CHAPTER NINE
9.1 Introduction

So far, we have considered for our study, either the original measurements taken on each piece of sculpture or the standardized measurements computed from the original measurements of different Pallava monuments. It has been found that in many cases we have only a limited number of art pieces available to us. It is quite possible that the sculptors of the Pallava period had carved more pieces than are available to us today. For example, we have considered 39 sculptures that are in well-preserved condition from the Kailasanatha Temple which is attributed to Pallava ruler, Rajasimha. If the sculptors of Rajasimha period had carved many more sculptures, is it possible to retrieve the measurements of those sculptures that are not available to us, making use of the available measurements? To find an answer to this question, in this chapter we have made an attempt to generate sets of measurements which could be attributed to Rajasimha's reign. To generate samples that are indistinguishable from the carvings of the Kailasanatha Temple, we have considered centroids and covariance matrix of the ten facial measurements, namely, facelength, morphological facelength, noselength, nosebreadth, nose-to-chin distance, eyelength,
eyebreadth, liplength, lipbreadth and facebreadth excluding ears. Simulation is carried out using an IBM 370/155 computer.

9.2 Method based on mean vector and covariance matrix

To generate samples that have the properties of Rajasimha's period, the mean vector and the covariance matrix of the variables mentioned in Section 9.1 are taken into consideration. Let us call mean vector and covariance matrix as $\mu$ and $\Sigma$ respectively. We shall assume that the data pertaining to the sample of the Kailasanatha Temple follow multivariate normal distribution. If we denote the observation vector as $X$, then $X$ follows multivariate normal with mean vector $\mu$ and a covariance matrix $\Sigma$.

i.e., $X \sim \mathcal{N}_p (\mu, \Sigma)$

where $p$ is the number of variables considered. By choosing a matrix $P$, it is always possible to make a non-singular linear transformation from $X$ to $Y$. The linear transformation is

$$Y = P(X - \mu)$$

where

$$Y \sim \mathcal{N}_p (0, P \Sigma P').$$

Then

$$X = P^{-1}Y + \mu.$$  

If $A$ is the eigenvector matrix of $\Sigma$, and if $A' = P$ then
the linear transformation becomes an orthogonal linear transformation and $A' \Sigma A = \Delta$, a diagonal matrix whose $i$th diagonal element is the $i$th characteristic root of $\Sigma$. Denoting the $i$th characteristic root of $\Sigma$ as $\alpha_i$, we find that

$$Y \sim N_p(0, \Delta),$$

and hence $$Y_i \sim N_1(0, \alpha_i)$$

Therefore

$$(Y_i/\sqrt{\alpha_i}) \sim N_1(0, 1).$$

In other words, $$\Delta^{-1/2}Y \sim N_p(0, I)$$

where $I$ is the identity matrix of order $p$.

Let $$Z = \Delta^{-1/2}Y$$

where $$Z \sim N_p(0, I),$$

and hence $$Z_i \sim N_1(0, 1).$$

But $$Y = A'(X - \mu).$$

Therefore $$Z = \Delta^{-1/2}A'(X - \mu)$$

i.e., $$X = A \Delta^{1/2}Z + \mu \ldots \ldots (i)$$

where $A' \Sigma A = \Delta$ and $Z_i \sim N_1(0, 1)$.

Pseudo-random numbers are generated and assuming that these are cumulative probabilities of the standard normals, ordinates are obtained by making use of the algorithm of Odeh and Evans (1974). These ordinate values are considered to be $Z_i$'s given in equation (i) since these
values are ordinates of the standard normals. Substituting the $Z_i$ values, the centroid of the variables, $\mu$, the eigenvector matrix $A$, and the diagonal matrix $\Delta$ in equation (i) we obtain a set of observation vectors $X$'s. These vectors possess all the characteristics of the mean vector and covariance matrix that are originally considered for the construction of these vectors.

In many pattern recognition problems, it is often necessary to generate samples using a given mean vector and a covariance matrix. By making use of linear transformations, the variables are made independent and uncorrelated to each other for the purpose of generation of samples, since it is very difficult to generate them when the variables are correlated (Fukunaga, 1972).

9.3 Generation of sets of measurements

By making use of the mean vector and the covariance matrix of the different facial measurements of the carvings of the Kailasanatha Temple, we have generated sets of measurements that are indistinguishable from those of the carvings of this temple. Mean vector and correlation matrix of the simulated data are given in Tables 9.1 and 9.2 respectively. The mean vector and the correlation matrix of the simulated data compare favourably with the corresponding
mean vector and the correlation matrix (Table 4.2) of the facial measurements of the sculptures of the Kailasanatha Temple. These measurements can be treated as the measurements of some of the sculptures of Rajasimha period that could have been carved by the master craftsmen who created the Kailasanatha Temple at Kancheepuram.