REVIEW ON
CUSUM CHARTS AND CHANGE POINT MODEL
CHAPTER II

REVIEW ON CUSUM CHARTS AND CHANGE POINT MODEL

The Cumulative Sum Charts or “CUSUM Charts” as it has come to be called is first introduced by British statistician PAGE, E.S. [1954], lead to a fewer researchers attention in this area of quality control. The contribution on CUSUM charts subsequent to the introduction by PAGE, E.S. and their state of art of literature on CUSUM charts does not seem to be reviewed and reported. The published work is also scantily in nature which promoted to make an attempt in this direction to review comprehensively the work on CUSUM charts and thus focus is emphasized in this chapter to review the art of literature right from PAGE’S contribution to till date and it is believed that the present literature survey would help researchers, to be aware of quantum of literature available in the theory of Cumulative Sum Charts. PAGE, E.S. [1955] contributed his might towards to the theory of CUSUM charts establishing the warning for the charts. KEMP.K.W.
[1958] suggested formulas for calculating the operating characteristic curve and the average sample number required for the sequential tests.

Control charts play an important role in SPC applications. Control charts are used to distinguish situations where only common causes of variations affect process outcomes from situations where, also, special causes are present. However, they do not indicate when special causes actually occurred. The Change Point Method formulation is an SPC technique aimed at knowing the time of process changes. The CPM can be used in three situations:

1. All parameters are known

2. Only the in control parameters known

3. None of the parameters is known.

The CPM formulation to be useful in finite horizon auto correlated data situation. The unknown parameters CPM formulation is attractive SPC tool in short run manufacturing environment, since it allows starting meaningful control charting at almost the begin of the run. However, they did not give any solution to reduce the numbers of control charts, and this is really one of the most important finite horizon SPC problems.
REVIEW OF LITERATURE - PERFORMANCE OF CUSUM CHART:

ALEX J. KONING [95], studied a general CUSUM chart for preliminary analysis of individual observations. A simulation study shows that certain implementations of these charts are highly effective in detecting assignable causes. Also explained the previously reported virtual equivalence between Girshick-Rubin[1952] and a CUSUM and generalize Girshick-Rubin to non-i.i.d. observations and non-constant hazard.

BISEL, A. F [9] proposes a novel idea for construction of V-mask for CUSUM charts not in the conventional fashion but suggested a semi-parabolic mask, which is used in detecting the shifts in the process average.

BISEL, A. F [10], continued his semi-parabolic procedure for construction of CUSUM charts by considering the correlations of the semi-parabolic masks.

BROOKS D AND EVANS D.A [13], discussed a classical method of studying a CUSUM control scheme of the decision interval type has been to regard the scheme as a sequence of sequential tests, to determine the average sample number for these component tests and to study ARL for the scheme. Examples are also given for the cases of a Poisson random variable and normal random variable.

CHARLES W. CHAMP AND WILLIAM H. WOODALL [15] gave a simple and efficient method, using Markov chains, to obtain the exact run-length properties of Shewart control charts with supplementary runs rules. Average run-length comparisons are
made among the Shewart $\bar{X}$ chart with supplementary runs rules, the basic Shewart $\bar{X}$ chart, and the cumulative sum (CUSUM) chart.

Cox [21], examined two systems of formulas for deriving the parameters of CUSUM chart, decision interval ‘$h$’ and reference value ‘$k$’ for selected values of the in control and out of control average run lengths.

Crowder, Stephen V. [26], presented a numerical procedure for the tabulation of average run lengths (ARLs) of a control chart for individual measurements in combination with a moving range chart based on two consecutive measurements. An exact expression for the ARL is given in terms of an integral equation, approximated using numerical Quadrature. ARLs are given for various settings of the control limits and shifts in the nominal level of the process mean and standard deviation. A control chart design strategy is presented.

Dong Han [33], compared the performance of Cumulative Score (CUSCORE), generalized likelihood ratio test (GLRT), and Cumulative sum (CUSUM) charts in detecting a dynamic mean change that finally approaches a steady-state value. Theoretical results in ARL comparisons are provided and proved that GLRT has the best performance among the three charts in detecting any mean change.

A. de Vries and B. J. Conlin [1], discusses statistical process control (SPC) charts to monitor production processes that have not been widely used in dairy management. Shewart and cumulative sum control charts were designed to determine true changes in Estrous Detection Efficiency (EDE) amidst normal
variation in dairy cattle. A stochastic simulation model was used to track performance over time of individual cows in herds of 100 and 1000 cows. Statistical process control charts detected changes in estrous detection efficiency soon enough to be potentially useful in dairy management.

DOUGLAS M. HAWKINS [58], A possible alternative to the hypothesis that the sequence X1, X2, …, Xn are i.i.d N(µ, σ2) random variables is that at some unknown instant the expectation µ shifts. The Likelihood ratio test for the alternative of a location shift is studied and its distribution under the null hypothesis found. Tables of standard fractiles are given, along with asymptotic results.

DOUGLAS M. HAWKINS [60], uses the running mean and standard deviation of all observations made on the process since start-up as substitutes for unknown true values of the process mean and standard deviation to determine the drifts from the conditions obtained at the start-up.

FELLNER, W.H [40] proposed a new average run length for cumulative sum schemes and proposed a standard algorithm for optimization.

GEORGE BOX AND JOSE RAMIREZ [45] developed CUSCORE statistics to identify a kind of departure in Shewhart charts, which can be used as an adjunct to the Shewhart charts. The resulting procedures relate to Wald-Barnard sequential tests and to CUSUM statistics, which are special cases of CUSCORE statistics.
Goel, A.L [49], is honored with a doctorate degree for his commanding work on comparative and economic investigations of Shewart charts and CUSUM charts.

Goel, A.L and Subba Rao, S [46] present an approach based on plotting the cumulative sums of queue lengths observed to control the traffic intensity in a queue. This procedure provides the test for detecting shifts in traffic intensity and different queue models are developed.

Goel, A.L and Wu, S.W [47] developed the determination of ARL, which is an important criterion in establishing the CUSUM procedure as it gives the probability of acceptance, through a counter Nomogram that suggest the limitations of CUSUM charts to control the normal mean in particular fashion.

Goel, A.L and Wu, S.W [48] studied the properties of the economically optimum design of constructing cumulative sum control charts.

Gonen Singer and Irad Ben-Gal [50], the classical funnel experiment was used by Deming to promote the idea of Statistical Process Control (SPC). The popular example illustrates that the implementation of simple feedback rules to stationary processes violates the independence assumption and prevents the implementation of conventional SPC. However, Deming did not indicate how to implement SPC in the presence of such feedback rules. This pedagogical gap is addressed in this paper by introducing a simple feedback rule to the funnel example that results in a nonlinear process to which the traditional SPC methods cannot be applied. The proposed
Performance of CUSUM Schemes and Change Point Model With Varying Distributions

method of Markov-based SPC (MSPC), which is a simplified version of the CSPC method, is shown to well monitor the modified process.

Hawkins, D.M [61], proposed a fast accurate approximation procedure to find out the average run lengths of CUSUM charts procedures.

Hung-Man Ngai and Jian Zhang [66], proposes a natural multivariate extension of the two-sided cumulative sum chart via projection pursuit. A modification is given for improving its performance for the special situation in which the process mean is already shifted at the time the charting begins. Simulation studies show that the new charts have slightly better performance than the competing charts in terms of the average delayed run length and standard deviation of the delayed run length, while performing a little worse in terms of the average run length.

James, O. Westgard [72], describes the adaptation of the decision limit cumulative sum method (CUSUM) to internal quality control in clinical chemistry. With the decision limit method, the CUSUM is interpreted against a numerical limit, rather than by use of a V-mask. The method can be readily implemented in computerized quality-control systems or manually on control charts. Computer simulation studies are used to determine the performance characteristics of several different CUSUM rules, alone and in combination with a Shewart rule. These studies indicate that improvements in existing quality-control systems should be possible by addition of this simple CUSUM method and by use of a combined Shewart-CUSUM control chart.
James C. Benneyan [68], developed an alternative Shewart-type statistical control chart for monitoring the number of cases between hospital-acquired infections and other adverse events. The author investigates the statistical properties of these new charts and illustrates several design consideration that significantly can improve their operating characteristics and sensitivity.

James C. Fu Fred A. Spiring and Hansheng Xie [69], developed a simple unified method based on a finite Markov chain approach for finding the run length distribution and ARL of a control scheme. In addition, the method yields the variance or standard deviation of the run length as a by product. Numerical results illustrating the results are given.

James M. Lucas, et al [71], considered Fast Initial Response (FIR) feature for CUCUM quality control schemes, which permits a more response to an initial out-of-control situation than does a standard CUSUM quality control scheme. A comparison is also made and showed that if the process starts out of control, the FIR feature has little effect, however, if the process mean is not at the desired level, an out-of-control signal will be faster when the FIR feature is used.

Johnson N.L [75], described a method for construction of CUSUM control charts for controlling the mean of a Weibull distribution. As a special case charts appropriate to exponentially distributed variables can be constructed.

Johnson, N.L and Kotz [76] studied various continuous and multivariate statistical distributions, which are useful to construct cumulative sum control charts.
JONES, L. ALLISON; WOODALL, WILLIAM H.; CONERLY, MICHAEL D [80], used a demerit rating system to simultaneously monitor counts of several types of defects in a complex product; the demerit statistic is a linear combination of the counts of these types of defects. The traditional recommendations are made to plot the demerit statistic on a control chart with symmetric 3-sigma control limits and this approach is reviewed. Also proposes an alternative method for determining control limits for the demerit control chart, which is based on the exact distribution of linear combinations of independent Poisson random variables.

JOSEPH J. PIGNATIELLO, JR [81], considered several distinct approaches for controlling the mean of a multivariate normal process including two new and distinct multivariate CUSUM charts, several multiple Univariate CUSUM charts, and a Shewart control chart. A Markov chain is used to evaluate the ARL performance of one of the charts while Monte Carlo simulation is used to evaluate the other multivariate schemes. ARL data are also presented.

KEITH M. BOWER, M.S.[86], In order to monitor a process, quality practitioners frequently use Shewart control charts(e.g. $\bar{X}$, R, P-charts, etc.), so named after the pioneering work of Dr. Walter Shewart, shown that if there are sharp, intermittent changes to a process, these types of charts are highly effective. However, if one is interested in a small, sustained shift in a process, other types of control charts may be preferred, for example the Cumulative Sum (CUSUM) chart.
KEMP, K.W [87], is the pioneering author in developing the nomograph for the CUSUM charts to calculate the Average Run Length (ARL).

KEMP, K.W [88], suggested some of the formal expressions, which are highly useful and applied to CUSUM charts in both practice and industrial situations.

KEN NISHINA AND SHINTARO NISHIYUKI [89], compared standard CUSUM charts (with head start value zero) with fast initial response (FIR) CUSUM charts proposed by Lucas and Crosier [1982] from the viewpoint of the false alarm probability function. The focus in this comparison is conservativeness, which is based on the relationship between the in control ARL and the false alarm probability function by regarding the CUSUM statistic as a Markov process. The comparison shows that standard CUSUM charts are conservative in contrast to FIR CUSUM charts.

KHAN, R. A [93], uses Wald's approximation to obtain average run length in CUSUM chart procedures.

KHAN, R. A [94], considered both one-sided and two-sided cumulative sum procedures of PAGE, E. S [1954] and showed it to be closely related to a sequential probability ratio tests and developed the generating functions of Cumulative Sum procedures.

L. ALLISON JONES, RIGDON, S.E. AND CHAMP, C.W [79], derived the run length distribution of the EWMA chart with estimated parameters. EWMA control chart is
Performance of CUSUM Schemes and Change Point Model With Varying Distributions

typically designed assuming that standards are given for the process parameters. In particular are rarely known, and control charts are constructed using estimates in place of the parameters. This practice can affect the control chart's run length performance in both in-control situations. Specifically, estimation can lead to substantially more false alarms that are frequent and yet reduce the sensitivity of the chart to detecting process changes the effect of estimation on the performance of the chart is discussed in a variety of practical scenarios.

**LEWIS VANBRACKLE [97]**, examines the statistical properties in detecting patterns of National Notifiable Disease Surveillance system. Control charts are applied, simulation and analytical techniques are used to study ARL characteristics of these control charts for various types of changes in the series. The ARLs for the highly correlated disease series are much longer than for the usual independent data case.

**LIM, T. O ET AL. [99]**, applied CUSUM charting to assess doctors' performance of endoscopic retrograde pancreatography, renal and breast biopsies, thyroidectomy, and instrumental delivery and showed at acceptable levels of performance, the CUSUM curve is flat, while at unacceptable levels of performance, the curve slopes upward and eventually crosses a decision interval. Also determined the usefulness and acceptability of CUSUM charting for assessing doctors' performance

**LUCAS, J. M [102]**, proposed a combined working scheme of Shewart CUSUM quality control charts citing their applications in practice.
Lucas, J.M [103]) has recommended a simple procedure for a modified V-mask procedure developed by Page, E.S.

Lucas, J.M [101], discussed the design and use of cumulative sum control charts to control quality.

Munford, A.G [115] proposed CUSUM schemes using a simple scoring system for controlling the mean of a normal distribution in the one and two-sided decision procedures, the ARL of the schemes are proved to be simple compared to that of cumulative sum schemes.

N.L. Johnson [74], considering the use of a CUSUM chart, in the way described Page and Barnard, as the application of two SPRT tests to the observed series taken in reversed order, some approximate formulae for properties of this procedure are obtained. These are simple probabilities of detection of a given change in average value to the position and slope of the critical limits. It is suggested that these results may sometimes be useful as guide to the limits to be used.

N.L. Johnson [77], described the methods of construction of CUSUM charts for folded normal variate. These charts are likely to be useful when the sign of an approximately normally distributed quantity is lost in measurement. Some assessment is given of the information lost by omission of the sign.

Nabar, S.B and Shoba, [116], develops CUSUM chart for inverse Gaussian distribution and this as many similarities to the Normal or Gaussian distribution, even the CUSUM chart shows remarkable similarity.
NAZATUL AINI ABD MAJID, MOHAMMED KHATIM HASAN, HAZURA MOHAMED AND ABD MALIK MD YUSOF [118], introduces an abdicative inference model and a new algorithm based on the adductive inference for handling control chart selection. Using adductive logic, a reverse system model is generated. This system model takes the observation data as inputs and produces a diagnosis by generating sets of data characteristics that explain the observation data.

NEIL B. MARKS AND TIMOTHY C. KREHBIEL [119], evaluated the design of individuals and moving range charts through extensive simulations. Recommendations are made concerning when to use the individuals chart only, when to use a combined individuals and moving range chart, and the optimal design parameters when the combined approach is used.

PAGE, E.S [126] developed CUSUM charts in a mathematical fashion using gauzing techniques, considering the importance of sampling techniques in the area of quality control.

PETER A. ROGERSON [142], provides an approximate formula to calculate the threshold directly from pre specified values of the reference value ($k$) and the in-control average run length ($ARL_0$). Formulas are also provided for choosing $k$ and $h$ from pre specified values of the in-control and out-of-control average run lengths.

PETER LORSCHEID [129], while in the Univariate case applying CUSUM techniques to quality control problems has been discussed intensively, in the multivariate case it is usually recommended monitoring the principle components of
the multivariate data simultaneously. But doing this, the multivariate character of the data is no longer present. This article treats the problems arising when CUSUM techniques are applied directly to the original multivariate data. Two approaches are presented to handle these problems: expectation-adjusted CUSUM and vector-modified CUSUM. While the first one is a heuristically solution of the problems, the second approach is based directly on sequential likelihood ratio theory. Experiences of the author have shown that the second approach reduces the number of observations that are necessary to monitor the process in the same magnitude as in the Univariate case.

POETRODJOJO et al [133], designed optimal CUSUM schemes to detect small and large increase in variability of a normal process. It is also shown that CUSUM with control statistics sample variance and sample standard deviation perform uniformly better than those with control statistic log S2. Fast initial response CUSUM properties are also presented.

REYNOLDS, M. R, JR AND ARNOLD, J. C [137], provided their contribution the theory of CUSUM charts by optimizing the one-sided Shewart charts with variable sampling intervals.

REYNOLDS, M. R, JR AND ARNOLD, J. C. [136], using the concepts of sampling theory establishes CUSUM charts with variable sampling intervals by treating standard CUSUMs for controlling the process mean which takes the samples from the
Performance of CUSUM Schemes and Change Point Model With Varying Distributions

process at fixed length sampling intervals using a control statistics by way of cumulative sums of differences between sample mean and target value.

RONALD B. CROAIER [143], presents the design procedures and average run lengths for two multivariate CUSUM quality control procedures. The first CUSUM procedures reduce each multivariate observation to a scalar and then form a CUSUM of the scalars. The second CUSUM procedure forms a CUSUM vector directly from the observations. These two procedures are compared with each other and with the multivariate Shewart chart. Other multivariate quality-control procedures are mentioned robustness, the initial response features for CUSUM schemes, and combined Shewart-CUSUM schemes are discussed.

RRL KANTAM, ET AL [83], applied sequential probability ratio procedures in construction of CUSUM charts for a variable process characteristic. The construction of mask and the values of ARL are also presented with distribution of the process variate is log-logistic distribution.

S. BERSIMIS J. PANARETOS AND S. PSARAKIS [7], Woodall and Montgomery [183] in a discussion paper, states that multivariate process control is one of the most rapidly developing sections of statistical process control. Nowadays, in industry, there are many situations in which the simultaneous monitoring or control, of two or more related quality - process characteristics is necessary. Process monitoring problems in which several related variables are of interest are collectively known as Multivariate Statistical Process Control (MSPC).This article
Performance of CUSUM Schemes and Change Point Model With Varying Distributions

has three parts. In the first part, the author discusses in brief the basic procedures for the implementation of multivariate statistical process control via control charting. In the second part, the author presented the most useful procedures for interpreting the out-of-control variable when a control charting procedure gives an out-of-control signal in a multivariate process. In the third part, the author presents the applications of multivariate statistical process control in the area of industrial process control, informatics, and business.

Saowanit Sukparungsee and Yupaporn Areepong [145], explicit formula for characteristics of EWMA as Average Run Length – the expectation of false alarm times and Average Delay time (AD) – the expectation of delay of true alarm times in case of Weibull distribution. Using the simple transformation technique, obtained the explicit expressions for evaluating ARL and AD when observations are Weibull by taking power of such observations. The accuracy of results is also compared with Monte Carlo simulations. Also presented the table of optimal parameter values for Weibull EWMA designs and the comparisons of performance of EWMA versus CUSUM charts are considered.

Shih-Hung Tai et al [149], develops the composite Shewart $\bar{X}$ and generally weighted average moving (GWMA) control chart to monitor process mean or variability. A simulation is conducted to evaluate the average run length (ARL) to false alarm and to monitor the shift in the process of the composite $X$-EWMA chart and a group of composite $X$-GWMA statistical control schemes. Extensive comparison shows that the $X$-GWMA control chart is more sensitive than the $X$-
EWMA control chart on monitoring the small shifts in the process mean or variability.

**Spencer Graves** [154], showed how the Girshick-Rubin [1952] monitor can be rewritten as a traditional one-sided CUSUM plus a correction term that generally provides fairly consistent bias and is often quite small.

**Stefan H. Steiner** [159], studied the run length properties of EWMAs with time-varying control limits are approximated using non-homogeneous Markov chains. Comparing the average run lengths of EWMA with time-varying control limits and results previously obtained for asymptotic EWMA charts shows that using time varying control limits is akin to the fast initial response (FIR) feature suggested for Cumulative Sum (CUSUM) charts. The methodology is illustrated assuming a normal process with known standard deviation where we wish to detect shifts in the mean.

**Stephen V. Crowder** [27], presented a numerical procedure using integral equations for the tabulation of moment of run length of exponentially weighted moving average charts. Both average run lengths and standard deviation of run lengths are presented for the two sided EWMA chart assuming normal observations, along with an example illustrating how to design such a chart. The procedure given extends easily to many non-normal cases and to one-sided versions of the EWMA chart.
TAYLOR, H.M [162], presented the very important aspect of cumulative sum chart namely their economic design of charting.

THE BRITISH STANDARDS INSTITUTION [1979-81], carried out a research project on data analysis and quality control using Cumulative Sum techniques proposing the decision rules and statistical tests for the development of CUSUM charts detailing their tabulation.

RENDTEL [135], investigated CUSUM-schemes with variable sampling interval and sample sizes are intended for situations where a production process switches at an unknown time from an in-control state to an out-of-control state. Suitable performance criteria are derived to compare CUSUM schemes with this additional feature.

VANDOBKEN DE BRUYN, C.S [169], suggested the cumulative theory and practice test procedures in his earlier work continued his contribution in establishing the nomographs for practical use and suggested the course material.

WALDMANN, K.H [171, 172], comes out with a solution for the bounds for concerned distributions of the Run length and these bounds are developed for both one-sided and two-sided decision schemes.

WOODALL, W. H [183], proposes a resolution of disagreements in order to improve the communication between practitioners and researchers. Disputes over the theory
and applications of SPC methods are frequent and often very tense; some of the controversies and issues are discussed.

WOODALL, W. H [179], proposed the distribution of run length for the one sided CUSUM procedures for continuous random variables and for the two-sided CUSUM procedures in [1984] [146] the Markovian chain approach is applied to establish the characteristics of CUSUM decision schemes.

Y. F. TAN AND A. H. POOI [161], derived iterative formulas for finding the run length distribution of two-sided CUSUM. The application of the iterative formulas is also illustrated in the normal two-sided CUSUM.

YAJUN MEI [187], constructed a simple counter example to the conjunction of Pollak [1985] and Yakir, Krieger and Pollak [1999], which states that PAGE's CUSUM procedure and the Shiryayev-Roberts procedure are asymptotically minimax optimal for dependent observations. An example is also discussed showing that the relationship between open-ended tests and change point detection procedures no longer holds for dependent observations.

YASHCHIN, E [188], accounted the performance of CUSUM control schemes for the serially correlated observations and in the same year [1989a], he recommends the analysis of CUSUMs and Marko type central schemes by using empirical distributions.
Performance of CUSUM Schemes and Change Point Model With Varying Distributions

Yoav Benjamini and Yechezkel Kling [190] address the use in SPC of the p-values, or their monotone transformation to the observed average run length value. We argue that the use of p-values in SPC carries with it major benefits and also demonstrate by offering modifications to the P-charts, mean & S charts, and CUSUM, the latter requiring special approximations to the p-value. Adjusted p-values for multiplicity control in SPC are used for examining ten control charts for five quality attributes running in parallel.

Yuan Guo and Kevin J. Dooley [192], describes how neural networks and Bayesian discriminate function techniques can be used to provide knowledge of how a product characteristic changed. The paper also addresses process change detection, feature vector selection, training patterns and error rates. Simulation experiments are used to test various hypotheses and compared the effectiveness of two proposed approaches against two similar heuristics.

Review on Change point Method:

Abdelmonem Snussi and Mohamed Limam [152], proposed the unknown parameters change point formulation in conjunction with residuals of various time series models as a statistical process control alternative for short run auto correlated data. Based on the average run length and standard deviation of the run length as criteria of control chart's performance, the proposed alternative is compared to other short run SPC techniques. Simulation results show that the change point
model formulation provides better shift detection properties than residual charts based on the Q statistics.

CHANGLIANG ZHOU [16] proposed a control chart based on the change-point model that is able to monitor linear profiles whose parameters are unknown, but can be estimated. Simulation results shows the proposed approach performs well across a range of possible shifts, and that it can be used during the start-up stages of a process. This chart can detect a shift in the intercept, slope or standard deviation.

DAVID V. HINKLEY [64], discusses the point of change in mean in a sequence of normal random variables can be estimated from a cumulative sum test scheme. The asymptotic distribution of this estimate and associated test statistics are derived and numerical results given. The relation to likelihood inference is emphasized. Asymptotic results are compared with empirical sequential results, and some practical implications are discussed.

DAVID V. HINKLEY [65], derived the asymptotic distribution of the maximum likelihood estimate and also the asymptotic distribution of the likelihood ratio statistic for testing hypotheses about the change-point. These asymptotic distributions are compared with some finite sample empirical distributions.

DOUGLAS M. HAWKINS AND PEIHUA QIU [54], SPC requires statistical methodologies that detect changes in the pattern of data over time. The common methodologies, such as Shewhart, CUSUM, and exponentially weighted moving
average charting, require the in-control values of the process parameters, but these are rarely known accurately. Using estimated parameters, the run length behavior changes randomly from one realization to another, making it impossible to control the run length behavior of any particular chart. A suitable methodology for detecting and diagnosing step changes based on imperfect process knowledge is the un-known parameter change point formulation. Long recognized as a Phase I analysis tool. He argue that it is also highly effective in allowing the user to progress seamlessly from the start of Phase I data gathering through Phase II SPC monitoring. Despite not requiring specification of the post-change process parameter values, its performance is never far superior for shifts away from the CUSUM shift for which the CUSUM chart is optimal. As another benefit, while change point methods are designed for step changes that persist, they are also competitive with the Shewart chart, the chart of choice for isolated non-sustained special causes.

DOUGLAS M. HAWKINS, PEIHUA QIU AND K. D. ZAMBA [57], applied Change-point methodologies to statistical process control are predicated on the possibility that a special cause induces a shift from an in-control statistical model to an out-of- control statistical model, and so are particularly attractive for persistent special causes. Along with indications of a loss of control, they provide estimates of when the shift occurred, and (if needed) of the before- and after-shift process parameters. A less obvious advantage is that some change-point proposals allow for the near-universal situation that the in-control distribution of process readings is not known exactly. This feature largely removes the need for extensive Phase I
calibration studies, and allows Phase II production use to start early. In addition to
the stand-alone use of change-point methodologies for both signalling and
diagnosing the effects of special causes, they have been proposed as tools for
following up signals given by other charting methods, when their likelihood
properties lead to good estimators of the time of occurrence and effect of the special
cause.

**Douglas M. Hawkins and K. D. Zamba** [56], Statistical process control (SPC)
involves ongoing checks to ensure that neither the mean nor the variability of the
process readings has changed. Conventionally, this is done by pairs of charts—
Shewart $\bar{X}$ and S (or R) charts, cumulative sum charts for mean and for variance, or
exponentially weighted moving average charts for mean and variance. The
traditional methods of calculating the statistical properties of control charts are
based on the assumption that the in-control true mean and variance were known
exactly, and use these assumed true values to set centerlines, control limits, and
decision intervals. The reality, however, is that true parameter values are seldom if
ever known exactly; rather, they are commonly estimated from a phase I sample.
The random errors in the estimates lead to uncertain run length distribution of the
resulting charts. An attractive alternative to the traditional charting methods is a
single chart using the unknown-parameter likelihood ratio test for a change in mean
and/or variance in normally distributed data. This formulation gives a single
diagnostic to detect a shift in mean, in variance, or in both, rather than two separate
diagnostics. Using the unknown parameter formulation recognizes the reality that at
best one has reasonable estimates of parameters and not their exact values. This
description implies an immediate benefit of the formulation, that the run behavior is controlled despite the lack of a large phase I sample. We demonstrate another benefit, that the change point formulation is competitive with the best of traditional formulations for detecting step changes in parameters.

KEOAGILE THAGA [91], referred CUSUM control chart as SS- CUSUM chart and is proposed that is capable of simultaneously detecting changes in both mean and standard deviation. This chart construction is based on sum of squares of the maximum standard CUSUM statistics. The ARLs are computed for comparison with other charts particularly with some recently developed single charts.

KEYUE DING AND YANHONG WU [92], investigates the biases of estimates of change point and change magnitude after CUSUM test. By assuming that the change point is far from the beginning and the in-control average run length of samples is large, second order approximations for the biases of both estimates are obtained by conditioning on detection, and biases of both estimates are very significant. Simulation studies show the approximations to be quite accurate in the case of detecting an increase in mean or variance when sampling from a normal distribution. The results demonstrate the fundamental differences between fixed sample size test and sequential test

MAHMOUD A. MAHMOUD, PETER A. PARKER, WILLIAM H. WOODALL, DOUGLAS M. HAWKINS [107], proposes a change point approach based on the segmented regression technique for testing the constancy of the regression parameters in a linear profile data set. The change point approach is based on the likelihood ratio
test for a change in one or more regression parameters, compared the performance of this method to that of the most effective Phase I linear profile control chart approaches using a simulation study. The advantages of the change point method over the existing methods are greatly improved detection of sustained step changes in the process parameters and improved diagnostic tools to determine the sources of profile variation and the location(s) of the change point(s).

NADLER, J AND ROBBINS, N. B [117], envisaged the important characteristics for detecting a change particularly in location parameter for a two-sided CUSUM decision procedure of Page, E.S.

SEONG-HEE KIM, CHRISTOS ALEXOPOULOS, AND KWOK-LEUNG TSUI [148], designed a distribution-free tabular CUSUM chart to detect shifts in the mean of an auto correlated process. The chart's Average Run Length (ARL) is approximated by generalizing Sigmund's ARL approximation for the conventional tabular CUSUM chart based on independent and identically distributed normal observations. Control limits for the new chart are also computed from the generalized ARL approximation and discussed the choice of reference value and the use of batch means to handle highly correlated processes. The new chart is compared with other distribution-free procedures using stationary test processes with both normal and non-normal marginal.

SMILEY W. CHENG AND KEOAGILE THAGA [151], proposed a Cumulative Sum (CUSUM) control chart capable of detecting changes in both the mean and the standard deviation for auto correlated data, referred to as the Max-CUSUM
chart for Auto correlated Process chart (MCAP chart), based on fitting a time series model to the data, and then calculating the residuals. The observations are represented as a first-order autoregressive process plus a random error term. The Average Run Lengths (ARL's) for fixed decision intervals and reference values, \((h, k)\) are also calculated. The proposed chart is compared with the combined Shewart-EWMA chart for auto-correlated data.

**Uwe Jensen and Constanze Lu"ttebohmert** [166], Change-point models describe formally the problem to decide whether a stochastic process is homogeneous in some sense or not. Reviewed and classified various change-point models with a focus on the more recent ones. In particular, considered change-point detection models in which the decision is based on sequentially observed data. Both, discrete and continuous models are presented. Finally, gave an overview of some parametric and nonparametric regression and hazard rate models with change-points.

**Vera Do Carmo, et al.** [170], implemented the applications of CUSUM and EWMA control charts in general, especially in order to detect small changes in the process average and is noticed that CUSUM control charts are more effective with the changes in the order of more 1.0 \(\sigma\) and above, and for all the alterations in the order of less 1.124 \(\sigma\) down.

**Wang** [189], considered CUSUM control chart based on data depth for detecting a shift in either in the mean vector, the covariance matrix or both of the process. The
Performance of CUSUM Schemes and Change Point Model With Varying Distributions

proposed new control chart can detect small sample, is preferable from a robustness point of view. A diagnostic aid is also given to estimate the location of the change.

WAYNE A. TAYLOR [163], studied change-point analysis as a powerful tool for determining whether a change has taken place. It also better characterizes the changes, control the overall error rate, is robust to outliers, is more flexible and is simpler to use. The paper also describes how to perform a change-point analysis and demonstrates its capabilities through a number of examples.

YOSHINOBU KAWAHARA and MASASHI SUGIYAMA [191], Change-point detection is the problem of discovering time points at which properties of time-series data change. This covers a broad range of real-world problems and has been actively discussed in the community of statistics and data mining. In this paper, we present a novel non-parametric approach to detecting the change of probability distributions of sequence data. Our key idea is to estimate the ratio of probability densities, not the probability densities themselves. This formulation allows us to avoid non-parametric density estimation, which is known to be a difficult problem. We provide a change-point detection algorithm based on direct density-ratio estimation that can be computed very efficiently in an online manner. The usefulness of the proposed method is demonstrated through experiments using artificial and real datasets.

ZHONGHUA LI [194], proposes a new self-starting approach which integrates the CUSUM of Q chart with the feature of adaptively varying the reference value, to
better detect a range of shifts with unknown process parameter. The simulation results show the proposed chart offers a balanced protection against shifts of different magnitudes and has comparable performance with the dynamic change-point control scheme. The choice of the chart parameters and making effect are also studied. A real example from the industrial manufacturing is used for demonstrating its implementation. Change point.