CHAPTER – IV

RELIABILITY MEASURES FOR THREE – COMPONENT SYSTEM IN THE PRESENCE OF CCS FAILURES AND HUMAN ERRORS
4. RELIABILITY MEASURES FOR THREE COMPONENT SYSTEM IN THE PRESENCE OF CCS FAILURES AND HUMAN ERRORS

4.1 INTRODUCTION

The common cause shock failures as well as human errors are identified to be the most dominant causes of failures which can severely degrade the system reliability. These are purely external causes, which affect multiple failures. Billinton and Allan [7] discussed the role of common cause shock failures.

A common cause failure is defined as any instance where multiple units or components fail due to a single cause. These failures are due to equipment design deficiencies, operations and maintenance errors, external environment, external catastrophe, functional deficiency common power source etc;
Human error may be defined as "a failure to perform a specified task (or the performance of a forbidden action), which could result in damage to equipment and property or disruption of scheduled operations" [35].

In this chapter, the formulae for reliability function and mean time between failures are derived for three component identical system which is affected by CCS failures as well as human errors in addition to individual failures. The impact of CCS failures along with human errors on the system is also established with certain numerical illustration.

4.2 NOTATIONS

\[ \begin{align*}
\lambda_i & : \text{rate of individual failure} \\
\lambda_c & : \text{rate of CCS failure} \\
\lambda_h & : \text{rate of human error} \\
c_1 & : \text{chance of individual failures} \\
c_2 & : \text{chance of CCS failures} \\
c_3 & : \text{chance of human errors} \\
\mu_0, \mu_1 & : \text{repair rates} \\
R_{sh}(t) & : \text{Reliability of the series system in } [0, t] \text{ with human error along with Common Cause Shock failures.} \\
R_{ph}(t) & : \text{Reliability of the parallel system in } [0, t] \text{ with human error as well as Common Cause Shock failures.} \\
E_{sh}(T) & : \text{MTBF of series system in the case of CCS failures and human errors.} \\
E_{ph}(T) & : \text{MTBF of parallel system in the case of CCS failures and human errors.}
\end{align*} \]
4.3 ASSUMPTIONS OF THE MODEL

1. The system has three s-independent and identical components.
2. The system is affected by both individual and CCS failures as well as human errors.
3. The individual failures, Common Cause Shock failures and human errors occur independently to each other.
4. The components are repaired singly.

4.4 THE MODEL

The Markov graph of the reliability is seen in fig. (4.1)

![Reliability Markov graph with CCS Failures and Human Errors - Three component identical system](image)

Fig. (4.1) : Reliability Markov graph with CCS Failures and Human Errors – Three component identical system
Under the assumptions stated, the Markov graph for reliability analysis is seen in Fig. (4.1). The quantities that appear in Fig. (4.1) are to be read as

\[
\lambda_{01} = 3\lambda_i c_1; \quad \lambda_{11} = 2\lambda_i c_1; \quad \lambda_{21} = \lambda_i c_1;
\]

\[
\lambda_{0c} = \lambda_c c_2; \quad \lambda_{0h} = \lambda_h c_3; \quad \mu_0 = \mu; \quad \mu_1 = 2\mu
\]

Keeping in view of the above assumptions, we formulate a Markov model to obtain the reliability function and MTBF of the system in the presence of CCS failures as well as human errors in addition to individual failures. The following Markovian equations that result in this model are as

\[
\begin{align*}
    p'_{0}(t) &= - (3\lambda_i c_1 + \lambda_c c_2 + \lambda_h c_3) p_{0}(t) + p_{1}(t) \mu \\
    p'_{1}(t) &= (3\lambda_i c_1) p_{0}(t) - (2\lambda_i c_1 + \mu) p_{1}(t) + 2\mu p_{2}(t) \\
    p'_{2}(t) &= (2\lambda_i c_1) p_{1}(t) - (\lambda_i c_1 + 2\mu) p_{2}(t) \\
    p'_{3}(t) &= (\lambda_c c_2 + \lambda_h c_3) p_{0}(t) + (\lambda_i c_1) p_{2}(t)
\end{align*}
\]

On simplification, the system of differential equations associated with the Markov graph (Fig. 4.1) is

\[
\begin{align*}
    p'_{0}(t) &= - (3\lambda_i c_1 + \lambda_c c_2 + \lambda_h c_3) p_{0}(t) + p_{1}(t) \mu \\
    p'_{1}(t) &= (3\lambda_i c_1) p_{0}(t) - (2\lambda_i c_1 + \mu) p_{1}(t) + 2\mu p_{2}(t) \\
    p'_{2}(t) &= (2\lambda_i c_1) p_{1}(t) - (\lambda_i c_1 + 2\mu) p_{2}(t) \\
    p'_{3}(t) &= (\lambda_c c_2 + \lambda_h c_3) p_{0}(t) + (\lambda_i c_1) p_{2}(t)
\end{align*}
\]
Using the Laplace transformation technique, the set of equations in (4.2) can be solved with the help of the initial conditions, given at \( t = 0 \), \( p_0(t) = 1 \) and \( p_1(t) = p_2(t) = p_3(t) = 0 \) and the solution is

\[
p_0(t) = \frac{[(\gamma_1^2 + \gamma_1 M + N) / (\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)] \exp(\gamma_1 t)}{- [(\gamma_2^2 + \gamma_2 M + N) / (\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)] \exp(\gamma_2 t) + [(\gamma_3^2 + \gamma_3 M + N) / (\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)] \exp(\gamma_3 t)} \tag{4.3}
\]

\[
p_1(t) = \frac{[(\lambda_1 c_1) (\gamma_1 + \lambda_1 c_1 + 2\mu) / (\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)] \exp(\gamma_1 t)}{- [(\lambda_1 c_1) (\gamma_2 + \lambda_1 c_1 + 2\mu) / (\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)] \exp(\gamma_2 t) + [(\lambda_1 c_1) (\gamma_3 + \lambda_1 c_1 + 2\mu) / (\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)] \exp(\gamma_3 t)} \tag{4.4}
\]

\[
p_2(t) = \frac{[6(\lambda_1 c_1)^2 / (\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)] \exp(\gamma_1 t)}{- [6(\lambda_1 c_1)^2 / (\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)] \exp(\gamma_2 t) + [6(\lambda_1 c_1)^2 / (\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)] \exp(\gamma_3 t)} \tag{4.5}
\]

\[
p_3(t) = 1 - \left[ p_0(t) + p_1(t) + p_2(t) \right] \tag{4.6}
\]

Where

\[
\begin{align*}
\gamma_1 &= -\gamma \sin \alpha - K_1 / 3 \\
\gamma_2 &= \gamma \sin (\pi/3 + \alpha) - K_1 / 3 \\
\gamma_3 &= \gamma \sin (-\pi/3 + \alpha) - K_1 / 3
\end{align*}
\tag{4.7}
\]

and \( \alpha = \theta \) is the solution of the equation.
\[
\sin 3\theta = -\frac{4q}{\gamma^3}
\]

Where \( q \) and \( \gamma \) are given by

\[
q = K_3 - K_1K_2 / 3 + 2K_i^3 / 27
\]

\[
\gamma = (2/3)(K_i^2 - 3K_2)^{\frac{1}{2}}
\]

and \( M, N, K_1, K_2 \) and \( K_3 \) are defined as follows

\[
M = (3\lambda_i c_1 + 3\mu)
\]

\[
N = (2(\lambda_i c_1)^2 + \lambda_i c_1 \mu + 2\mu^2)
\]  

\[
K_1 = (6\lambda_i c_1 + \lambda_h c_3 + \lambda_c c_2 + 3\mu)
\]

\[
K_2 = [11(\lambda_i c_1)^2 + 3\lambda_h c_2 \lambda_i c_1 + 3\lambda_c c_2 \lambda_i c_1 + 7\lambda_i c_1 \mu + 3\lambda_h c_3 \mu
\]

\[
+ 3\lambda_c c_2 \mu + 2\mu^2]
\]

\[
K_3 = [6(\lambda_i c_1)^3 + 2(\lambda_i c_1)^2 \lambda_h c_2 + 2(\lambda_i c_1)^2 \lambda_c c_2 + 2\lambda_c c_2 \mu^2 + \lambda_i c_1 \lambda_h c_3 \mu
\]

\[
+ \lambda_i c_1 \lambda_c c_2 \mu + 2\lambda_h c_3 \mu^2]
\]

\( \gamma_1, \gamma_2 \) and \( \gamma_3 \) are always negative

\( \forall \lambda_i \geq 0 \)

\( c_1, c_2 \) & \( c_3 \in (0, 1) \)
4.5 RELIABILITY FUNCTION – CCS FAILURES AND HUMAN ERRORS

The three component system would be either series configuration or parallel configuration. Therefore reliability of the system in the interval (0, t) can be derived using the equations (4.3) - (4.6)

4.5.1 RELIABILITY FUNCTION – SERIES CONFIGURATION

For three component identical system, the reliability function in the case of series configuration is obtained as

\[
R_{sh}(t) = p_0(t) = \left[ \frac{(\gamma_1^2 + \gamma_1 M + N)}{(\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)} \right] \exp(\gamma_1 t) - \left[ \frac{(\gamma_2^2 + \gamma_2 M + N)}{(\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)} \right] \exp(\gamma_2 t) + \left[ \frac{(\gamma_3^2 + \gamma_3 M + N)}{(\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)} \right] \exp(\gamma_3 t) \quad \text{-------- (4.10)}
\]

Which reduces to

\[
R_{sh}(t) = \exp \left[ - (3\lambda_1 c_1 + \lambda_2 c_2 + \lambda_3 c_3) t \right] \quad \text{-------- (4.11)}
\]

By substituting \( \gamma_1, \gamma_2 \) and \( \gamma_3 \) as seen in (4.10) and putting \( p_0 = 0 \) in the model, as we do not consider any transition from state ‘1’ to ‘2’ and from state ‘1’ to ‘0’ because state ‘1’ is failure state and repair is not considered for series system.
The reliability expression given in (4.11) will however agree with the expression, by synchronizing the present model into two unit system, already derived [10] in the CCS failures case, assuming that no human errors are affecting the system (i.e. $\lambda_h = 0$ or $c_3 = 0$).

**ILLUSTRATION: (4.1)**

For the series configuration, the reliability values are obtained by taking chance of individual failures $c_1 = 0.5$, chance of CCS failures $c_2 = 0.25$, chance of human errors $c_3 = 0.25$ and various rates of individual, CCS failures and human errors are given in table (4.1) & (4.2). Reliability curves are also plotted as against time $t$ and shown in Fig. (4.2) & (4.3).
### Table: (4.1) Reliability Function - Series Configuration - CCS Failures and Human Errors

\( \mu = 0, c_1 = 0.5, c_2 = 0.25, c_3 = 0.25 \)

<table>
<thead>
<tr>
<th>Time ( t )</th>
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<th>( \lambda_i = 0.005 )</th>
<th>( \lambda_i = 0.01 )</th>
<th>( \lambda_i = 0.02 )</th>
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<td>( \lambda_c = 0.01 )</td>
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<td>( \lambda_c = 0.04 )</td>
</tr>
<tr>
<td></td>
<td>( \lambda_h = 0.01 )</td>
<td>( \lambda_h = 0.02 )</td>
<td>( \lambda_h = 0.04 )</td>
<td>( \lambda_h = 0.08 )</td>
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<td>0.835270</td>
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<td>0.886934</td>
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### Table: (4.2) Reliability Function - Series Configuration - CCS Failures and Human Errors

\( \mu = 0, c_1 = 0.5, c_2 = 0.25, c_3 = 0.25 \)

<table>
<thead>
<tr>
<th>Time ( t )</th>
<th>( \lambda_i = 0.05 )</th>
<th>( \lambda_i = 0.10 )</th>
<th>( \lambda_i = 0.20 )</th>
<th>( \lambda_i = 0.40 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda_c = 0.10 )</td>
<td>( \lambda_c = 0.20 )</td>
<td>( \lambda_c = 0.40 )</td>
<td>( \lambda_c = 0.80 )</td>
</tr>
<tr>
<td></td>
<td>( \lambda_h = 0.15 )</td>
<td>( \lambda_h = 0.30 )</td>
<td>( \lambda_h = 0.60 )</td>
<td>( \lambda_h = 1.20 )</td>
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</table>
\( \lambda_i, \lambda_c \) & \( \lambda_h \) are failure rates / unit time

Fig. (4.2) : System Reliability Plots for Series Configuration
Fig. (4.3) : System Reliability Plots for Series Configuration

$\lambda_i, \lambda_c \text{ and } \lambda_h$ are failure rates / unit time
4.5.2 RELIABILITY FUNCTION – PARALLEL CONFIGURATION

The reliability function for three unit identical parallel system is obtained by considering the probability of successful states i.e. either ‘0’ or ‘1’ or ‘2’, because in either of these states the system is said to be in trouble free operation during the time (0, t). Thus

\[ R_{ph}(t) = p_0(t) + p_1(t) + p_2(t) \]

\[ R_{ph}(t) = \left[ T_1 \right] \exp (\gamma_1 t) - \left[ T_2 \right] \exp (\gamma_2 t) + \left[ T_3 \right] \exp (\gamma_3 t) \]  \hspace{1cm} (4.12)

where

\[ T_1 = \left[ (\gamma_1^2 + \gamma_1 M + N) + (3 \lambda c_1) (\gamma_1 + \lambda c_1 + 2 \mu) + 6(\lambda c_1)^2 \right] / (\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3) \]

\[ T_2 = \left[ (\gamma_2^2 + \gamma_2 M + N) + (3 \lambda c_1) (\gamma_2 + \lambda c_1 + 2 \mu) + 6(\lambda c_1)^2 \right] / (\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3) \]

\[ T_3 = \left[ (\gamma_3^2 + \gamma_3 M + N) + (3 \lambda c_1) (\gamma_3 + \lambda c_1 + 2 \mu) + 6(\lambda c_1)^2 \right] / (\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3) \]

M, N, and \( \gamma_1, \gamma_2, \gamma_3 \) are given in (4.8) and (4.7) respectively.

If the human errors do not affect the system (i.e. \( \lambda_h = 0 \) or \( c_3 = 0 \)) then the expression for Reliability (4.12) agrees with the result, when three unit system reduced to two unit system, which is already developed [10] under the influence of CCS failures in addition to individual failures.

ILLUSTRATION: (4.2)

As in the case of parallel system, the reliability values are obtained with chance of individual failures \( c_1 = 0.5 \), chance of CCS failures \( c_2 = 0.25 \), chance of human errors \( c_3 = 0.25 \) and various failure rates and service rates mentioned in the tables and shown in table (4.3) & (4.4). System reliability plots also drawn against time ‘t’ and shown in Fig. (4.4) and (4.5).
### Table: (4.3) Reliability Function – Parallel Configuration –
CCS Failures and Human Errors

\( \mu = 0.01, c_1 = 0.5, c_2 = 0.25, c_3 = 0.25 \)

<table>
<thead>
<tr>
<th>Time</th>
<th>( \lambda_i = 0.025 )</th>
<th>( \lambda_i = 0.05 )</th>
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### Table: (4.4) Reliability Function – Parallel Configuration –
CCS Failures and Human Errors

\( \mu = 2, c_1 = 0.5, c_2 = 0.25, c_3 = 0.25 \)

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<td>0.314720</td>
<td>0.117156</td>
<td>0.025586</td>
</tr>
</tbody>
</table>
\( \lambda_i, \lambda_c \) & \( \lambda_h \) are failure rates / unit time when \( \mu = 0.01 \) repair / unit time

Fig. (4.4): System Reliability Plots for Parallel Configuration
\( \lambda_1, \lambda_c \) & \( \lambda_h \) are failure rates / unit time
when \( \mu = 2 \) repairs / unit time

**Fig. (4.5) : System Reliability Plots for Parallel Configuration**
4.6 MTBF – CCS FAILURES AND HUMAN ERRORS

The expressions for mean time between failures of the three component identical system for both series and parallel configurations in the presence of CCS failures as well as human errors in addition to individual failures are derived and presented in this section.

4.6.1 MEAN TIME BETWEEN FAILURE – SERIES CONFIGURATION

For three unit identical series system, the expected life time during which an item performs its function successfully under the influence of CCS failures along with human errors is thus

\[ E_{sh}(T) = \int_0^\infty R_{sh}(t) \, dt \]

Using the result in (4.10), the expression reduces to

\[ E_{sh}(T) = \frac{1}{(3\lambda_1 c_1 + \lambda_c c_2 + \lambda_h c_3)} \quad (4.13) \]

4.6.2 MEAN TIME BETWEEN FAILURE – PARALLEL CONFIGURATION

The expression for Mean time between failures of three component parallel system under the influence of CCS failures along with human errors can be derived as

\[ E_{ph}(T) = \int_0^\infty R_{ph}(t) \, dt \]

\[ = \int_0^\infty [ T_1 \exp(\gamma_1 t) - T_2 \exp(\gamma_2 t) + T_3 \exp(\gamma_3 t) ] \, dt \]

99
On simplification, the above expression reduces to

\[ E_{ph}(T) = - \left( \frac{6\lambda_i c_1 l_1 + N + 9(\lambda_i c_1)^2}{\gamma_1 \gamma_2 \gamma_3} \right) \]  \hspace{1cm} \text{--------- (4.14)}

Where \( \gamma_1, \gamma_2, \gamma_3 \) and \( N \) are given in (4.7) and (4.8) respectively.

**ILLUSTRATION: (4.3)**

The values of Mean time between failures (MTBF) of the three component identical system are obtained in the presence of CCS failures as well as human errors in addition to individual failures for both series and parallel configurations and shown in table (4.5) & (4.6).
### Table: (4.5) MTBF - Three Component System - Parallel Configuration

<table>
<thead>
<tr>
<th>μ</th>
<th>λ₁</th>
<th>λ₂</th>
<th>λ₃</th>
<th>λ₁</th>
<th>λ₂</th>
<th>λ₃</th>
<th>λ₁</th>
<th>λ₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0025</td>
<td>348.526314</td>
<td>167.829195</td>
<td>82.863241</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.005</td>
<td>379.981656</td>
<td>174.263157</td>
<td>83.914598</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0075</td>
<td>418.353136</td>
<td>181.722236</td>
<td>85.437010</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>465.575561</td>
<td>189.990828</td>
<td>87.131578</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.015</td>
<td>524.830907</td>
<td>199.110277</td>
<td>88.945201</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.02</td>
<td>601.506091</td>
<td>209.176568</td>
<td>90.861118</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.03</td>
<td>704.489304</td>
<td>220.320284</td>
<td>92.875652</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.04</td>
<td>850.028056</td>
<td>232.787780</td>
<td>94.995414</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>0.05</td>
<td>1071.340587</td>
<td>246.687039</td>
<td>97.220285</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.06</td>
<td>1448.693387</td>
<td>262.415453</td>
<td>99.555139</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table: (4.6) MTBF - Three Component System - Series and Parallel Configuration

<table>
<thead>
<tr>
<th>λ₁ = 0.1, μ = 1, c₁ = 0.5, c₂ = 0.25, c₃ = 0.25</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>λₖ</th>
<th>λₗ</th>
<th>Eₗ(λₖ)</th>
<th>Eₗ(λₗ)</th>
<th>Eₗ(λₖ)</th>
<th>Eₗ(λₗ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>3.333333</td>
<td>550.705176</td>
<td>3.076923</td>
<td>6.609125</td>
</tr>
<tr>
<td>0.6</td>
<td>0.3</td>
<td>3.333333</td>
<td>550.705176</td>
<td>2.666667</td>
<td>5.140670</td>
</tr>
<tr>
<td>0.7</td>
<td>0.4</td>
<td>3.333333</td>
<td>550.705176</td>
<td>2.352941</td>
<td>4.206132</td>
</tr>
<tr>
<td>0.8</td>
<td>0.5</td>
<td>3.333333</td>
<td>550.705176</td>
<td>2.105263</td>
<td>3.559114</td>
</tr>
<tr>
<td>0.9</td>
<td>0.6</td>
<td>3.333333</td>
<td>550.705176</td>
<td>1.904762</td>
<td>3.084616</td>
</tr>
<tr>
<td>1.0</td>
<td>0.7</td>
<td>3.333333</td>
<td>550.705176</td>
<td>1.739130</td>
<td>2.721755</td>
</tr>
<tr>
<td>1.2</td>
<td>0.8</td>
<td>3.333333</td>
<td>550.705176</td>
<td>1.538461</td>
<td>2.313525</td>
</tr>
<tr>
<td>1.4</td>
<td>0.9</td>
<td>3.333333</td>
<td>550.705176</td>
<td>1.379310</td>
<td>2.011782</td>
</tr>
<tr>
<td>1.6</td>
<td>1.0</td>
<td>3.333333</td>
<td>550.705176</td>
<td>1.250000</td>
<td>1.779666</td>
</tr>
<tr>
<td>1.8</td>
<td>1.1</td>
<td>3.333333</td>
<td>550.705176</td>
<td>1.142857</td>
<td>1.595570</td>
</tr>
<tr>
<td>2.0</td>
<td>1.2</td>
<td>3.333333</td>
<td>550.705176</td>
<td>1.052632</td>
<td>1.445988</td>
</tr>
</tbody>
</table>
4.7 DISCUSSION

The reliability measures namely Reliability R (t) and MTBF (E(T)) of three component identical system in the presence of CCS failures as well as human errors along with individual failures are established and presented in this chapter. We observed from tables (4.1) – (4.4) that the reliability of the system for both series and parallel configurations decreased rapidly when the rate of individual and CCS failures along with human errors increases. It is also seen from table (4.5) that the MTBF values increases as the service rate (μ) increases. Further, it is observed from table (4.6), that the values of MTBF decreases as the failure rates (CCSF & human error) increases. This necessitates the inclusion of human errors as well as CCS failures when the system is under the influence of CCS failures along with human errors, to assess correctly the reliability measures.
4.8 AVAILABILITY FOR 3-COMPONENT SYSTEM WITH CCS FAILURES AND HUMAN ERRORS

4.8.1 INTRODUCTION

This chapter emphasizes the Common Cause Shock Failures (CCSF) and human errors on the availability of a three component system (both time dependent and steady-state). In this chapter, frequency of failures of a system which is another important measure in the reliability analysis is also discussed. The definition of availability function appears to be very similar to that of reliability function but both of them have different meanings. Reliability emphasizes fault-free operation in the interval (0, t) whereas availability is concerned with the status of the system at the epoch of time \( t \). The availability measure is very important in the case of repairable systems. Therefore, we allow repair even after the system is found in failed state for deriving the availability.

In this chapter, we derived the expression for availability (both transient and steady-state) and the frequency of encountering the different states of the 3-component identical system in the context of CCS failures as well as human errors in addition to individual failures and also the study includes numerical examples to illustrate the model.
4.8.2 NOTATIONS

\( \lambda_i, \lambda_c \text{ and } \lambda_h \) : the failure rates of individual, CCS and human errors respectively

\( c_1, c_2 \text{ and } c_3 \) : the chance of individual, CCS failures and human errors respectively

\( \mu_0, \mu_1, \mu_2 \) : repair rates

\( A_{sh}(t) \) : availability of series configuration in the presence of CCS failures and human errors

\( A_{ph}(t) \) : availability of the parallel configuration in the presence of CCS failures as well as human errors

\( f_{sh}(\text{down}) \) : frequency of down state in the presence of CCS failures as well as human errors for series configuration

\( f_{ph}(\text{down}) \) : frequency of down state in the presence of CCS failures as well as human errors for parallel configuration

4.8.3 ASSUMPTIONS OF THE MODEL

1. The system consists of three s-independent and identical components.

2. The components in the system may fail individually or common cause shocks or human errors or occurrence of all these failures simultaneously at the rates of \( \lambda_i, \lambda_c \text{ and } \lambda_h \) respectively. The chances of these failures are \( c_1, c_2 \text{ and } c_3 \) s.t \( c_1 + c_2 + c_3 = 1 \).

3. The system is affected by individual and CCS failures as well as human errors.

4. Common cause failure, human error and individual failure rates are constant.

5. The failed components are repaired singly and repair times follow an exponential distribution.
4.8.4 THE MODEL

The Markov graph of the three component Availability system is given in Fig. (4.8.1)

Fig. (4.8.1): Availability Markov graph – three-component identical system with individual and CCS failures as well as human errors.
Under the assumptions given in section (4.8.3), we have formulated a Markov graph which represents a three unit identical system given in Fig. (4.8.1) to derive the availability function \( A(t) \) and the frequency of encountering the different states in the presence of individual failures, CCS failures as well as human errors. The numerals in figure (4.8.1) denote the state numbers.

The quantities \( \lambda_{01}, \lambda_{11}, \lambda_{21}, \lambda_{oc}, \lambda_{oh}, \mu_0, \mu_1 \) and \( \mu_2 \) that appeared in fig. (4.8.1) are defined as follows:

\[
\begin{align*}
\lambda_{01} &= 3\lambda_i c_i, \\
\lambda_{11} &= 2\lambda_i c_i, \\
\lambda_{21} &= \lambda_i c_i, \\
\lambda_{oc} &= \lambda_c c_2, \\
\lambda_{oh} &= \lambda_h c_3, \\
\mu_0 &= \mu, \\
\mu_1 &= 2\mu \text{ and} \\
\mu_2 &= 3\mu
\end{align*}
\]

(4.8.1)

The Markov equations associated with the system states are

\[
\begin{align*}
p_0(t + dt) &= p_0(t) \left[ 1 - (\lambda_{01} + \lambda_{oc} + \lambda_{oh}).dt \right] + p_1(t) \mu_0. dt \\
p_1(t + dt) &= \lambda_{01}. p_0(t). dt + p_1(t) \left[ 1 - (\lambda_{21} + \mu_0).dt \right] + p_2(t) \mu_1. dt \\
p_2(t + dt) &= \lambda_{11}. p_1(t). dt + p_2(t) \left[ 1 - (\lambda_{21} + \mu_1).dt \right] + \mu_2 . p_3(t). dt \\
p_3(t + dt) &= (\lambda_{oc} + \lambda_{oh}). p_0(t). dt + \lambda_{21}. p_2(t). dt + p_3(t) \left[ 1 - \mu_2 . dt \right]
\end{align*}
\]

(4.8.2)
On simplification, the set of differential equations are
\[
\begin{align*}
\dot{p}_0(t) &= -(\lambda_{o1} + \lambda_{oc} + \lambda_{oh}) \cdot p_0(t) + p_0 \cdot p_1(t) \\
\dot{p}_1(t) &= \lambda_{o1} \cdot p_0(t) - (\lambda_{11} + \mu_0) \cdot p_1(t) + \mu_1 \cdot p_2(t) \\
\dot{p}_2(t) &= \lambda_{11} \cdot p_1(t) - (\lambda_{21} + \mu_1) \cdot p_2(t) + \mu_2 \cdot p_3(t) \\
\dot{p}_3(t) &= (\lambda_{oc} + \lambda_{oh}) \cdot p_0(t) - \lambda_{21} \cdot p_2(t) - \mu_2 \cdot p_3(t)
\end{align*}
\]
\[
\text{(4.8.3)}
\]

Using the Laplace transformation, the set of equations stated in (4.8.3) can be solved
with the help of the initial conditions, given at \( t = 0 \), \( p_0(t) = 1 \), \( p_1(t) = p_2(t) = p_3(t) = 0 \)
and the solution is
\[
p_0(t) = \left[ \frac{(\gamma_1^3 + \gamma_1^2 \gamma_1 + \gamma_1 + I)}{\gamma_1 (\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)} \right] \exp(\gamma_1 t)
- \left[ \frac{(\gamma_2^3 + \gamma_2^2 \gamma_2 + \gamma_2 + I)}{\gamma_2 (\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)} \right] \exp(\gamma_2 t)
+ \left[ \frac{(\gamma_3^3 + \gamma_3^2 \gamma_3 + \gamma_3 + I)}{\gamma_3 (\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)} \right] \exp(\gamma_3 t)
- \frac{I}{\gamma_1 \gamma_2 \gamma_3}
\]
\[
\text{(4.8.4)}
\]
\[
p_1(t) = \left[ \frac{(\gamma_1^2 \gamma_2 + \gamma_1 \gamma_2 + I)}{\gamma_1 (\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)} \right] \exp(\gamma_1 t)
- \left[ \frac{(\gamma_2^2 \gamma_2 + \gamma_2 \gamma_2 + I)}{\gamma_2 (\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)} \right] \exp(\gamma_2 t)
+ \left[ \frac{(\gamma_3^2 \gamma_3 + \gamma_3 \gamma_3 + I)}{\gamma_3 (\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)} \right] \exp(\gamma_3 t)
- \frac{I}{\gamma_1 \gamma_2 \gamma_3}
\]
\[
\text{(4.8.5)}
\]
\[
p_2(t) = \left[ \frac{(\gamma_1 \gamma_3 + I)}{\gamma_1 (\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)} \right] \exp(\gamma_1 t)
- \left[ \frac{(\gamma_2 \gamma_3 + I)}{\gamma_2 (\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)} \right] \exp(\gamma_2 t)
+ \left[ \frac{(\gamma_3 \gamma_3 + I)}{\gamma_3 (\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)} \right] \exp(\gamma_3 t)
- \frac{I}{\gamma_1 \gamma_2 \gamma_3}
\]
\[
\text{(4.8.6)}
\]
\( P_3(t) = 1 - [ p_0(t) + p_1(t) + p_2(t) ] \)  

\begin{align*}
\gamma_1, \gamma_2 \text{ and } \gamma_3 \text{ in equations (4.8.4) – (4.8.6) are defined as} \\
\gamma_1 &= -\gamma \sin \alpha - \Lambda_1 / 3 \\
\gamma_2 &= \gamma \sin (\pi/3 + \alpha) - \Lambda_1 / 3 \\
\gamma_3 &= \gamma \sin (-\pi/3 + \alpha) - \Lambda_1 / 3
\end{align*}  

\text{and } \alpha = \theta \text{ is the solution of the equation.} 

\[ \sin 30 = -\frac{4q}{y^3} \]  

Where \( q \) and \( y \) are given by 

\[ q = A_3 - A_1 A_2 / 3 + 2A_1^3 / 27 \]  

\[ y = (2/3) (A_1^2 - 3A_2)^{1/3} \]  

Where \( A_1, A_2 \) and \( A_3 \) are defined as follows 

\[ A_1 = (\mu_0 + \mu_1 + \mu_2 + \lambda_0 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_{0h} ) \]  

\[ A_2 = (\mu_0 \lambda_2 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{0h} + \lambda_{0h} + \lambda_{0h} + \lambda_{0h} + \lambda_{0h}) \]  

\[ A_3 = (\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9, \lambda_{0h}, \lambda_{0h}, \lambda_{0h}, \lambda_{0h}) \]  

\[ G_1 = (\mu_0 + \mu_1 + \mu_2 + \lambda_1 + \lambda_2) \]  

\[ G_2 = \lambda_0 \]  

\[ (4.8.9) \]
\[ H_1 = (\mu_0 \mu_1 + \lambda_{11} \mu_2 + \lambda_{11} \lambda_{21} + \mu_1 \lambda_{21} + \mu_0 \lambda_1) \]
\[ H_2 = (\lambda_{01} \mu_1 + \lambda_{01} \mu_2 + \lambda_{01} \lambda_{21}) \]
\[ H_3 = (\lambda_{01} \lambda_{11} + \lambda_{0h} \mu_2 + \lambda_{0c} \mu_2) \]

\[ I_1 = (\mu_0 \mu_1 \mu_2) \]
\[ I_2 = (\lambda_{01} \mu_1 \mu_2 + \mu_1 \mu_2 \lambda_{0c} + \mu_1 \mu_2 \lambda_{0h}) \]
\[ I_3 = (\mu_2 \mu_0 \lambda_{0c} + \mu_2 \lambda_{11} \lambda_{0c} + \lambda_{01} \lambda_{11} \mu_2 + \mu_2 \mu_0 \lambda_{0h} + \mu_2 \lambda_{11} \lambda_{0h}) \]

and the quantities \( \lambda_{01}, \lambda_{11}, \lambda_{21}, \lambda_{0c}, \lambda_{0h}, \mu_0, \mu_1 \) and \( \mu_2 \) as given in (4.8.1) are to be substituted using (4.8.8) – (4.8.11)

### 4.8.5 AVAILABILITY FUNCTION – CCS FAILURES AND HUMAN ERRORS

Availability is a performance criterion for repairable systems that accounts for both the reliability and maintainability properties of a system. In this section, we consider to study the availability function for three component identical system which is affected by CCS failures as well as human errors in addition to individual failures for both time dependent and steady-state availabilities.

#### 4.8.5.1 TIME-DEPENDENT AVAILABILITY – CCS FAILURES AND HUMAN ERRORS

We derive the time-dependent availability for both series and parallel configurations in the case of CCS failures as well as human errors in addition to individual failures.
A) AVAILABILITY FUNCTION – SERIES CONFIGURATION

For the case of series system, the time-dependent availability is given by

\[ A_{sh}(t) = p_0(t) = \left\{ \frac{(\gamma_1^3 + \gamma_1^2 G_1 + H_1 \gamma_1 + I_1)}{\gamma_1 (\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)} \right\} \exp(\gamma_1 t) \]

\[ - \left\{ \frac{(\gamma_2^3 + \gamma_2^2 G_1 + H_1 \gamma_2 + I_1)}{\gamma_2 (\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)} \right\} \exp(\gamma_2 t) \]

\[ + \left\{ \frac{(\gamma_3^3 + \gamma_3^2 G_1 + H_1 \gamma_3 + I_1)}{\gamma_3 (\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)} \right\} \exp(\gamma_3 t) \]

\[ - \frac{I_1}{(\gamma_1 \gamma_2 \gamma_3)} \]

\[ \text{------- (4.8.12)} \]

Where \( G_1, H_1, I_1 \) and \( \gamma_1, \gamma_2, \gamma_3 \) are given in (4.8.9) - (4.8.11) and (4.8.8) respectively. For series case, no transition is allowed from state '1' to state '2' and state '2' to state '3'. Hence \( \lambda_{11} = \lambda_{21} = 0 \) and also substitute the quantities \( \lambda_{01}, \lambda_{11}, \lambda_{21}, \lambda_{02}, \lambda_{03}, \mu_0, \mu_1 \) and \( \mu_2 \) as given in (4.8.1). The expression (4.8.12) reduced to

\[ A_{sh}(t) = D_1 \exp(\gamma_1 t) - D_2 \exp(\gamma_2 t) + D_3 \exp(\gamma_3 t) - \frac{6\mu^3}{(\gamma_1 \gamma_2 \gamma_3)} \]

\[ \text{------- (4.8.13)} \]

Where

\[ D_1 = \left\{ \frac{(\gamma_1^3 + 6\mu \gamma_1^2 + 11\mu^2 \gamma_1 + 6\mu^3)}{\gamma_1 (\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)} \right\} \]

\[ D_2 = \left\{ \frac{(\gamma_2^3 + 6\mu \gamma_2^2 + 11\mu^2 \gamma_2 + 6\mu^3)}{\gamma_2 (\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)} \right\} \]

\[ D_3 = \left\{ \frac{(\gamma_3^3 + 6\mu \gamma_3^2 + 11\mu^2 \gamma_3 + 6\mu^3)}{\gamma_3 (\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)} \right\} \]

And

\[ \gamma_1 = -\gamma \sin \alpha - A_1 / 3 \]

\[ \gamma_2 = \gamma \sin (\pi/3 + \alpha) - A_1 / 3 \]

\[ \gamma_3 = \gamma \sin (-\pi/3 + \alpha) - A_1 / 3 \]

\[ \text{------- (4.8.14)} \]

and \( \alpha = 0 \) is the solution of the equation.
\[ \sin 30 = -4q / \gamma^3 \]

Where \( q \) and \( \gamma \) are given by

\[ q = A_3 - A_1A_2 / 3 + 2A_1^3 / 27 \]
\[ \gamma = (2/3) \left( A_1^2 - 3A_2 \right)^{1/2} \]

Where \( A_1, A_2 \) and \( A_3 \) are defined as follows

\[ A_1 = (6\mu + 3\lambda_i c_1 + \lambda_c c_2 + \lambda_h c_3) \]
\[ A_2 = (15\lambda_i \mu + 6\lambda_c c_2 \mu + 6\lambda_h c_3 \mu + 11\mu^2) \]
\[ A_3 = (11\lambda_h c_3 \mu^2 + 6\mu^3 + 11\lambda_c c_2 \mu^2 + 18 \lambda_i c_1 \mu^2) \]

In this series system, the availability expression given in (4.8.13) agrees with the result, by synchronizing the present model into two component system, already developed [82] when CCS failures in addition to individual failures are acting on the system. (i.e. no human error is affecting the system \( \lambda_h = 0 / c_3 = 0 \))

**ILLUSTRATION: (4.4)**

The values of availability of the series system are obtained as a function of time ‘t’ by taking chance of individual failures \( c_1 = 0.5 \), chance of CCS failures \( c_2 = 0.25 \), chance of human errors \( c_3 = 0.25 \) and various failure rates and service rates and given in table (4.7) and table (4.8). Availability plots are also drawn against time ‘t’ and appended.
### Table: (4.7) Availability in the case of CCS failures and human errors — Series system

$c_1 = 0.5, c_2 = 0.25, c_3 = 0.25$

<table>
<thead>
<tr>
<th>Time</th>
<th>$R_{sh}(t)$</th>
<th>$A_{sh}(t)$</th>
<th>$R_{sh}(t)$</th>
<th>$A_{sh}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\mu = 10$</td>
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<td>0.997641</td>
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</tr>
<tr>
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<td>0.996385</td>
<td>0.997641</td>
<td>0.438235</td>
</tr>
<tr>
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<td>0.997641</td>
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<tr>
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<td>0.996385</td>
<td>0.997641</td>
<td>0.063928</td>
</tr>
</tbody>
</table>
\( \lambda_1 = 0.01, \lambda_2 = 0.02 & \lambda_3 = 0.03 \) are failure rates / unit time

\( \lambda_1 = 0.1, \lambda_2 = 0.2 & \lambda_3 = 0.3 \) are repair rates / unit time

Fig. (4.8.2): System Availability Plots for Series Configuration

113
<table>
<thead>
<tr>
<th>Time</th>
<th>$R_{sh}(t)$</th>
<th>$A_{sh}(t)$</th>
<th>$R_{sh}(t)$</th>
<th>$A_{sh}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$\mu = 5$</td>
<td>$\mu = 10$</td>
</tr>
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</tr>
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</tr>
<tr>
<td>3</td>
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</tr>
<tr>
<td>4</td>
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<td>0.868477</td>
</tr>
<tr>
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<tr>
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<td>0.063928</td>
<td>0.929677</td>
<td>0.004087</td>
<td>0.868477</td>
</tr>
</tbody>
</table>

Table (4.8) Availability in the Case of CCS Failures and Human Errors – Series System

$c_1 = 0.5, c_2 = 0.25, c_3 = 0.25$
Fig. (4.8.3): System Availability Plots for Series Configuration

\[ \lambda_i = 0.1, \lambda_e = 0.2 \text{ & } \lambda_h = 0.3 \text{ failure rates / unit time} \]

\[ \lambda_i = 0.2, \lambda_e = 0.4 \text{ & } \lambda_h = 0.6 \text{ failure rates / unit time} \]
B) AVAILABILITY – PARALLEL CONFIGURATION

The time dependent availability for a parallel system in the case of CCS failures as well as human errors in addition to individual failures can be obtained as:

\[ A_{ph}(t) = p_0(t) + p_1(t) + p_2(t) \]

\[ = (X_1 + Y_1 + Z_1) \exp(\gamma_1 t) - (X_2 + Y_2 + Z_2) \exp(\gamma_2 t) + (X_3 + Y_3 + Z_3) \exp(\gamma_3 t) \]

\[ + (X_4 + Y_4 + Z_4) \quad \text{(4.8.15)} \]

Where

\[ X_1 = \frac{(\gamma_1^3 + \gamma_1^2 G_1 + H_1 \gamma_1 + I_1)}{\gamma_1 (\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)} \]

\[ X_2 = \frac{(\gamma_2^3 + \gamma_2^2 G_1 + H_1 \gamma_2 + I_1)}{\gamma_2 (\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)} \]

\[ X_3 = \frac{(\gamma_3^3 + \gamma_3^2 G_1 + H_1 \gamma_3 + I_1)}{\gamma_3 (\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)} \]

\[ Y_1 = \frac{(\gamma_1^2 G_2 + \gamma_1 H_2 + I_2)}{\gamma_1 (\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)} \]

\[ Y_2 = \frac{(\gamma_2^2 G_2 + \gamma_2 H_2 + I_2)}{\gamma_2 (\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)} \]

\[ Y_3 = \frac{(\gamma_3^2 G_2 + \gamma_3 H_2 + I_2)}{\gamma_3 (\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)} \]

\[ Z_1 = \frac{H_3 + I_3}{\gamma_1 (\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)} \]

\[ Z_2 = \frac{H_3 + I_3}{\gamma_2 (\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)} \]

\[ Z_3 = \frac{H_3 + I_3}{\gamma_3 (\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)} \]

\[ X_4 = -I_1 / (\gamma_1 \gamma_2 \gamma_3) \]

\[ Y_4 = -I_2 / (\gamma_1 \gamma_2 \gamma_3) \]

\[ Z_4 = -I_3 / (\gamma_1 \gamma_2 \gamma_3) \]
The quantities $G_1, G_2, H_1, H_2, H_3, I_1, I_2, I_3$ and $\gamma_1, \gamma_2, \gamma_3$ associated with $X_1, X_2, X_3, X_4, Y_1, Y_2, Y_3, Y_4, Z_1, Z_2, Z_3, Z_4$ are defined in (4.8.9), (4.8.10), (4.8.11) and (4.8.8) respectively. Even in the parallel system, the expression given in (4.8.15) agrees with the formula when the present model (3-unit system) reduced to two unit system, already developed [82] when the system is affected by CCS failures in addition to individual failures (implies no human error is affecting the system i.e. $\lambda_h = 0 / c_3 = 0$).

**ILLUSTRATION: (4.5)**

The values of availability in the case of parallel system are obtained by taking chance of individual failures $c_1 = 0.5$, chance of CCS failures $c_2 = 0.25$, chance of human errors $c_3 = 0.25$ and different rates of failures and service rates as given in tables (4.9) & (4.10) and the availability plots are also drawn and shown in Fig. (4.8.4) & (4.8.5).
**Table: (4.9) Availability in the Case of CCS Failures and Human Errors – Parallel System**

\(c_1 = 0.5, \ c_2 = 0.25, \ c_3 = 0.25\)

<table>
<thead>
<tr>
<th>Time</th>
<th>(\lambda_i = 0.05, \lambda_c = 0.1, \lambda_h = 0.2)</th>
<th>(\lambda_i = 0.1, \lambda_c = 0.2, \lambda_h = 0.3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(R_{sh}(t))</td>
<td>(A_{ph}(t))</td>
</tr>
<tr>
<td></td>
<td>(\mu = 10)</td>
<td>(\mu = 10)</td>
</tr>
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<td>1.000000</td>
</tr>
<tr>
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</tr>
<tr>
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<td>0.997716</td>
</tr>
<tr>
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<td>0.997716</td>
</tr>
<tr>
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<td>0.997716</td>
</tr>
<tr>
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<td>0.697624</td>
<td>0.997716</td>
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<td>6</td>
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<td>0.997716</td>
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<tr>
<td>7</td>
<td>0.604088</td>
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</tr>
<tr>
<td>9</td>
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<td>0.997716</td>
</tr>
<tr>
<td>10</td>
<td>0.486765</td>
<td>0.997716</td>
</tr>
</tbody>
</table>
\( \lambda_i = 0.05, \lambda_r = 0.1 \& \lambda_h = 0.2 \) are failure rates / unit time

\( \mu = 15 \text{ repairs / unit time} \)

\( \mu = 10 \text{ repairs / unit time} \)

\( \mu = 10 \text{ repairs / unit time} \)

Fig. (4.8.4) : System Availability Plots for Parallel Configuration
**Table (4.10) Availability in the Case of CCS Failures and Human Errors - Parallel System**

\[ c_1 = 0.5, c_2 = 0.25, c_3 = 0.25 \]

<table>
<thead>
<tr>
<th>Time ( t )</th>
<th>( \lambda_i = 0.2, \lambda_c = 0.4, \lambda_h = 0.6 )</th>
<th>( \lambda_i = 0.5, \lambda_c = 1, \lambda_h = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>( R_{ph}(t) ) ( \mu = 10 )</td>
<td>( A_{ph}(t) ) ( \mu = 10 ) ( \mu = 15 )</td>
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<td>1.000000</td>
</tr>
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<tr>
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</tr>
<tr>
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<td>0.486366 0.992397 0.994868</td>
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<td>0.031477</td>
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<td>0.015780</td>
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<td>7</td>
<td>0.186245 0.992397 0.994868</td>
<td>0.007911</td>
</tr>
<tr>
<td>8</td>
<td>0.146510 0.992397 0.994868</td>
<td>0.003966</td>
</tr>
<tr>
<td>9</td>
<td>0.115252 0.992397 0.994868</td>
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</tr>
<tr>
<td>10</td>
<td>0.090663 0.992397 0.994868</td>
<td>0.000997</td>
</tr>
</tbody>
</table>
\( \lambda_1 = 0.2, \lambda_c = 0.4 \) & \( \lambda_h = 0.6 \) are failure rates / unit time
\( \mu = 15 \) repairs / unit time
\( \mu = 10 \) repairs / unit time

\( \lambda_1 = 0.5, \lambda_c = 1 \) & \( \lambda_h = 2 \) are failure rates / unit time

Fig. (4.8.5) : System Availability Plots for Parallel Configuration
4.8.5.2 STEADY-STATE AVAILABILITY – CCS FAILURES AND HUMAN ERRORS

We developed the steady state availability for both series and parallel configurations in the presence of CCS failures as well as human errors in addition to individual failures and are presented in this section.

A) AVAILABILITY – SERIES CONFIGURATION

The steady-state availability of series system can be obtained by using the final value theorem of Laplace transformation

\[
A_{\text{sh}}(\infty) = \lim_{t \to \infty} A_{\text{sh}}(t) = \lim_{t \to \infty} [ p_0(t) ]
\]

\[
= \lim_{s \to 0} [ s p_0^*(s) ]
\]

\[
A_{\text{sh}}(\infty) = - \frac{1}{1 / (\gamma_1 \gamma_2 \gamma_3)} \quad \text{-----------------(4.8.16)}
\]

Where \( I_1 \) and \( \gamma_1, \gamma_2, \gamma_3 \) are defined in (4.8.11) & (4.8.14) respectively.

B) AVAILABILITY – PARALLEL CONFIGURATION

The steady-state availability of the parallel system is derived because of the long run usage of the system (i.e. \( t \to \infty \))

\[
A_{\text{ph}}(\infty) = \lim_{t \to \infty} A_{\text{ph}}(t) = \lim_{t \to \infty} [ p_0(t) + p_1(t) + p_2(t) ]
\]
Using the final value theorem of the Laplace transformation, the availability would be

\[ A_{ph}(\infty) = - \frac{(I_1 + I_2 + I_3)}{(\gamma_1 \gamma_2 \gamma_3)} \]  

\[ \text{------------------------ (4.8.17)} \]

Where \( I_1, I_2, I_3 \) and \( \gamma_1, \gamma_2, \gamma_3 \) are given in (4.8.11) and (4.8.8) respectively.

### 4.8.6 FREQUENCY OF ENCOUNTERING THE DIFFERENT STATES – CCS FAILURES AND HUMAN ERRORS

The frequency of encountering the different states of the system in the presence of Common cause shock failures as well as human errors in addition to individual failures is developed in terms of the steady state probabilities of the different states.

\[ p_0 = - \frac{I_1}{(\gamma_1 \gamma_2 \gamma_3)} \]
\[ p_1 = - \frac{I_2}{(\gamma_1 \gamma_2 \gamma_3)} \]
\[ p_2 = - \frac{I_3}{(\gamma_1 \gamma_2 \gamma_3)} \]
\[ p_3 = - \frac{I_4}{(\gamma_1 \gamma_2 \gamma_3)} \]

Where \( I_4 = \left[ \lambda_{01} \lambda_{11} \lambda_{21} + (\lambda_{0c} + \lambda_{0b}) (\lambda_{11} \lambda_{21} + \mu_0 \lambda_{21} + \mu_0 \mu_1) \right] \)

and \( I_1, I_2, I_3 \) and \( \gamma_1, \gamma_2, \gamma_3 \) are given in (4.8.11) and (4.8.8) respectively. The quantities \( \lambda_{01}, \lambda_{11}, \lambda_{21}, \lambda_{0c}, \lambda_{0b}, \mu_0, \mu_1 \) and \( \mu_2 \) are given in (4.8.1)
4.8.6.1 FREQUENCY OF DOWN STATE – SERIES CONFIGURATION

The frequency of down-state in the presence of CCS failures as well as human errors along with individual failures.

\[ f_{sh} \text{ (down)} = - I_1 \left( 3 \lambda_i c_1 + \lambda_c c_2 + \lambda_h c_3 \right) / (\gamma_1 \gamma_2 \gamma_3) \]  \hspace{1cm} (4.8.18)

Where \( I_1 \) and \( \gamma_1, \gamma_2, \gamma_3 \) are given in (4.8.11) and (4.8.8)

4.8.6.2 FREQUENCY OF DOWN STATE – PARALLEL CONFIGURATION

The frequency of down-state of the parallel system in the case of CCS failures and human errors along with individual failures is derived as

\[ f_{ph} \text{ (down)} = - 3 \mu \left[ (\lambda_c c_2 + \lambda_h c_3) (2\mu^2 + \lambda_i c_1 \mu + 2 (\lambda_i c_i)^2) + 6 (\lambda_i c_1)^3 \right] / (\gamma_1 \gamma_2 \gamma_3) \]  \hspace{1cm} (4.8.19)

Where \( \gamma_1, \gamma_2, \gamma_3 \) are defined in (4.8.8)

ILLUSTRATION: (4.6)

The values of frequency of down-state of the system for both series and parallel configurations are obtained by taking chance of individual failures \( c_1 = 0.5 \), chance of CCS failures \( c_2 = 0.25 \), chance of human errors \( c_3 = 0.25 \) and service rate \( \mu = 5 \) repairs / unit time for various rates of failures as presented in table (4.11)
**Table: (4.11) Frequency of Down-State — Three Component System — Series and Parallel Configurations**

<table>
<thead>
<tr>
<th>$\mu = 5$</th>
<th>$c_1 = 0.5$, $c_2 = 0.25$, $c_3 = 0.25$</th>
<th>Series configuration</th>
<th>Parallel configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda_i$</td>
<td>$\lambda_c$</td>
<td>$\lambda_h$</td>
</tr>
<tr>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.027297</td>
</tr>
<tr>
<td>0.02</td>
<td>0.04</td>
<td>0.06</td>
<td>0.054186</td>
</tr>
<tr>
<td>0.03</td>
<td>0.06</td>
<td>0.09</td>
<td>0.080674</td>
</tr>
<tr>
<td>0.04</td>
<td>0.08</td>
<td>0.12</td>
<td>0.106770</td>
</tr>
<tr>
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</tr>
<tr>
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<td>0.157814</td>
</tr>
<tr>
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<td>0.182778</td>
</tr>
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</tr>
<tr>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.255521</td>
</tr>
</tbody>
</table>

**Note:** The values in the table represent the frequency of down-state for both series and parallel configurations with given parameters $\lambda_i$, $\lambda_c$, and $\lambda_h$, and $\mu = 5$. The configurations are used to analyze the system's reliability and performance.
4.8.7 DISCUSSION

The availability measures for three component identical system, which is affected by common cause shock failures as well as human errors in addition to individual failures, such as time-dependent availability, steady state availability and frequency of down state of the system for both series and parallel configurations are developed. The reliability function and availability function are compared in the case of CCS failures as well as human errors and it is observed that \(A(t) \geq R(t)\) remains valid in the present model. This is verified with various values of service rate \(\mu\) and with various levels of chance of individual, CCS failures and human error rates, i.e. \(c_1 = 0.5, c_2 = 0.25, c_3 = 0.25\) we observed from tables (4.7 - 4.10), the system availability decreases as the rate of individual, CCS and human error failures increases. This confirms the fact that the influence of failure rates reduces the availability of the system.

The frequency of down-state is computed for various values of failure rates and presented. It is observed from table (4.11) that the frequency of the down-state for both series and parallel configurations increases with increase in rate of individual, CCS and human error failures. However, it is also observed that the frequency of down-state in the series system is more than the parallel system. By and large, the availability measures and frequency of down-state of three-component series and parallel system in the context of CCS failures as well as human errors are reduced and hence these failures need to be consider in the present work.