CHAPTER-8

PROPERTY ASSESSMENT THROUGH AN IMPRECISE
GOAL PROGRAMMING MODEL

8.1 OBJECTIVE

In this chapter, we propose a model to estimate the market value of residential properties where the selling price is an imprecise estimate of the real value, and when the number of observed residential transactions is small. Thus, the price paid for a property may lie between a lower and an upper limit. The real estate appraiser's experience, judgement and knowledge will be introduced explicitly in the proposed estimation model through a learning process by using the satisfaction functions concept. This model was utilized to estimate the market value of a number of properties in a residential area of the town of Gajuvaka situated in the Regional Municipality of Vishakapatnam, for taxation purposes.

8.2 DATA OF THE PROBLEM

This study was carried out to estimate the assessed value (market value) for taxation purposes of a group of properties in a residential area of the town of Gajuvaka situated in the Regional Municipality of Vishakapatnam. The amount of property tax is determined on the basis of the assessed value of the property. The most common methods utilized to assess the market value of properties are given below:

(i) the selling price approach, which consists of taking into account the transactions of similar type for neighbouring properties;

(ii) the construction cost approach, which is based on the cost of new construction or the replacement cost of the property;

(iii) the revenue approach, which is based on an average annual net revenue generated by the property.

These approaches help to set the most probable market value of a real estate. The selling
price approach is more popular than the other methods. In utilizing this approach, the appraiser will analyse a number of real estate transactions. These transactions are often executed in varying situations such as regular sale, non arms-length sale (relatives, friends, business associate or others), sale involving a public organization, sale involving a financial institution (for example, a repossession), sale involving joint ownership, sale as a result of the owner's transfer, sales after a major renovation, or direct purchase from an agent.

The situation in which a property is sold has an impact on the selling price. In many cases, the price paid does not necessarily reflect the market value of the real estate being sold. For example, the selling price of a repossessed home is often less than its market value. In this case, the mortgage holder is more concerned with a quick recovery of his investment and does not take the time needed to sell the property in a better condition in order to obtain a higher price. In such a situation, the appraiser is aware of the fact that the selling price is adversely affected and that the "real" market value of the property could be considerably different than the paid price. In other words, the price of the real estate transaction does not always accurately reflect the market value of the property.

Setting aside the circumstances of the sale, the report on the strengths of the real estate market also plays an important role in determining the selling price of a property. In fact these reports are directly related to the interactions which could exist between the buyer and the seller. The paid price is the result of a negotiation, sometimes lengthy and exhausting, where the buyer and the seller are individually seeking to meet their own interests. In this situation, the qualifications, the experience and the ability to convince the other party determine the price. In other words, the market forces have some impact on the final price of a property. Appraisers agree with the fact that the list price of real estates do not always reflect the market value. Taxpayers will often challenge the high level of property taxes claiming that their property is over-evaluated.
In Table 8.1, we present data regarding a number of transactions carried out in the Vishakapatnam municipality. This data has been provided by the Real Estate Appraisal Company of Saichitra, whose specialty is, among other things, real estate appraisals. The data in Table 8.1 were compiled for twenty properties sold during the period of January to December, 2006.

In Table 8.1, the columns numbered from 1 to 10 have the following definitions:

1. Living space in square meters,
2. The age of the house in years,
3. Basement living space in square meters,
4. The number of additional washrooms,
5. Carport area in square meters,
6. Size of shed in square meters,
7. Size of attached garage in square meters,
8. Selling price \( y_i \) excluding the value of the land (in Rupees) (The observed selling price of the properties was given by the appraiser and the appraiser has excluded from the price of each property the value of the land)
9. Lower limit \( y_i^l \) associated with observation \( i \).
10. Upper limit \( y_i^u \) associated with observation \( i \).

The observed selling prices \( y_i \) presented in column (8) of Table 8.1 may be outside the intervals presented by the columns (9) & (10). Exceptionally in this study, all the values of column (8) are within the interval \([y_i^l, y_i^u] \) (for \( i = 1,2,\ldots,20 \)). Thus, the values of \( \zeta \), in program 2 are equal to the values of \( y_i \) [see column 8 of Table 8.1].
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8.3 MODEL DEVELOPMENT

In this chapter we will propose the use of the goal programming model (GP) with satisfaction functions as an estimation tool for valuation purposes. The GP model is a particular case of the distance function model which consists of minimizing the sum deviations between the obtained solutions and the fixed goals for the objectives under consideration. Charnes et al. [1986] and Sueyoshi [1986] have introduced the possibility of using the GP as a parametric estimation tool where the deviations or the distances between the observed values and the estimated values are to be minimized. The most popular methods of parametric estimation are the “least square method” (LS) and the “least absolute value method” (LAV). Formally, the LS method is based on the optimization of the following program:

$$\min_{\beta_o, \beta_j} \sum_{i=1}^{n} \left( y_i - \left( \beta_o + \sum_{j=1}^{m} x_{ij} \beta_j \right) \right)^2$$

where $\beta_o$ and $\beta_j$ ($j = 1, 2, \ldots, m$) are the parameters to be estimated and $y_i$ (for $i = 1, 2, \ldots, n$) are the observed values. The LAV method can be written as follows:

$$\min_{\beta_o, \beta_j} \sum_{i=1}^{n} \left| y_i - \left( \beta_o + \sum_{j=1}^{m} x_{ij} \beta_j \right) \right|$$

This second model corresponds to the GP model, which after the transformation proposed by Charnes et al. [1986] can be written as follows:

Program 1: Min. $Z = \sum_{i=1}^{n} \left( \delta_i^+ + \delta_i^- \right)$

Subject to $\beta_o + \sum_{j=1}^{m} x_{ij} \beta_j + \delta_i^- - \delta_i^+ = y_i$ (for $i = 1, 2, \ldots, n$),

$\beta_o$ and $\beta_j$ are free (unrestricted in sign), (for $j = 1, 2, \ldots, m$), $\delta_i^+, \delta_i^- \geq 0$, (for $i = 1, 2, \ldots, n$).
Aouni et al. [1997] and Kettani et al. [1998] indicated that the LAV method has some advantages compared to the LS method. In fact, contrary to the LS method, the LAV method is less affected by outlier values.

This estimation process generally includes the following steps:

(i) Identifying the factors (independent variables) that play a significant role in explaining the behaviour of the dependent variable, and

(ii) Specifying the general response function which enables the manager to put the behaviour of the dependent variable in terms of the identified factors to estimate the parameters of the response function.

This is done by a sample analysis that describes the observable behaviour of the variable under study. This model does not require any particular assumptions regarding the data generating process, such as normality requirement distribution and large sample sizes. On the flip side, this model does not permit any statistical analysis of the significance of its findings.

This chapter proposes the use of generalized penalty function forms for deviations by introducing the concept of “satisfaction functions”, developed by Martel and Aouni [1990] to determine the estimated values of parameters in the context where the observed values reflect the fuzziness of the real value of the property. In such a situation, the analyst needs to specify a lower limit and an upper limit for the selling price of each property. The formulations of the imprecise goal programming model developed by Martel and Aouni [1993, 1998], and Aouni [1998] will be utilized to integrate the appraiser’s judgement, knowledge, and experience to estimate the market value of a group of properties in a residential area of the town of Gajuvaka situated in the Regional Municipality of Vishakapatnam, for taxation purposes.

Real Estate Estimation with Imprecise Information

In this section we will use the formulation developed by Aouni et al. [1997] where they deal with the imprecision of the information in the estimation process. This formulation considers that the observed values $y_i$ (for $i = 1, 2, \ldots, n$) of the dependent variable are
within an interval. Real estate evaluation is one of many applications of the model where
the appraiser wish to estimate the market value of properties by using imprecise
information given by the paid price of the real estate. To deal with such a situation, the
appraiser will establish an interval within which the market value of the property may be
situated. The values $y_{i}^{'}$ & $y_{i}^{''}$ are respectively the lower and upper limits of the interval.
Thus, the values of the dependent variable (or the goals) are defined by the
interval $[y_{i}^{'} , y_{i}^{''}]$.
The aim of the estimation process is to determine the contribution of the explaining
variables $x_{ij}$ in order that the estimated value $\hat{y}_{i}$ falls, as much as possible, within the
interval $[y_{i}^{'} , y_{i}^{''}]$. We consider that when the $\hat{y}_{i}$ value is inside this interval, there is no
estimation error. If it is outside of the interval (i.e. $\hat{y}_{i}$ is smaller than $y^{'}_{i}$ or greater
than $y^{''}_{i}$), then the sum of the absolute distances between $\hat{y}_{i}$ and $y^{'}_{i}$ or $y^{''}_{i}$ are to be
minimized. To obtain the values of the estimation parameters, we will define, for each
observation (or a number of observations), a satisfaction function where the appraiser
expresses his/her tolerance regarding the distance $\delta_{i}$ between the observed $y_{i}$ and the
estimated $\hat{y}_{i}$ values. As developed by Martel and Aouni\cite{Martel and Aouni1990}, the general form of
the satisfaction functions is as presented in the following Figure8.1.
Figure 8.1: The Satisfaction Function

With this function, the appraiser will be completely satisfied when the deviations $\delta_i$ from the goals are located within the interval $[0, \alpha_{id}]$ (where $\alpha_{id}$ represents indifference threshold). There will be no penalty for deviations within the indifference interval and the satisfaction level will be at its maximal level 1. For deviations above $\alpha_{id}$, the appraiser satisfaction function decreases monotonously (but not necessarily linearly). The deviations that exceed the veto threshold $\alpha_{iv}$ are not acceptable. Within the interval $[\alpha_{id}, \alpha_{iv}]$, the appraiser will be dissatisfied (i.e. his/her satisfaction level is zero) but the solutions are not necessarily the same for the positive $\left(F_i^+(\cdot)\right)$ and negative $\left(F_i^-(-)\right)$ deviations.
The estimation model of Aouni et al. [1997] for imprecise variables is as follows:

Program 2: Max. $Z = \sum_{i=1}^{n} [W_i^- F_i^-(\delta_i^-) + W_i^+ F_i^+(\delta_i^+)]$

Subject to:

$\beta_0 + \sum_{j=1}^{m} x_{ij} \beta_j + \delta_i^- - \delta_i^+ = \zeta_i$ (for $i = 1, 2, \ldots, n$),

$\sum_{j=1}^{m} C_{ik} \beta_j \leq c_k$ (for $k = 1, 2, \ldots, K$; the system constraints, if necessary),

$\zeta_i \in [y_{il}^l, y_{il}^u]$ (for $i = 1, 2, \ldots, n$)

$0 \leq \delta_i^- \leq \alpha_{il}^-$ (for $i = 1, 2, \ldots, n$)

$0 \leq \delta_i^+ \leq \alpha_{il}^+$ (for $i = 1, 2, \ldots, n$)

$\beta_0$ and $\beta_j$ are unrestricted variables (for $j = 1, 2, \ldots, m$) (estimation parameters).

The goals $\zeta$, (the property's selling price) can be any value with in the interval $[y_i^l, y_i^u]$. In fact, if the observed value $y_i \in [y_i^l, y_i^u]$ then $\zeta_i = y_i$, but if $y_i$ is outside this interval then $\zeta_i$ will be equal to $y_i^l$ or $y_i^u$ depending on whether $y_i < y_i^l$ or $y_i > y_i^u$. Since there is no estimation error within the interval $[y_i^l, y_i^u]$, the indifference thresholds should be such that $\alpha_{il}^+ \geq y_i^u - \zeta_i$ and $\alpha_{il}^- \geq \zeta_i - y_i^l$ ($\alpha_{il}^+$ and $\alpha_{il}^-$ are the indifference thresholds associated to the positive and the negative deviations respectively). The attributes (explaining variables) are denoted by $x_{ij}$ (for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$) and the general response function is $\beta_0 + \sum_{j=1}^{m} x_{ij} \beta_j$. In this formulation, the coefficients $W_i^+$ and $W_i^-$ are weights that the appraiser should associate to the data $i$ (observed values) (for $i = 1, 2, \ldots, n$) depending on whether the deviations are negative or positive.
Utilizing the interval \([y', y'']\) as well as his satisfaction functions, the appraiser has the possibility to introduce explicitly his experience and judgement in order to estimate the property's market value in an imprecise environment. At any time during the estimation process, the appraiser may his satisfaction functions. The main objective of this learning process is to help the appraiser to explicitly express his tolerance level.

This formulation enables the appraiser to estimate the contribution of the property's attributes to evaluation of its market value where the selling prices reflect the fuzziness of the real market value of the property. Moreover, the appraiser can introduce explicitly his knowledge, judgement and experience through different types of satisfaction functions. Martel and Aouni \(^{158}[1990]\) proposed a list of six types of satisfaction functions. This list is neither exhaustive nor restrictive. Indeed, the appraiser can define other types of satisfaction functions that represent better his tolerances.

Moreover, compared to the statistical approaches, our estimation model provides a less naïve and simplistic treatment of the data when particular observations may cause fuzziness in the estimation. In fact, there may be more reliable observations compared to others and the estimation method has to allow the appraiser to consider such effects.

This model has been utilized to estimate the market values of a small set of residential properties where the appraiser realizes that the observed selling prices do not reflect the real market values of the properties.

During the estimation process, we have presented and explained to the appraiser the six different satisfaction functions that are proposed by Aouni \(^{159}[1998]\) and Martel and Aouni \(^{158}[1990]\). The appraiser concluded that the satisfaction function of type V is an appropriate one regarding the twenty properties under study. This function represents adequately his satisfaction. He has indicated also that it was easy for him to understand the meaning of each threshold. The shape of this function is presented in Figure 8.2.
Figure 8.2: The Appraiser's Satisfaction Function

In order to obtain the final values of the thresholds $\alpha_{id}$, $\alpha_{lo}$ & $\alpha_{iv}$, the appraiser has tried different values and for each set of the thresholds, we have computed the mean absolute deviations (MAD). The MAD measures the precision of the model estimation where the average of absolute differences between the estimated values and the observed values is to be minimised. The MAD formula is as follows: $\text{MAD} = \frac{\left(\sum_{i=1}^{n} |\hat{y}_i - y_i|\right)}{n}$, where $\hat{y}_i$ denotes the estimated value and $y_i$ the observed value. The last set of the thresholds that the appraiser retained was obtained through a learning process where the final choice of these parameters was guided by the preceding tests. Table-8.2 indicates the appraiser-defined values of indifference, dissatisfaction and veto threshold levels associated with the twenty observations.
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<td>3480</td>
<td>6980</td>
<td>13960</td>
</tr>
<tr>
<td>20</td>
<td>2350</td>
<td>4650</td>
<td>9300</td>
<td>2350</td>
<td>4650</td>
<td>9300</td>
</tr>
</tbody>
</table>
The definition of different thresholds is as following:

- $a_{id}^-$: indifference threshold relative to negative deviations;
- $a_{io}^-$: dissatisfaction threshold relative to negative deviations;
- $a_{iv}^-$: veto threshold relative to negative deviations;
- $a_{id}^+$: indifference threshold relative to positive deviations;
- $a_{io}^+$: dissatisfaction threshold relative to positive deviations;
- $a_{iv}^+$: veto threshold relative to positive deviations;

It is not necessary that the threshold values $a_i^+$ and $a_i^-$ be equal. The appraiser will choose the threshold values that best reflect his tolerance for both positive and negative deviations.

The veto thresholds ($a_{iv}^-$ and $a_{iv}^+$) for each observation ($I = 1, 2, \ldots, 20$) were set by the appraiser at twice the dissatisfaction threshold levels ($a_{io}^-$ and $a_{io}^+$). For example, for the first property they were set at $a_{iv}^- = 2a_{io}^- = 2(4,500) = 9,000$ and $a_{iv}^+ = 2a_{io}^+ = 2(4,500) = 9,000$. Therefore, the appraisal parameters leading to these negative and positive deviations, which exceed Rs 9,000 from the selling price of the property, are not taken into consideration by the appraiser. In addition, the appraiser has given the same importance to the twenty observations. Thus, the relative importance coefficients in program 2 are all equal (i.e., $W_i^+ = W_i^- = 1$, for $i = 1, 2, \ldots, 20$).

For the first observation [Table 8.2], the appraiser satisfaction functions related to the positive deviations are the same as the satisfaction function presented in Figure 8.2 where the values of the indifference ($a_{id}$), the dissatisfaction ($a_{io}$) and the veto ($a_{iv}$) thresholds are as follows:
The corresponding satisfaction function \( F^+_1(\delta^+_1) \) and \( F^-_1(\delta^-_1) \) can be written as following:

\[
F^+_1(\delta^+_1) = \begin{cases} 
1 & \text{if } 0 \leq \delta^+_1 \leq 2300, \\
2.045 - 0.00045\delta^+_1 & \text{if } 2300 \leq \delta^+_1 \leq 4500, \\
0 & \text{if } 4500 \leq \delta^+_1 \leq 9000.
\end{cases}
\]

\[
F^-_1(\delta^-_1) = \begin{cases} 
1 & \text{if } 0 \leq \delta^-_1 \leq 2300, \\
2.045 - 0.00045\delta^-_1 & \text{if } 2300 \leq \delta^-_1 \leq 4500, \\
0 & \text{if } 4500 \leq \delta^-_1 \leq 9000.
\end{cases}
\]

The equivalent representation of the above functions \( F^+_1(\delta^+_1) \) and \( F^-_1(\delta^-_1) \) require the introduction of the 0-1 integer variable \( \lambda_k = \{0,1\} \) (for \( k = 1,2,\ldots,6 \)). Thus, the formulation of the estimation model, by considering only the first observation, can be detailed as follows:

**Program 3:**

Max \( Z = \)

\[
\hat{\lambda}_1(1) + \lambda_2(2.045 - 0.00045\delta^+_1) + \lambda_3(0) + \lambda_4(1) + \lambda_5(2.045 - 0.00045\delta^-_1) + \lambda_6(0)
\]

Subject to: \( \beta_0 + 99.2 \beta_1 + 27 \beta_2 + 49.6 \beta_3 + 36.6 \beta_7 + \delta^+_1 - \delta^-_1 = 45201, \)

\[
2300\lambda_2 + 4500 \lambda_3 - \delta^+_1 \leq 0,
\]

\[
\delta^+_1 - 2300\lambda_1 - 4500\lambda_2 - 9000 \lambda_3 \leq 0,
\]

\[
2300\lambda_5 + 4500 \lambda_6 - \delta^-_1 \leq 0,
\]

\[
\delta^-_1 - 2300\lambda_3 - 4500\lambda_6 - 9000 \lambda_6 \leq 0,
\]

\[
\delta^+_1 \leq 9000, \ \delta^-_1 \leq 9000; \ \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 = 1,
\]

\( \lambda_k = \{0,1\} \) (for \( k = 1,2,\ldots,6 \)); \( \delta^+_1 \) and \( \delta^-_1 \geq 0, \)

\( \beta_0 \) and \( \beta_j \) (\( j = 1,2,\ldots,7 \)) are free.
Solving the program 3 leads to the optimisation of a non-linear objective function expressed in 0-1 variables. In fact, this program contains the two following non-linear terms: 0.00045δ₁⁺λ₂ and 0.00045 δ₁⁻λ₃. The linearization procedure developed by Oral and Kettani [1992] and modified by Aouni [1998] was used to obtain an equivalent linear formulation of program 3.

Program 4: Max. \( Z = \lambda_1 + 2.045\lambda_2 + \xi_1 + \lambda_4 + 2.045\lambda_5 + \xi_2 \)

Subject to: \( \beta_0 + 99.2\beta_1 + 27\beta_2 + 49.6\beta_3 + 36.6\beta_7 + \delta_1^- - \delta_1^+ = 45201, \)

\( 2300\lambda_2 + 4500\lambda_3 - \delta_1^+ \leq 0, \)

\( \delta_1^+ - 2300\lambda_1 - 4500\lambda_2 - 9000\lambda_3 \leq 0, \)

\( 2300\lambda_5 + 4500\lambda_6 - \delta_1^- \leq 0, \)

\( \delta_1^- - 2300\lambda_5 - 4500\lambda_6 - 9000\lambda_6 \leq 0, \)

\( \delta_1^+ \leq 9000, \delta_1^- \leq 9000; \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 = 1, \)

\( \lambda_k = \{0,1\} \text{ (for } k = 1,2,\ldots,6), \delta_1^+ \text{ and } \delta_1^- \geq 0, \)

\( \beta_0 \text{ and } \beta_j (j = 1,2,\ldots,7) \) are free.

Programs 3 and 4 are equivalent in the sense that they have the same optimal solution. The optimization of program 4 can be obtained by using QM for WINDOWS Package. We proceed in the same way for the nineteen other observations. In this chapter we have emphasis on how the model developed by Aouni et al. [1997] was utilized to estimate the market value of a number of residential properties than to consider in depth the optimization process.
RESULT AND ANALYSIS

The appraisal problem for twenty cases was solved by utilizing the model presented in program 2. The formulated mathematical program is non-linear. The linear transformation procedure developed by Oral and Kettani\textsuperscript{186} [1992] and modified by Aouni\textsuperscript{11} [1998] was used to obtain an equivalent linear program. The linearization procedure is illustrated, in Programs 3 & 4, through the first observation from Table-8.1. The optimization of the linear equivalent formulation (for the 20 observations under study) was performed by using QM for WINDOWS package. The computed optimal values of the estimation parameters are as follows: $\beta_0 = 16,018.96; \beta_1 = 385.42; \beta_2 = -714.12; \beta_3 = 34.92; \beta_4 = 6,304.74; \beta_5 = 172.69; \beta_6 = 470.12; \beta_7 = 355.42$ and $Z = 14.85$.

The estimated values of parameters $\beta_0$ and $\beta_j (j = 1,2,\ldots,7)$ indicate the unit contribution of the explanatory variable $j$ in the estimation of the market values of the properties studied. These contributions are summarized as follows:

- **Living space**: Rs 385.42 per square meter;
- **Age of the house**: -Rs 714.12 per year (negative sign indicates that age decrease the market value of the property);
- **Basement living space**: Rs 34.92 per square meter;
- **Additional washrooms**: Rs 6,304.74 per washroom;
- **Carport space**: Rs 172.69 per square meter;
- **Size of shed**: Rs 470.12 per square meter;
- **Size of attached garage**: Rs 355.12 per square meter;

- The value of the constant of estimation $\beta_0 = Rs 16,018.96$ indicates the part of the property’s value that can not be explained by the attributes considered by the appraiser.

The value of the objective function $Z$ (where $Z = 14.85$) represents the total satisfaction achieved for the sample of twenty properties. The satisfaction is measured based on the
differences between the selling prices of the properties and their market values as estimated by the model, in accordance with the thresholds defined for each property. This indicates a high level of satisfaction at 74.25% (14.85/20). The objective function of the proposed model indicates the percentage of the properties whose deviation between the estimated value and the observed value falls inside the indifference zone, a region where the appraiser is completely satisfied. Since we have only 20 observations and for each observation the maximum level of the satisfaction function is one(1), the objective function Z can reach the maximum level of 20. This level of satisfaction may be considered as a counterpart for $R^2$ or level of significance produced by statistical models.

We have also used these parameters to estimate the market value of an additional property having the following characteristics:

- Living space = 97.10 square meters;
- Age of the house = 19 years;
- The number of additional washrooms = 0 (none);
- Size of carport = 33.90 square meters;
- Size of shed = 11.60 square meters;
- Size of attached garage = 0 (none)

The actual selling price, excluding the value of the land, for this property was Rs 54,275.

The estimated market value of this property = $16018.96 + (385.42)(97.10) + (-14.12)(19) + (34.92)(55.90) + (172.69)(33.90) + (470.12)(11.60) + (355.12)(0) = Rs 53,134.57 = Rs 53,135$

The appraiser estimates that the value of this property is within the range of Rs 51,561 to Rs 56,989. The estimated value calculated by using our model is in fact within this interval. The difference in the monetary value between the estimated market value and the actual selling price of this property is in the order of Rs 1,140 which represents 2.10% of the selling price. The difference is less than the acceptable deviation of ±5% established by professionals.
During this experiment, the appraiser judged that the results obtained by this model were in fact satisfactory. Moreover, he appreciated the fact that the model enabled him to integrate, explicitly, his experience regarding the real estate evaluation and his knowledge of the residential sector. In addition, this expert expressed his interest in using the model on a regular basis for real estate evaluation.

In order to compare the ordinary “least-squares method” (OLS) and the standard goal programming (SGP) (program 1), we used the same data (Table 8.1) to compute the estimation parameters. The results of these two models are based on the assumption that the property sale price $y_i$ is a precise value. Moreover, these two models do not explicitly incorporate the appraiser’s experience and judgement which differentiates them from the imprecise goal programming (IGP) model (program 2) that we propose in this chapter.

The MAD was used to compute the estimation precision of each model. The best estimation parameters ($\beta_0, \beta_j$ for $j = 1, 2, \ldots, 7$) and the mean absolute deviations of the three models are summarised in Table 8.3.

<table>
<thead>
<tr>
<th>Estimation Parameters</th>
<th>OLS Model</th>
<th>SGP</th>
<th>IGP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>5932.54</td>
<td>4888.5</td>
<td>16018.96</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>497.29</td>
<td>490.2</td>
<td>385.42</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-764.97</td>
<td>-732.95</td>
<td>-714.12</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>15.98</td>
<td>9.13</td>
<td>34.92</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>4651.48</td>
<td>5368.66</td>
<td>6304.74</td>
</tr>
<tr>
<td>$\beta_5$</td>
<td>177.84</td>
<td>193.78</td>
<td>172.69</td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>524.22</td>
<td>572.37</td>
<td>470.12</td>
</tr>
<tr>
<td>$\beta_7$</td>
<td>353.97</td>
<td>389.95</td>
<td>355.12</td>
</tr>
<tr>
<td>MAD</td>
<td>1889.36</td>
<td>1704.16</td>
<td>2076.66</td>
</tr>
</tbody>
</table>
The MAD of the SGP model is smaller than that of the OLS model. This means that in the case where the observed values of the property selling price are precise, it will be preferable to use the SGP model for estimating the market value of the 20 properties under study. However, it is not appropriate to compare the results of OLS and SGP to those obtained by the IGP model. In the IGP there are no estimation errors (penalties) for the deviations between the observed estimated values within the indifference interval. Moreover, through the appraiser satisfaction function, the deviations have not the same impact within the interval \([\alpha_{id}, \alpha_{io}]\) and his/her satisfaction decreases monotonously.

8.5 CONCLUSION

The parametric appraisal model proposed in this chapter allows the appraiser to incorporate his experience and judgement in a situation where the values of the dependent variable are not precise and are expressed in the form of an interval. The values of the appraisal parameters are obtained with the help of a learning process whereby the appraiser plays an active role in developing the model. Besides, the satisfaction function allows him/her to further express his/her confidence in the observed values. As well, he/she can carry out a sensitivity analysis in order to determine the impact of the variation in the discrimination thresholds on the appraisal parameters of the satisfaction functions. This model allows for a more accurate and less naïve processing of historical data (particularly when the number of real estate sales data is relatively small) where the appraiser can, for example, place less emphasis on certain observations, which he/she considers to be weaker than the others.