grating of the spectrograph at the KTT was described. The critical parameters, the retardance and the positional error in the waveplate, the positional error in the GTP and the compensating polaroid, to characterise the polarimeter was found out. These parameters will be used in the actual observation in order to eliminate the spurious polarisation signal produced by the polarimeter alone.
Table 3.3: Measurement of the mis-alignments of the optical axis of the polaroid and QWP and the retardance error of QWP.

<table>
<thead>
<tr>
<th>Component</th>
<th>Parameters for an ideal polarimeter</th>
<th>Parameters measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>QWP - Axis</td>
<td>45° from the slit direction</td>
<td>43.0±0.3° from the slit direction</td>
</tr>
<tr>
<td>QWP - Retardance</td>
<td>90°</td>
<td>95.5±0.5°</td>
</tr>
<tr>
<td>Analyser - Axis</td>
<td>45° from the slit direction</td>
<td>45.0±0.3° from the slit direction</td>
</tr>
<tr>
<td>Compensating polaroid</td>
<td>45° from the slit direction</td>
<td>44.0±0.5° from the slit direction</td>
</tr>
</tbody>
</table>
Figure 3.14: The grating response to the input linearly polarised light. The x-axis is the azimuth of the polarised light and the y-axis is the normalised intensity of the grating response. The scattered data points near azimuth angle 300° is because of passing clouds.