CHAPTER-4
CAPITAL BUDGETING IN THE PUBLIC SECTOR:
A ZERO-ONE GOAL PROGRAMMING APPROACH

4.1 OBJECTIVE
This chapter presents a capital budgeting model to deal with a wide range of public sector areas. Generalized, the model could aid in capital budgeting in A.P. state government universities, penal systems, and water resource planning (MCH). The model's objective function is not limited to single goals measured in commensurable units, it can be handle multiple conflicting goals measured in incommensurable units. While the example demonstrates local government capital budgeting, the model is flexible enough to be applied throughout the public sector.

4.2 DATA OF THE PROBLEM
The example shows the capital budgeting in A.P. state government universities, penal systems, and water resources planning (MCH) examining 19 investment proposals. Total investment in the projects is limited both by a ceiling on total expenditures and by specific allowable increases in annual operating expenses. Major goals include law enforcement, fire protection, community intellectual development, housing, recreation, clean streets, and satisfaction of political and social pressures. Costs and technical coefficients associated with each goal are presented in the Table-4.1.
<table>
<thead>
<tr>
<th>Investment Proposal</th>
<th>Cost</th>
<th>Incremental Annual Operating Expenses</th>
<th>Estimated Reduction in Major Crimes/1000pop</th>
<th>Estimated Reduction in Annual Juvenile Arrests/1000Juveniles</th>
<th>Estimated Reduction in Major Fires Annually/1000 people</th>
<th>Estimated Reduction in Rs value of Property Lost to Fires Annually/1000 people</th>
<th>No. of Books Loaned Out/10 People</th>
<th>Reduction % Dropout prior to H.S. Graduation</th>
<th>Increase in Persons Expressing Satisfaction with Street Cleanliness Appearances -5% Sample Survey</th>
<th>No. of Days Citizens Use of Recreational Facilities</th>
<th>Low Cost Housing Units Provided</th>
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<tbody>
<tr>
<td>X1 New High school</td>
<td>Rs 45,00,000</td>
<td>Rs 10,00,000</td>
<td>0.3</td>
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<td>X6 Park in North West section</td>
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<td>X10 Youth Recreational Center</td>
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<td>X11 Ice Rink</td>
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<td>3,00,000</td>
<td>0.5</td>
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<td>6.3</td>
<td>12,000</td>
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<td>X14 Subsidized Urban Renewal Low cost Housing</td>
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<td>X16 Additional Firestation &amp; Equipment</td>
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<td>14.6</td>
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<td>X18 Trash Truck Prototype A</td>
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<td>X19 Trash Truck Prototype B</td>
<td>3,50,000</td>
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</tbody>
</table>
4.3 INTEGER GOAL PROGRAMMING MODEL

Goal programming is a mathematical programming approach in which the objective function is expressed in terms of deviations from the stated goals. The deviation variables are scalar weighted and/or ordinally ranked. Goal programming differs from linear programming in that it does not require translating multiple and conflicting goals (which may be measured in incommensurable units) into a unidimensional objective criterion. It allows these goals to be measured in unlike units and to be treated in both a sequential and/or simultaneous manner. It is a realistic tool that allows the public administrator to represent the particular community’s policies and desires in incommensurable units and to solve problems using hierarchical optimization. Profit or utility measurements simply do not apply in the public sector. Arbitrary conversion of unlike units into a unidimensional objective function yields meaningless results. It is impossible, for example, to determine Rupees benefits associated with the purchase of fire fighting machinery while at the same time considering benefits associated with saved lives, protection of property, and reduced suffering. Goal programming solves this problem through hierarchical optimization procedure in which weights are implicitly assigned by creating preemptive priorities. This avoids the direct conversion or assignment of weights to goals which is necessary in linear programming.

4.3.1 System Constraints and Goals Constraints as developed as Follows:

In addition to the indivisibility requirements for the decision variables, several of the projects \( (X_1, X_2, X_3, X_4, X_7, X_8, X_{16}, X_{17}, X_{18}, X_{19}) \) are mutually exclusive, as they perform similar functions. Also, project \( X_{11} \) is not possible financially unless project \( X_{10} \) is also accepted. This leads to the following set of system constraints:

\[
X_1 + X_2 + d_1^- - d_1^+ = 1.0 \tag{4.1.a}
\]

\[
X_3 + X_4 + d_2^- - d_2^+ = 1.0 \tag{4.1.b}
\]
\[ X_7 + X_8 + d_3^- - d_3^+ = 1.0 \]  
\[ X_{16} + X_{17} + d_4^- - d_4^+ = 1.0 \]  
\[ X_{18} + X_{19} + d_5^- - d_5^+ = 1.0 \]  
\[ X_{11} - X_{10} + d_6^- - d_6^+ = 0.0 \]

where \( X_i \) = projects 'i' expressed as a zero-one value, \( d_g^+ \) = the positive deviational or slack variable from the \( g \)th goal or constraint; and \( d_g^- \) = the negative deviational or slack variable from the \( g \)th goal or constraint. The deviational variables are introduced to change the goal equations into equalities.

**Capital Budget Goal**

Public distaste for fiscal deficits and tax increases dictates a limit on funds available to new projects. Satisfying the budget ceiling is given the top priority after the system constraints. Satisfying all other goals involves trade-offs because of limited funds. Assuming a maximum total limit on expenditures of Rs7,50,00,000 the capital budgeting goal becomes

\[
\text{Rs}45,00,000 X_1 + \text{Rs}15,00,000 X_2 + \text{Rs}25,00,000 X_3 + \text{Rs}7,00,000 X_4 \\
+ \text{Rs}4,00,000 X_5 + \text{Rs}3,00,000 X_6 + \text{Rs}20,00,000 X_7 + \text{Rs}9,00,000 X_8 \\
+ \text{Rs}3,00,000 X_9 + \text{Rs}15,00,000 X_{10} + \text{Rs}5,00,000 X_{11} + \text{Rs}6,00,000 X_{12} \\
+ \text{Rs}30,00,000 X_{13} + \text{Rs}20,00,000 X_{14} + \text{Rs}15,00,000 X_{15} + \text{Rs}25,00,000 X_{16} + \\
\text{Rs}15,00,000 X_{17} + \text{Rs}2,50,000 X_{18} + \text{Rs}3,50,000 X_{19} + d_7^- - d_7^+ = \text{Rs}7,50,00,000 \ldots (4.2)
\]

**Annual Operating Expenses Goal**

In addition to satisfying an absolute capital budgeting constraint, the public planners wish to limit the increase in annual operating expenses caused by new project acceptance to Rs 2,20,00,000. This goal can be formulated as follows:

\[ T10192 \]
\[ Rs1,00,000X_1 + Rs5,00,000X_2 + Rs6,00,000X_3 + Rs3,00,000X_4 + Rs1,00,000X_5 + Rs1,00,000X_6 + Rs5,00,000X_7 + Rs3,00,000X_8 + Rs2,50,000X_9 + Rs3,50,000X_{10} + Rs1,50,000X_{11} + Rs1,50,000X_{12} + Rs3,00,000X_{13} + Rs3,00,000X_{14} + Rs3,00,000X_{15} + Rs6,00,000X_{16} + Rs4,50,000X_{17} + Rs1,00,000X_{18} + Rs1,00,000X_{19} + d_8^- - d_4^+ = Rs2,20,00,000..............(4.3) \]

**Political-Social Goal**

To satisfy political and social forces, city planners feel it necessary to accept a minimum of three projects among projects \(X_1, X_2, X_3, X_4, X_{15}, X_{16}\) and \(X_{17}\). These involve police, fire, and school improvements and subsidized housing for senior citizens. This goal can be written as follows.

\[ X_1 + X_2 + X_3 + X_4 + X_{15} + X_{16} + X_{17} + d_9^- - d_9^+ = 3...........................................(4.4) \]

**Law Enforcement Goal.**

In order to quantify and still capture the essence of public sector goals, it may be appropriate to formulate multiple goals in order to properly represent one objective. It may be appropriate to quantify the objective of law enforcement as both a desired reduction in major crimes and a reduction in juvenile delinquency as measured by juvenile arrests. Therefore, if the community's specific goal is a reduction of 2.6 major crimes per 1,000 inhabitants per year in addition to a reduction of 12.0 juvenile arrests per 1,000 juveniles per year, these goals can be formulated as follows:

\[ 0.3X_1 + 0.1X_2 + 6.4X_3 + 2.0X_4 + 0.1X_5 + 0.1X_6 + 0.3X_{10} + 0.1X_{11} + 0.1X_{12} + 0.5X_{13} + 0.4X_{14} + 0.2X_{15} + d_{10}^- - d_{10}^+ = 2.6...........................................(4.5.a) \]

and

\[ 10.3X_1 + 4.1X_2 + 1.2X_3 + 0.6X_4 + 1.6X_5 + 1.4X_6 + 0.1X_7 + 0.1X_8 + 0.1X_9 + 8.5X_{10} + 3.9X_{11} + 4.2X_{12} + 1.2X_{13} + 1.0X_{14} + 0.3X_{15} + d_{11}^- - d_{11}^+ = 12.0...........................................(4.5.b) \]
Fire Protection Goal

Fire protection is another major municipal goal. Specifically, city officials wish to reduce fires by 11.0 major fires annually per 1,000 inhabitants, in addition to cutting the annual Rupee value of property lost to fires per 1,000 inhabitants by Rs 30,00,000. Again, because of the difficulty in adequately quantifying this goal, two goals are used as surrogates for the fire protection goal, illustrating the flexibility of the goal programming approach.

This goal can be formulated as:

\[ 6.3X_{13} + 5.3X_{14} + 2.4X_{15} + 14.6X_{16} + 9.4X_{17} + d_{12}^- - d_{12}^+ = 11.0 \] \( (4.6.a) \)

\[ \text{Rs}12,000X_{13} + \text{Rs}10,000X_{14} + \text{Rs}5,200X_{15} + \text{Rs}36,000X_{16} + \text{Rs}20,000X_{17} + d_{13}^- - d_{13}^+ = \text{Rs}30,00,000 \] \( (4.6.b) \)

Recreation Facilities Goals

The city planners also wish to increase the community’s recreational facilities by 75,000 citizen-day use units. This would involve development of recreational facilities such as parks, ice rinks, and swimming pools, renovation of the old Municipal High school and its athletic facilities or building of a new model high school, as well as urban renewal projects that include recreational facilities. This goal can be quantified as follows:

\[ 15,000X_1 + 12,000X_2 + 20,000X_5 + 15,000X_6 + 75,000X_{10} + 25,000X_{11} + 25,000X_{12} + 5,000X_{13} + 3,000X_{14} + 2,000X_{15} + d_{14}^- - d_{14}^+ = 75,000 \] \( (4.7) \)

Community Intellectual Development Goal

The city also wishes to provide for community intellectual development by making possible an increase of 1.25 in the number of library books loaned per ten inhabitants, in addition to reducing the high school dropout ratio by 1.25%. Again two incommensurable goals will be used as a surrogate for a single goal that is difficult to quantify. This goal is quantified as
\[0.50X_1 + 0.30X_2 + 3.75X_7 + 1.90X_8 + 0.75X_9 + d_{15}^- - d_{15}^+ = 1.25\] 

\[\text{(4.8.a)}\]

and

\[3.90X_1 + 1.05X_2 + 0.20X_7 + 0.10X_8 + 0.05X_9 + d_{16}^- - d_{16}^+ = 1.25\] 

\[\text{(4.8.b)}\]

**Public Housing Goal**

City planners wish to increase by 2,000 the number of low-cost housing units provided by the municipality:

\[2,000X_{13} + 1.300X_{14} + 700X_{15} + d_{17}^- - d_{17}^+ = 2,000\] 

\[\text{(4.9)}\]

These units would replace current substandard housing by creating planned neighborhoods with small parks, wide well lighted streets, and easy access to public transportation.

**City Cleanliness Goal**

The city also wishes to achieve an increase of 7.5% in the number of people expressing satisfaction with the cleanliness and appearance of the city streets, based upon a 5% random sample survey. This can be represented as follows:

\[0.1X_1 + 0.1X_3 + 1.0X_5 + 1.0X_6 + 0.1X_7 + 0.2X_{10} + 0.1X_{11} + 3.1X_{13}\]

\[+ 2.6X_{14} + 1.4X_{15} + 6.9X_{18} + 4.3X_{19} + d_{18}^- - d_{18}^+ = 7.5\] 

\[\text{(4.10)}\]

To satisfy this goal it would be necessary to accept projects dealing with urban renewal and more efficient trash collection.

**4.3.2 Objective Function**

Priorities were established for each goal based on desires of residents. The system constraints involving mutually exclusive and contingent projects are placed at the first priority level. The capital budgeting goal is designated the second priority goal, the annual operating expenses goal priority three, and the political-social goal priority four. The law enforcement goal is established as the fifth priority, within this priority level the
crime reduction and juvenile arrests goals are weighted equally. The sixth priority is the fire protection goals of reducing major fires and property losses. As both of these goals are surrogates for the fire protection goal, they are both placed at the same priority level with the deviation variable associated with the satisfaction of the major fire goal weighted by a factor of 2,000 since that is the average Rupee value of property lost in a major fire. The recreation facilities are placed at the seventh priority, while the community intellectual development goal is at the eighth priority, with the library book usage and high school dropout reduction goals equally weighted. The ninth priority level is assigned to the public housing goal and the tenth to the city cleanliness goal. The priority designations resulted in the following function:

Minimize $Z = P_1 \sum_{i=1}^{6} d_i^+ + P_2 d_7^+ + P_3 d_8^+ + P_4 d_9^- + P_5 (d_{10}^- + d_{11}^-) + P_6 (2000 d_{12}^- + d_{13}^-) + P_7 d_{14}^- + P_8 (d_{15}^- + d_{16}^-) + P_9 d_{17}^- + P_{10} d_{18}^-$.
4.4 RESULT AND ANALYSIS

The solution will be obtained by using QM for WINDOWS package may interpreted as follows:

Table-4.2

The zero-one goal programming model yielded the following results:

\[ X_2 = 1.0 \quad X_4 = 1.0 \quad X_5 = 1.0 \quad X_6 = 1.0 \quad X_9 = 1.0 \]

\[ X_{12} = 1.0 \quad X_{14} = 1.0 \quad X_{17} = 1.0 \quad \text{All other } X_i = 0.0 \]

The first seven goals (P_i through P_7) are completely attained, while Priorities 8 through 10 were attained partially:

Priority 1: Satisfied. The system constraints were satisfied.

Priority 2: Satisfied. The Rs7,50,00,000 budget goal was met, with proposed budget totaling Rs7,30,00,000.

Priority 3: Satisfied. The annual operating expense goal of RS 2,20,00,000 was met, with a proposed increase in operating expenses of Rs 2,15,00,000.

Priority 4: Satisfied. The political-social goal of accepting at least three projects from the given subset of projects was met exactly.

Priority 5: Satisfied. Projects chosen are expected to produce a reduction of 2.8 major crimes and 13.0 juvenile arrests per 1,000 inhabitants.

Priority 6: Satisfied. Projects chosen are expected to produce a reduction of 14.7 major fires and Rs 30,00,000 in property losses per 1,000 inhabitants per year.

Priority 7: Satisfied. The recreational facilities goal of an increase in the community's recreational facilities by 50,000 citizen-day use units was exactly achieved.

Priority 8: Underachieved. The desired increase of 1.25 in the number of library books loaned per ten inhabitants fell short by 0.20, and the desired reduction in the percentage of high school dropouts of 1.25% fell short by 0.15%.

Priority 9: Underachieved. The desired increase of 2,000 low cost housing units fell short by 700 units.

Priority 10: Underachieved. The desired increase of 7.5% in the number of people expressing satisfaction with the cleanliness, appearance, and condition of the city streets fell short by 2.8%.