CHAPTER-5
DETERMINATION OF OPTIMAL MACHINING PARAMETERS
USING GOAL – PROGRAMMING TECHNIQUE

5.1 OBJECTIVE
Machining economics has been analyzed frequently as an effort to provide optimal machining condition from the standpoint of the minimum-cost criterion, the maximum-production-rate criterion, and the maximum profit-rate criterion. These objectives were used separately to find the optimal machining conditions, particularly optimal machining speed. There are some situations in which some of these objectives may compete with each other. Added to this there are sometimes other constraints that are set by the working condition. The purpose of this chapter is to use multiple objectives optimization technique which balances a set of conflicting goals and stays within such restrictions.

5.2 OBJECTIVES AND CONSTRAINTS
In this chapter metal removal rate and tool life for Raghu industry in Hyderabad, are to be considered as the conflicting objective functions in the goal programming model defined as follows:

Metal Removal Rate Objective
Metal removal rate is a measurement of how fast metal is removed from a component; it can be computed by multiplying the cross-sectional area of the chip by the travel speed of the tool along the length of the component. Since metal removal rate in turning can be expressed in the form:

\[ \text{Metal Removal Rate (MRR)} = p.D.d.f.N \]

Where \( D \) = outer diameter, \( d \) = depth of cut, \( f \) = feed rate (ipm), and \( N \) is the spindle speed which is given by

\[ N = \frac{12v}{p.D} \]
Hence material removal rate becomes

\[ MRR = 12dxf \]

Where \( d \) = depth of cut (in.), \( f \) = feed rate (ipm), and \( v \) = cutting speed (fpm).

**Tool Life Objective**

Most studies have used Taylor’s tool life equation to determine the optimal cutting conditions. The generalized Taylor’s tool life (TL) equation as a function of cutting speed and feed rate for a given depth of cut can be expressed as:

\[ TL = S/[v^n (f)^m] \]

Where \( S, n \) and \( m \) are constants. Notice that MRR objective is directly proportional to feed and speed, while the TL objective is inversely proportional to feed and speed. If the feed rate and cutting speed are decreased, the MRR is reduced and the tool life is prolonged. Thus these two objectives are conflicting and hence one must find a balance between the metal removal rate and tool life. In other words, the optimal solution must include some compromise between these objectives.

**Constraints**

There are sometimes constraints that are usually set by the working condition in including maximum metal removal speed, feed rate, horsepower etc. In general, the raw material for a component is usually constrained by design and/or application, tooling and the cutting conditions. Hence they are fixed and there is a little or no control over these items. The following constraints will be used in this chapter, but the other constraints (surface finish requirement) can be easily added as required.

1. **Feed Rate**

Determining appropriate feed rate depends on many factors including tooling, cutting force restriction, or surface finish requirements, but it is quite natural to assume that it has two ranges as follows:

\[ f_{\text{min}} \leq f \leq f_{\text{max}} \]
2. Cutting Speed

It is also quite natural to assume that the cutting speed can be expressed between these two ranges as follows:

\[ v_{min} \leq v \leq v_{max} \]

or

\[ \frac{\pi DN_{min}}{12} \leq v \leq \frac{\pi DN_{max}}{12} \]

where \( N_{min} \) and \( N_{max} \) are the minimum and maximum spindle speed on the machine.

3. Horsepower

In general the machining resistance is given by the power function of machining speed and feed rate. This must not be greater than the motor power of the machine tool. For a given depth of cut in inches, the horsepower available at the spindle can be expressed as:

\[ vf^\alpha \leq K \]

where \( \alpha \) and \( K \) are fixed and constant.

5.3 GOAL PROGRAMMING

Goal programming is a technique designed to solve problems involving multiple objectives within the general framework of linear programming. This technique requires user to select a set of goals that have to be achieved for the various objectives. It can be viewed as an approach that strives toward several objectives simultaneously. The basic approach of this technique is accomplished in three stages as follows:

1. a specific numeric goal for each objective and its ranking order are established
2. an objective function for each objective is formulated
3. a solution that minimizes the sum of deviations of these objective functions from their respective goals is sought.

This technique has got a lot of attention in optimization literature and has been shown to have many practical applications. Examples of related applications are resource allocation in higher education, production planning and inventory control, cell formation in flexible manufacturing systems, reducing project completion time in project management, choosing the best investment portfolio in financial management, and
production smoothing under just-in-time manufacturing environment.

5.3.1 Model Formulation

To follow the first step of the goal programming approach, we need to determine a specific numeric goal for each objective and its ranking order. Suppose the primary goal is for the MRR which must be greater than $MRR_i$ cubic inches/min, and the secondary goal is for the TL that must have at least $TL_i$ minutes, or

$$12vfd \geq MRR_i$$
$$\frac{S}{v^{1/n} f^{1/m}} \geq TL_i$$

In goal programming context, these ranked goals are called preemptive priorities, because the decision maker is not willing to sacrifice any amount of achievement of the higher priority goal for the lower priority goal. To continue the formulation of the model, a goal equation must be developed for each goal. The goal equation for the first goal is written as:

$$12vfd + d^-_1 - d^+_1 = MRR_i$$

The two new variables, $d^-_1$ and $d^+_1$ are called deviational variables. Both two variables are used to represent the amount of metal removal less than and higher than the $MRR_i$, respectively. The superscript (+) and (-) are used to indicate the positive or negative deviation from the target value. Similarly, the goal equation for the secondary goal can be formulated as:

$$\frac{S}{v^{1/n} f^{1/m}} + d^-_2 - d^+_2 \geq TL_i$$

To complete the formulation of the model, the objective function must be constructed. This is accomplished by reflecting the goal constraints in our objective function. This objective function minimizes the sum of deviations of each individual objective function from their respective goals. Since we do not want the amount of metal removal to be to less than $MRR_i$, we let the objective function minimize the $d^-_1$. This means that the model tends to minimize the value of $d^-_1$ as the first step, before addressing any other
This can be done by creating a new form of objective function to minimize $P_1d_1^-$ where $P_1$ designates the first-ranked priority goal. As a result, this objective function tends to make $P_1d_1^-$ equal to zero or the minimum possible amount.

Repeating the same steps, we now formulate our second-ranked priority goal which states the tool life that must have at least TL1 minutes. Since underachievement is represented by $d_2^-$, the objective must also minimize this deviational variable. Therefore, the objective function related to this goal is to minimize $P_2d_2^-$, where $P_2$ designates the second-ranked priority goal. In solving this model, this second-ranked goal will not be achieved until goal one has been considered. Hence, the goal programming objective function is:

$$\text{Min } P_1d_1^- + P_2d_2^-$$

Where $P_1$ and $P_2$ are not numerical weights on the deviation variables, but simply designates that $P_1 >>>> P_2$; that is $P_1$ is infinitely larger than $P_2$.

Since goal programming handles multiple objectives within the general framework of linear programming, it is required to express the problem as a linear goal programming model.

Thus the problem objectives and constraints can be written as:

$$\log v + \log f \geq \log(MRR_1/12d)$$

$$\frac{1}{n}\log v + \frac{1}{n}\log f \geq \log\left(\frac{S}{TL_1}\right)$$

5.3.2 Complete Goal Programming Model

We now write the complete goal programming model for the problem as follows:

$$\text{Min } P_1d_1^- + P_2d_2^-$$
Subject to:

Metal removal goal \[ \log v + \log f + d_1^- - d_1^+ = \log \left( \frac{\text{MRR}_1}{12d} \right) \]

Tool life goal \[ \frac{1}{n} \log v + \frac{1}{n} \log f + d_2^- - d_2^+ = \log \left( \frac{S}{TL_1} \right) \]

Feed rate constraint \[ \log f_{\text{min}} \leq \log f \leq \log f_{\text{max}} \]

Cutting speed constraint \[ \log v_{\text{min}} \leq \log v \leq \log v_{\text{max}} \]

Horsepower constraint \[ \log v + \alpha \log f \leq \log K \]

Nonnegativity constraint \[ \log v, \log f, d_1^-, d_1^+, d_2^-, d_2^+ \geq 0 \]

5.4 RESULT AND ANALYSIS

The solution will be obtained by using QM for WINDOWS package may be interpreted as follows:

<table>
<thead>
<tr>
<th>Priorities</th>
<th>Goal Attainment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>Achieved</td>
</tr>
<tr>
<td>$P_2$</td>
<td>Achieved</td>
</tr>
</tbody>
</table>

\[ d_1^- = d_1^+ = 0, \quad d_2^- = 4, \quad d_2^+ = 0 \]

The only difference between a linear programming problem and a goal programming problem is in solving sequence problems. This means that the goal programming involves solving a sequence of linear programming models with different objective functions; $P_1$ goals are considered first, $P_2$ goals second, $P_3$ goals third, and so on. The number of linear programs that must be solved in sequence is determined by the number of priority levels. Once a solution for the first formulation at the highest goal priority level is achieved, the value of the deviational variable, that is the objective, is added to the model as a constraint, and the second-priority deviational variable becomes the new objective. The sequential process continues until all the priorities are implemented or until there is no better solution.