APPENDIX A.1

A.1.1 FORTRAN PROGRAMME FOR FITTING A nTH DEGREE POLYNOMIAL
EMPLOYING THE PRINCIPLE OF 'LEAST-SQUARE'.

It has been reported in Section 4 that observed deflection obtained from photo-negatives for the urethane rubber beam model showed fluctuations. For smoothing the observed deflection, it was already reported, Curve fitting technique was employed. A Fortran programme for fitting a nth degree polynomial employing the principle of 'least-square' is presented below:

```fortran
PROGRAM FOR FITTING A POLYNOMIAL EQN BY LEAST SQRS
DIMENSION A(150,150),B(150,150),X(150),Y(150,2),C(150,1150),YC(150,2,12),SDN(20)
OPEN (UNIT =2,DEVICE='DSK',FILE='D.DAT')
PRINT 205
FORMAT(/40X,'SPAN=15 CM CENTRAL IMPACT DCSP//')
MM=0
READ (2,100,END=99) LL
IF (MM-LL)10,98,98
READ (2,200,END=99)N,M,L
IF(MM)25,15,25
DO 16 1=1 , N
READ(2,330,END=99)X(I),(Y(I,0),=1,L)
FORMAT (14X,F14.4,14X,F14.4)
FORMATION OF COEFFICIENT MATRIX OF THE NORMAL EQUATIONS
DO 42 1= 1 , C(I,1)=1.
MP1 = M + 1
DO 43 0=2, MP1
DO 67 I=1,N
A ( i, j ) = A ( i, j ) + C ( i, k )* Y ( i, k )
FORMATION OF THE CONSTANT MATRIX
DO 45 J= 1 , L
B ( I, J ) = B ( I, J ) + C ( I, K )* Y ( K, J )
CALL SUBROUTINE SIMUL TO SOLVE NORMAL EQUATIONS
```

205 FORMAT(/40X,'SPAN=15 CM CENTRAL IMPACT DCSP//')
CALL SIMUL (A,MP1,B,L,DET)
PRINT 600,N
PRINT 700,M
DO 50 J=1,L
DO 50 I=1,MP1
IM1=I-1
50 PRINT 800,IM1,B(I,J)
S = 0.0
DO 901 J=1,L
YC (I,J,M)=0.
DO 902 K=1,MP1
KM1=K-1
902 YC(I,J,M)=YC(I,J,M) + B(K,J)*X(I)**KM1
SDY = (YC(I,J,M)-Y(I,J))**2
S = S + SDY
SDN(M) = SORT (S/N)
PRINT 903,M,SDN(M)
903 FORMAT(/7X,'DEGREE OF POLYNOMIALS=',I4,7X,1,'STANDARD DEVIATION=',F15.8)
MM=MM+1
GO TO 5
100 FORMAT(14)
200 FORMAT(314)
600 FORMAT(/1D OF GIVEN DATA POINTS=',I4)
700 FORMAT(/7X,'DEGREE OF POLYNOMIALS=',I4)
800 FORMAT(/5X,I2,'DEGREE COEFFICIENT=',E14.8)
98 SMALL = SDN(M)
905 MM1 =MM1-1
904 IF(MM1-5) 904,904,905
905 IF(SMALL.LE.SDN(MM1)) GO TO 906
906 SMALL =SDN(MM1)
MM =MM1
GO TO 906
907 FORMAT(/7X,'DEGREE OF POLYNOMIALS=',I4,7X,1'SMALLEST STANDARD DEVIATIONS ,F15.8)
PRINT 912
PRINT 205
911 PRINT 912,I,X(I),(Y(I,J),J=1,L),(YC(I,J,MM1),J=1,L)
912 FORMAT(/7X,'NO. OF POINT=',I4,7X,'TIME=',F10.5
1,7X,'OBS.DEFLECTION=',F10.5,7X,'CURFIT.DEFLECTION=',F10.5)
CLOSE(UNIT=2)
GO TO 999
99 STOP
END
SUBROUTINE SIMUL(A,N,B,M,DET)
DIMENSION A(150,150),B(150,150),IPV(150),IND(150,2),PIV(150)
COMMON IPV,IND,PIV
EQUIVALENCE (IROU,IROW),(ICOL,ICOL)

STATEMENTS FOR INITIALIZATION

DO 13= 1,N
IPV(J) =0
13 CONTINUE

C STATEMENTS FOR INITIALIZATION
57 DET =1.
DO 19 I =1,N
IPV(I) =0
19 CONTINUE

C SEARCH FOR PIVOT ELEMENT
Z=0
• DO 19 J=1,N
IF(IPV(J)=1) 13,19,13
13 DO 32 K = 1, N
IF(IPV(K)=1) 43,32,81
43 IF(ABS(Z)=ABS(A(J,K))) 83,32,32
83 IROU =J
ICOL=K
Z =A(J,K)
32 CONTINUE
19 CONTINUE
IPV(ICOL)= IPV(ICOL) +1

C TO PUT PIVOT ELEMENT ON THE DIAGONAL
73 DET = -DET
DO 21 L =1,N
Z = A(IROW,L)
21 A(IROW,L) = A(ICOL,L)
IF(M) 109,109,33
33 DO 2 L = 1, M
Z = B(IROW,L)
2 B(IROW,L) = B(ICOL,L)
109 IND(I,1) = IROW
IND(I,2) = ICOL
PIV(I) = A(ICOL,ICOL)
DET = DET*PIV(I)

C TO DEVIDE PIVOT ROW BY PIVOT ELEMENT
A(ICOL,ICOL)= 1.
DO 207 L =1,N
207 A(ICOL,L) = A(ICOL,L)/PIV(I)
IF(M) 347,347,65
66 DO 551 L= 1,1*1
551 B(ICOL,L)=B(ICOL,L)/PIV(I)

C TO REDUCE NONPIVOT ROWS
347 DO 35 LI = 1,N
35 IF(LI=ICOL)22,35,22
22 \quad Z = A(LI, ICOL)
A(LI, ICOL) = 0
DO 189 L = 1, N
189 \quad A(LI, L) = A(LI, L) - A(ICOL, L) * Z
DO 68 L = 1, M
68 \quad B(LI, L) = B(LI, L) - B(ICOL, L) * Z
CONTINUE
C
INTERCHANGE OF COLUMNS
DO 31 = 1, N
31 \quad L = N - I + 1
IF(IND(L, 1) - IND(L, 2)) 29, 3, 29
29 \quad JROW = IND(L, 1)
JCOL = IND(L, 2)
DO 49 K = 1, N
Z = A(K, JROW)
A(K, JROW) = A(K, JCOL)
A(K, JCOL) = Z
CONTINUE
RETURN
END
A.1.2 EQUATIONS FOR BEST CURVE FITTED DEFLECTION HISTORIES

It has been already reported in Section 4.5 that the results from computer programming for best curve fitted deflection histories are presented graphically in Figs. 4.21 to 4.26. The equation of beam deflection (Y) may be expressed in nth degree polynomial as below:

\[ Y = A_0 + A_1 t + A_2 t^2 + A_3 t^3 + A_4 t^4 + A_5 t^5 + A_6 t^6 + \ldots \ldots + A_{12} t^{12} + \ldots \]

in which,

- \( A_0, A_1, A_2, A_3, \ldots \) - Coefficients of polynomial, and
- \( t \) - Time in millisecond from the onset of collision.

The coefficients for the best fitted polynomial for deflection histories are presented in Table A.1.1 for central- and quarter-span-impact loading situations. The abbreviations followed in this table are as follows:

- **DQLSP** - Deflection at left quarter span point
- **DQRSP** - Deflection at right quarter span point
- **DCSP** - Deflection at central span point
- \( l \) - Beam-span in millimetre.
- \( n \) - Degree of polynomial
<table>
<thead>
<tr>
<th>l (mm)</th>
<th>n</th>
<th>Item</th>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>$A_5$</th>
<th>$A_6$</th>
<th>$A_7$</th>
<th>$A_8$</th>
<th>$A_9$</th>
<th>$A_{10}$</th>
<th>$A_{11}$</th>
<th>$A_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>8</td>
<td>DQLSP</td>
<td>$7.91E-1$</td>
<td>$3.63E1$</td>
<td>$5.51E1$</td>
<td>$3.98E1$</td>
<td>$1.42E1$</td>
<td>$2.16E0$</td>
<td>$2.15E2$</td>
<td>$2.07E-2$</td>
<td>$3.09E-3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>10</td>
<td>DQLSP</td>
<td>$1.29E-1$</td>
<td>$3.24E1$</td>
<td>$1.12E-1$</td>
<td>$1.94E-1$</td>
<td>$2.05E-2$</td>
<td>$1.77E-4$</td>
<td>$1.99E-5$</td>
<td>$3.64E-6$</td>
<td>$2.33E-7$</td>
<td>$6.49E-10$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>DCSP</td>
<td>$3.34E-1$</td>
<td>$9.34E0$</td>
<td>$3.25E0$</td>
<td>$1.58E-1$</td>
<td>$1.59E-2$</td>
<td>$2.31E-3$</td>
<td>$2.03E-5$</td>
<td>$2.07E-7$</td>
<td>$1.87E8$</td>
<td></td>
<td>$2.02E-10$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>12</td>
<td>DQLSP</td>
<td>$1.06E0$</td>
<td>$7.29E1$</td>
<td>$1.37E1$</td>
<td>$3.25E1$</td>
<td>$1.25E2$</td>
<td>$1.36E3$</td>
<td>$3.95E5$</td>
<td>$3.76E7$</td>
<td>$7.89E8$</td>
<td>$3.94E-9$</td>
<td>$4.79E40$</td>
<td>$3.09E-11$</td>
<td>$5.83E-13$</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>DCSP</td>
<td>$2.22E-1$</td>
<td>$1.31E1$</td>
<td>$3.08E1$</td>
<td>$1.43E-1$</td>
<td>$2.32E-2$</td>
<td>$2.53E-3$</td>
<td>$6.73E-5$</td>
<td>$3.39E-6$</td>
<td>$2.33E-7$</td>
<td>$3.09E-9$</td>
<td>$2.54E40$</td>
<td>$2.52E-11$</td>
<td>$7.82E-13$</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>DQLSP</td>
<td>$2.06E-1$</td>
<td>$2.62E2$</td>
<td>$1.00E3$</td>
<td>$3.35E3$</td>
<td>$1.18E3$</td>
<td>$2.32E2$</td>
<td>$1.57E8$</td>
<td>$2.35E3$</td>
<td>$1.14E3$</td>
<td>$1.83E3$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>12</td>
<td>DCSP</td>
<td>$8.34E-1$</td>
<td>$1.45E1$</td>
<td>$1.00E3$</td>
<td>$7.84E3$</td>
<td>$1.35E4$</td>
<td>$9.77E3$</td>
<td>$3.21E4$</td>
<td>$3.69E3$</td>
<td>$1.44E4$</td>
<td>$4.39E4$</td>
<td>$1.19E4$</td>
<td>$2.22E4$</td>
<td>$5.46E3$</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>DQRS</td>
<td>$5.81E-1$</td>
<td>$2.10E1$</td>
<td>$1.37E2$</td>
<td>$1.03E3$</td>
<td>$4.47E2$</td>
<td>$4.66E3$</td>
<td>$4.58E3$</td>
<td>$3.54E3$</td>
<td>$2.91E3$</td>
<td>$1.27E3$</td>
<td>$1.62E3$</td>
<td>$1.05E3$</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>8</td>
<td>DQLSP</td>
<td>$3.90E-1$</td>
<td>$1.91E1$</td>
<td>$2.32E0$</td>
<td>$4.57E-1$</td>
<td>$1.36E-2$</td>
<td>$1.97E-3$</td>
<td>$7.63E-5$</td>
<td>$2.14E-6$</td>
<td>$2.63E-8$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>DCSP</td>
<td>$2.35E-1$</td>
<td>$1.73E0$</td>
<td>$1.28E0$</td>
<td>$2.92E-2$</td>
<td>$2.53E-3$</td>
<td>$5.18E-5$</td>
<td>$1.16E-6$</td>
<td>$3.75E-8$</td>
<td>$9.22E-9$</td>
<td>$2.56E-9$</td>
<td>$1.18E-10$</td>
<td>$3.22E-12$</td>
<td>$2.63E-13$</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>DQRS</td>
<td>$4.82E-1$</td>
<td>$1.19E1$</td>
<td>$1.82E0$</td>
<td>$2.21E-1$</td>
<td>$1.13E-2$</td>
<td>$2.43E-4$</td>
<td>$5.59E-6$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>DQLSP</td>
<td>$1.97E-1$</td>
<td>$1.67E0$</td>
<td>$1.93E-1$</td>
<td>$1.549E-2$</td>
<td>$2.59E-3$</td>
<td>$3.21E-4$</td>
<td>$3.14E-4$</td>
<td>$2.85E-6$</td>
<td>$7.86E-8$</td>
<td>$1.24E-8$</td>
<td>$2.99E-10$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>DCSP</td>
<td>$3.01E0$</td>
<td>$9.18E-1$</td>
<td>$1.04E0$</td>
<td>$1.10E-1$</td>
<td>$1.61E-3$</td>
<td>$1.37E-3$</td>
<td>$3.72E-5$</td>
<td>$1.31E-5$</td>
<td>$1.49E-7$</td>
<td>$2.89E-9$</td>
<td>$2.58E-13$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>12</td>
<td>DQRS</td>
<td>$5.99E-1$</td>
<td>$1.71E0$</td>
<td>$2.57E0$</td>
<td>$1.51E-1$</td>
<td>$1.50E-2$</td>
<td>$1.67E-3$</td>
<td>$1.77E-5$</td>
<td>$3.1E-6$</td>
<td>$4.9E-7$</td>
<td>$1.47E-8$</td>
<td>$2.33E-40$</td>
<td>$1.74E-41$</td>
<td>$4.6E-13$</td>
</tr>
</tbody>
</table>

* Negative co-efficient, E = Exponent
APPENDIX A.2

A.2.1 NUMERICAL SOLUTION OF 'TIMOSHENKO-EQUATION'

It has been reported in Section 5.1 that a transducer was fabricated for measurement of contact-force which was induced by the impact of a freely falling mass on the beam. It was also mentioned therein the calibration constant (i.e. the kilogram equivalence of 1 Volt of the oscilloscope trace) of the force-transducer was determined from comparing the corresponding impulse integrals of the experimental and analytical force-histories. The force-histories were obtained, for this purpose, for a perspex beam (overall size: 145.3 mm x 25.2 mm x 11.5 mm thick) simply supported over a span of 120 mm and impacted centrally by two different strikers of weight 14.02 gm and 17.53 gm. The strikers were released from a height of 176.4 mm. The contact velocities for the two strikers were estimated to be 1.626 and 1.460 m/s, respectively. The theoretical contact force-histories were determined from the numerical solution of 'Timoshenko-equation' for the above two cases. The details regarding the analytical solution will be presented in this appendix.

Timoshenko(26), accepting contact pressure(p) equation of Hertz, wrote instantaneous relationship between complete displacement(D) of the striker from the onset of collision, local deformation (α) and the central deflection (Y) of the simply supported rectangular beam impacted centrally. Effect of shear, rotatory inertia, internal damping, lateral contraction in beam and loss of energy due to collision were neglected in the
relationship given by him as follows:

\[ v_0 \int_0^T \frac{dt_1}{m} \left( \int_0^t Pdt \right) = H. P \frac{2}{3} \sum_{i=1, 3, 5, \ldots} 1 + \frac{1}{1 + \frac{2g}{\sqrt{\pi} a}} \int_0^T P \sin \frac{i^2 \xi^2 a^2}{2} a(t-t_1) \frac{i^2 \xi^2}{1} dt_1, \quad \ldots(\text{A.2.1}) \]

\[ \text{and } Y = \sum_{i=1, 3, 5, \ldots} 1 + \frac{1}{1 + \frac{2g}{\sqrt{\pi} a}} \int_0^T P \sin \frac{i^2 \xi^2 a^2}{2} a(t-t_1) \frac{i^2 \xi^2}{1} dt_1, \quad \ldots(\text{A.2.2}) \]

where,

\[ l = \text{span of the beam} \]
\[ a = (EIg/AY)^{1/2}, \quad \ldots \quad \ldots \quad \ldots(\text{A.2.3}) \]
\[ A = \text{cross-sectional area of the beam} \]
\[ T = \text{time measured from the beginning of the impact} \]
\[ t_1 = \text{time ranging from zero to } T \text{ with an increment of } dt_1 \]

For the above two cases mentioned (in the first para) the 'Timoshenko-Equation' was solved numerically by sub-dividing the interval of time from zero to \( T \) into small elements and then following a step by step method of integration as given by Timoshenko (26). Generally a good accuracy could be achieved if the small element of time \( t_1 \) was taken of the order of \( 1/180 \)th part of the time period of fundamental mode of vibration of the simply supported beam. The Timoshenko-Equation, for brevity, may be expressed as follows:
in which, \( X = p_n \) (reaction of the beam on the striker after nth interval of time), unknown,

\[ B = \frac{1}{H} \left[ \left( \frac{1}{3} \Delta_n \right) / (Elx^3) + \left( \frac{3\pi^2}{4m} \right) \right], \]

\[ C = \frac{1}{H} \left[ \left( \frac{1}{3} / Elx^3 \right) \sum_{j=1}^{n-1} \left( \Delta_{n-j+1} \cdot p_j + \left( \frac{1}{3} / Elx^3 \right) \Delta_{n-j+1} \cdot p_j \right) \right. \]

\[ + \left. \left( \frac{3\pi^2}{4m} \right) \cdot \Delta_{n-1} \right] \]

\[ \text{and} \]

\[ \Delta_{n-j+1} = \sum_{n-j}^{\infty} - \sum_{n-j+1}^{\infty} \left[ \sum_{i=1,3,5,\ldots}^{\infty} \frac{\cos \left( \frac{1}{2} \pi \frac{(n-j)}{i^4} \right)}{i^4} - \sum_{i=1,3,5,\ldots}^{\infty} \frac{\cos \left( \frac{1}{2} \pi \frac{(n-j+1)}{i^4} \right)}{i^4} \right] \]

For the present study Equation (A.2.4) was solved by an iterative procedure. In the solution an imaginary value of \( X \) was physically interpreted as the case of no contact between the striker and the beam.

After determining the contact force-history, \( p_n (p_n = X) \), the histories of central deflection \( (Y) \) of the beam and displace-
ment (D) of the striker were determined from the following relationships:

\[ d_n = d_{n-1} + v_{n-1} - \left(3t^2/4m\right)\left(p_n + p_{n-1}\right), \quad \ldots \quad (A.2.6) \]

\[ v_n = v_{n-1} - \left(t/2m\right)\left(p_n + p_{n-1}\right), \quad \ldots \quad (A.2.7) \]

\[ \alpha_n = H \cdot (p_n)^{2/3}, \quad \ldots \quad (A.2.8) \]

And

\[ \gamma_n = \left(1^2/L^4\right) \sum_{j=1}^{n} \left(\Delta_{n-j+1}\right) + \sum_{j=2}^{n} \left(\Delta_{n-j+1}\right) \cdot p_{j-1} \quad (A.2.9) \]

Where, \( \alpha_n \) = contact deformation after nth interval of time,

\[ \gamma_n = \text{central deflection of the beam after nth interval of time} \]

By considering deformation at the contact point Timoshenko et al. (27) proved that Hertz's coefficient (H) appearing in Equation (A.2.4), could be evaluated using the following equation:

\[ H = \sqrt{\frac{3}{16} \left(9\pi^2/16\right) \left(H_1 + H_2\right)^2 (1/R_1 + 1/R_2)}, \quad \ldots \quad (A.2.10) \]

\[ H_1 = \frac{(1-\nu_1^2)}{J_1 \cdot E_1} = \text{constant for the material of striker tip} \]

\[ H_2 = \frac{(1-\nu_2^2)}{J_2 \cdot E_2} = \text{constant for the material of beam} \]

in which, \( R_1, R_2 \) = Radii of hemispherical striker-tip, top edge of the beam.
For the numerical solution of 'Timoshenko-Equation' (A.2.4) a Fortan programme used is presented below:

```fortran
DIMENSION DEL(180),P(180)
OPEN(UNIT=2,DEVICE='DSK',FILE='IMPACT DAT',FORM='INPUT')
OPEN(UNIT=3,DEVICE='DSK',FILE='NUM SOL.DAT',FORM='OUTPUT')
 FORMAT(52X,'IMPACT///')
 PRINT 101
 WRITE(3,101)
 READ(2,8,END=1000)(DEL(I),I=1,90)
 READ(2,8,END=1000)(DEL(I),I=91,180)
 READ(2,7,END=1000)HC,BS,YB,SM,PI,TOW,BM,V
 FORMAT(9E15.6)
 FORMAT(90E15.6)
 WRITE(3,103)
 FORMAT(48X,'SPAN',9X,'M1',9X,'BM',9X,'V///')
 PRINT 103
 WRITE(3,104)BS,SM,BM,V
 FORMAT(44X,F8.4,2X,F8.4,2X,F8.4///)
 FORMAT(5X,'N',7X,'O',16X,'V',12X,'ALP',12X,'Y///')
 PRINT 104,N,D,V,ALP,Y,P(1),BB,AA,R
 WRITE(3,105)N,D,V,ALP,Y,P(1),BB,AA,R
 FORMAT(X,I5,8E15.6)
 K=2
 N=2
 SUM1=0.0
 SUM2=0.0
 DO 6 J=1,N-1
 N1=N-J+1
 TEM1=DEL(N1)*P(J)
 SUM2=SUM2+TEM1
 DO 5 J=3,N
 N3=N-J+1
 TEM2=DEL(N3)*P(J1)
 SUM1=SUM1+TEM2
 TEM3=3*S**3/(YB*SM*PI**4)
 TEM4=0.75*(TOW**2)/BM
 B=TEM3*DEL(1)+TEM4
 DO 20 J=1,N-1
 TEM1=DEL(N3)*P(J)
 SUM1=SUM1+TEM1
 DO 30 J=2,N-1
 TEM2=DEL(N3)*P(J1)
 SUM2=SUM2+TEM2
 A=TEM3*(SUM1+SUM2)+TEM4*(N-1)-V*TOW-D
```

AA=A/HC

GO TO 70

IF(P(N))27,27,28

P(N)=0.

D=D+V*TOW-TEM*4*(P(N)+P(N-1))

V=V-0.5*TOW*(P(N)+P(N-1))/BM

ALP=HC*P(N)**0.66667

Y=TEM3*(D*L(1)*P(N)+SUM1+SUM2)

PRINT 9,N,D,Y,ALP,Y,P(N),BB,AA,R

WRITE(3,9),N,D,Y,ALP,Y,P(N),BB,AA,R

N=N+1

IF(100-4)1000,25,25

IF(AA3)27,27,54

X1=SQRT(AA3)

R=(X1**2)-(-BB*X1-AA)**3

IF(R)72,73,74

X1=X1-0.1

R=X1**2-1.0*(-BB*X1-AA)**3

IF(R)72,73,75

P(N)=X1

GO TO (88,55)

STOP

END
A.2.2 (i) ELASTIC CONSTANTS OF BEAM AND STRIKER MATERIALS,

(ii) DATA AND

(iii) RESULTS.

(i) Elastic Constants of Beam and Striker Materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Radius of tip (cm)</th>
<th>Poisson's ratio, $\nu$</th>
<th>Young's Modulus, $E$ (kg/cm$^2$)</th>
<th>$H_1$ (for striker) or $H_2$ (for beam) (cm$^2$/gm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Striker, 14.02 gm</td>
<td>0.6750</td>
<td>0.314</td>
<td>0.680 x 10$^5$</td>
<td>4.22 x 10$^-9$</td>
</tr>
<tr>
<td>Striker, 17.53 gm</td>
<td>0.6775</td>
<td>0.314</td>
<td>0.680 x 10$^5$</td>
<td>4.22 x 10$^-9$</td>
</tr>
<tr>
<td>Beam of Peapex (PMMA)</td>
<td>infinity</td>
<td>0.37</td>
<td>0.452 x 10$^5$</td>
<td>6.08 x 10$^-9$</td>
</tr>
</tbody>
</table>

(ii) Data:

The quantity of the form $\Delta_{n-j+1}$ appearing in Equation(A.2.4) could be evaluated by suitable sub-routine programme in the main programme. Again term $\Delta_{n-j+1}$ is independent of beam parameters and can be evaluated for different values of $'j'$ for a particular chosen value of $'n'$. Timoshenko (26) evaluated and tabulated the values of $\Sigma_{n-j+1}$ and $\Delta_{n-j+1}$ (correct up to five places of decimals) for different values of $'j'$ assuming the value of $'n'$ to be 360. In the present study the values of $\Delta_{n-j+1}$ calculated by Timoshenko (originally) were accepted. Other data as mentioned in the 'Read' statements are presented below:
(iii) Results:

The results of the numerical solution of the 'Timoshenko-Equation' for the above case I and case II are presented graphically in Figs. A.2.1 and A.2.2, respectively. In Fig. A.2.1 results regarding time histories of contact force ($P_n$), central deflection ($Y$) at the lower fibre of the perspex beam are shown by 'firm-line' and 'chain-dot-line', respectively and the same for striker displacement ($d$) for the 14.02 gm striker is shown by 'dotted line'.

### Table 1

<table>
<thead>
<tr>
<th>Case I, striker 14.02 gm</th>
<th>Case II, striker 17.53gm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contact velocity=162.6 cm/sec</td>
<td>Contact velocity=146cm/sec</td>
</tr>
<tr>
<td>$HC$ (Hertz's constant)</td>
<td>$9.554520 \times 10^{-6}$</td>
</tr>
<tr>
<td>($cm/gm^2/3$)</td>
<td></td>
</tr>
<tr>
<td>$BS$ (Beam-span)</td>
<td>12.0</td>
</tr>
<tr>
<td>($cm$)</td>
<td></td>
</tr>
<tr>
<td>$W$ (Young's Modulus for Beam)</td>
<td>$0.452 \times 10^8$</td>
</tr>
<tr>
<td>($gm/cm^2$)</td>
<td></td>
</tr>
<tr>
<td>$SM$ (Second M.O.I. of beam)</td>
<td>1.549620</td>
</tr>
<tr>
<td>($cm^4$)</td>
<td></td>
</tr>
<tr>
<td>$PI$</td>
<td>3.141593</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$TOW$ (Sec)</td>
<td>$3.623153 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$BM$ (mass of striker)</td>
<td>$1.430612 \times 10^{-2}$</td>
</tr>
<tr>
<td>($gm\cdot sec^2/cm$)</td>
<td></td>
</tr>
<tr>
<td>$V$ (contact velocity)</td>
<td>162.6</td>
</tr>
<tr>
<td>($cm/sec$)</td>
<td></td>
</tr>
</tbody>
</table>
FIG. A.2.1 ANALYTICAL CONTACT FORCE BETWEEN STRIKER & BEAM (P), ANALYTICAL BEAM DEFLECTION (Y), & ANALYTICAL STRIKER DISPLACEMENT (D)

TIME IN MILLISECOND:
14.02 gm Striker.

Size (mm):
145.2 x 25.2 x 11.5, 1.626 m/s
FIG.A.2.2 ANALYTICAL CONTACT FORCE BETWEEN STRIKER & BEAM (P), ANALYTICAL BEAM DEFLECTION (Y) & ANALYTICAL STRIKER DISPLACEMENT (D).
From the Fig. A.2.1 it is clearly observed that two subimpacts occurred. The duration of the first subimpact occurred was 0.1812 ms (from the onset of collision) and the striker (14.02 gm) was separated from the beam face at this instant of time, showing clearly greater beam deflection than the striker displacement. Again, at time 0.3880 ms (from the onset of collision) the striker came in contact with the beam. Thereby the second subimpact occurred in the beam. The second subimpact was effective for a duration of 0.3880 ms to 0.5470 ms and thereafter the striker moved upward from the beam face signifying the end of duration of impact.

Similarly time-histories (for the same beam struck by 17.53 gm striker) of contact force, central beam deflection (Y) of lower fibre and striker displacement (D) are shown (Fig. A.2.2) by 'firm-line', 'chain-dot-line' and 'dotted line', respectively. It is also clear from this figure that there were two sub-impacts occurred. The first subimpact existed for 0.1992 ms (from the onset of collision), signifying separation of the striker from the beam from this instant of time. And at time 0.3440 ms the striker, again, came in contact with the beam producing second subimpact which existed upto 0.5390 ms. And thereafter the striker moved upward from the beam face signifying the end of duration of impact.