CHAPTER 5

Effects of Hall currents on MHD flow of a couple stress through a composite medium in a parallel plate channel in presence of effect of inclined magnetic field
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Effects of Hall currents on MHD flow of a couple stress through a composite medium in a parallel plate channel in presence of effect of inclined magnetic field

5.1. Introduction:

The flow between parallel plates is a classical problem that has important applications in Magneto Hydro Dynamic (MHD) power generators and pumps, accelerators, aerodynamic heating, electrostatic precipitation polymer technology, petroleum industry, purification of crude oil and fluid droplets, sprays, designing cooling systems with liquid metal, centrifugal separation of matter from fluid and flow meters. Hartman and Lazarus [3] studied the influence of a transverse uniform magnetic field on the flow of a viscous incompressible electrically conducting fluid between two infinite parallel stationary and insulating plates. Then the problem was extended in numerous ways.
Closed form solutions for the velocity fields were obtained [1, 2 & 11] under the different physical effects. In the above mentioned cases the hall term was ignored in applying Ohm’s Law as it has no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of magneto-hydrodynamics is to words a strong magnetic field, so that the influence of electro-magnetic force is noticeable by Cramer et al [2]. Under these conditions, the hall current is important and it has a marked effect on the magnitude and direction of the current density and consequently on the magnetic force. Pop [5], Sato [9], Yamanishi [13], Sherman and Sutton [10] have discussed the hall effects on the steady hydro magnetic flow between two parallel plates.

Tani [12] studied the hall effect on the steady motion of electrically conducting and viscous fluids in channels. Linga Raju T and Ramana Rao [4] studied steady viscous incompressible fluid flow between two parallel walls in the presence of a uniform magnetic field applied transversely to the flow and when rotated at an angular velocity about an axis perpendicular to the walls, taking hall current into account. Rao and Krishna [6] studied hall effects on the non-torsionally generated unsteady hydro magnetic flow in semi-infinite expansion of an electrically conducting viscous rotating fluid. Krishna and Rao [7 & 8] discussed the Stokes and Ekmann problems in magneto-hydrodynamics taking hall effects into account. The effects of hall current on the hydrodynamic boundary layers and shear stress are discussed. In this paper, we discuss the hall effects on steady hydro magnetic flow of couple stress fluid through a porous medium in a rotating parallel plate channel.

In this chapter, the steady hydro magnetic flow of an incompressible electrically conducting couple stress fluid in a parallel plate channel bounded on one side by a porous bed under the influence of a uniform inclined magnetic field of strength \( H_0 \) inclined at an angle of inclination \( \alpha \) with the normal to the boundaries taking hall current into account is studied.
and analysed. The perturbations are created by a constant pressure gradient along the plates. The equations for the couple stress fluid flow in non-porous region are based on Stokes constitutive equations while in the porous bed the equations are based on Brinkman's model. The exact solutions of the velocity in the clean fluid and the porous medium consists of steady state are analytically derived. Its behaviour computationally discussed with reference to the various governing parameters with the help of graphs. The shear stresses on the boundaries and the mass flux are also obtained analytically and their behaviour is computationally discussed.

5.2. Formulation and Solution of the Problem:

Consider an incompressible viscous and electrically conducting couple stress fluid in a parallel plate channel bounded by a porous bed on the lower side. The fluid is driven by a uniform pressure gradient parallel to the channel plates and the entire flow field is subjected to a uniform inclined magnetic field of strength $H_0$ inclined at an angle of inclination $\alpha$ with the normal to the boundaries in the transverse $xy$-plane. Equation of motion along $x$-direction the $x$-component current density $\mu_e J_x H_0$ and the $z$-component current density $\mu_e J_z H_0$.

Choose a Cartesian system $O(x, y, z)$ such that the boundary walls are at $z=0$ and $z=1$ and are assumed to be parallel to $xy$-plane. The fluid medium consists of two zones namely zone 1 and zone 2. Zone 1 consists of clean fluid governed by Navier-Stokes equations and zone 2 corresponds to the flow through porous bed governed by Brinkman's equations. At the interface the fluid satisfies the continuity condition of velocity and stress. The boundary plates are assumed to be parallel to $xy$-plane and the magnetic field of strength $H_0$ inclined at an angle of inclination $\alpha$ to the $z$-axis in the transverse $xz$-plane. This inclined magnetic field on the axial flow along the $x$-direction gives rise to the current density along $y$-direction in view of Ohm's law. Also the inclined magnetic field in the presence of current density exerts
a Lorentz force with components along O(x, z) direction. The component along z-direction induces a secondary flow in that direction while its x-components changes perturbation to the axial flow.

The steady hydro magnetic equations governing the couple stress fluid in zone 1 under the influence of a uniform inclined magnetic field of strength $H_0$ inclined at an angle of inclination $\alpha$ with reference to a frame are

\[
\frac{\eta \, d^4 u}{\rho \, dz^4} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{d^2 u}{dz^2} - \frac{\mu_e J_x H_0 \sin \alpha}{\rho} 
\tag{5.2.1}
\]

\[
\frac{\eta \, d^4 w}{\rho \, dz^4} = \nu \frac{d^2 w}{dz^2} + \frac{\mu_e J_x H_0 \sin \alpha}{\rho} 
\tag{5.2.2}
\]

The Brinkman equations are governing flow through porous medium with respect to the frame in zone 2 are

\[
\frac{\eta \, d^4 u_p}{\rho \, dz^4} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu_{eff} \frac{d^2 u_p}{dz^2} - \frac{\mu_e J_x^p H_0 \sin \alpha}{\rho} - \frac{\nu}{k} u_p 
\tag{5.2.3}
\]

\[
\frac{\eta \, d^4 w_p}{\rho \, dz^4} = \nu_{eff} \frac{d^2 w_p}{dz^2} + \frac{\mu_e J_x^p H_0 \sin \alpha}{\rho} - \frac{\nu}{k} w_p 
\tag{5.2.4}
\]

Where, $(u, w)$ and $(u_p, w_p)$ are the velocity components along O(x, z) directions respectively. $\rho$ is the density of the fluid, $\mu_e$ is the magnetic permeability, $\nu$ is the coefficient of kinematic viscosity, $\nu_{eff}$ the coefficient of the effective kinematic viscosity, $k$ is the permeability of the medium, $H_0$ is the applied magnetic field. When the strength of the magnetic field is very large, the generalized Ohm's law is modified to include the Hall current, so that
\[ J + \frac{\omega_c \tau_c}{H_0} J \times H = \sigma (E + \mu_e q \times H) \]  

(5.2.5)

Where, \( q \) is the velocity vector, \( H \) is the magnetic field intensity vector, \( E \) is the electric field, \( J \) is the current density vector. \( \omega_c \) is the cyclotron frequency, \( \tau_c \) is the electron collision time, \( \sigma \) is the fluid conductivity and, \( \mu_e \) is the magnetic permeability.

In equation (5.2.5) the electron pressure gradient, the ion-slip and thermo-electric effects are neglected. We also assume that the electric field \( E=0 \) under assumptions reduces to

\[ J_x - m J_z \sin \alpha = -\sigma u_{ec} H_0 w \sin \alpha \]  

(5.2.6)

\[ J_z + m J_x \sin \alpha = -\sigma u_{ec} H_0 \mu \sin \alpha \]  

(5.2.7)

where \( m = \omega_c \tau_c \) is the hall parameter.

On solving equations (5.2.6) and (5.2.7) obtain

\[ J_x = \frac{\sigma u_{ec} H_0 \sin \alpha}{I + m^2 \sin^2 \alpha} (u_m \sin \alpha - w) \]  

(5.2.8)

\[ J_z = \frac{\sigma u_{ec} H_0 \sin \alpha}{I + m^2 \sin^2 \alpha} (u + w m \sin \alpha) \]  

(5.2.9)

Using the equations (5.2.8) and (5.2.9), the equations of the motion with reference to rotating frame are given by

\[ \frac{\eta d^4 u}{\rho dz^4} = -\frac{1}{\rho} \frac{\partial \rho}{\partial x} + \nu \frac{d^4 u}{dz^4} - \frac{\sigma u_{ec}^2 H_0^2 \sin \alpha}{\rho (I + m^2 \sin^2 \alpha)} (u + w m \sin \alpha) \]  

(5.2.10)

\[ \frac{\eta d^4 w}{\rho dz^4} = \nu \frac{d^4 w}{dz^4} + \frac{\sigma u_{ec}^2 H_0^2 \sin \alpha}{\rho (I + m^2 \sin^2 \alpha)} (u m \sin \alpha - w) \]  

(5.2.11)

and the equations of motion governing flow through a porous medium with respect to a rotating frame are given by
\[
\frac{\eta}{\rho} \frac{d^4 u_p}{dz^4} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{d^2 u_p}{dz^2} - \frac{\sigma_{\mu_e}^2 H_0^2 \sin^2 \alpha}{\rho(1 + m^2 \sin^2 \alpha)} (u_p + w_p m \sin \alpha) - \frac{v}{k} u_p
\]
(5.2.12)

\[
\frac{\eta}{\rho} \frac{d^4 w_p}{dz^4} = \nu \frac{d^2 w_p}{dz^2} + \frac{\sigma_{\mu_e}^2 H_0^2 \sin^2 \alpha}{\rho(1 + m^2 \sin^2 \alpha)} (u_p m \sin \alpha - w_p) - \frac{v}{k} w_p
\]
(5.2.13)

Let \( q = u + iw, \ q_p = u_p + iw_p \)

Now combining the equations (5.2.10) and (5.2.11), we obtain

\[
\frac{\eta}{\rho} \frac{d^4 q}{dz^4} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{d^2 q}{dz^2} - \frac{\sigma_{\mu_e}^2 H_0^2 \sin^2 \alpha}{\rho(1 + m^2 \sin^2 \alpha)} (1 - im \sin \alpha) q
\]
(5.2.14)

and combining equations (5.2.12) and (5.2.13), we obtain,

\[
\frac{\eta}{\rho} \frac{d^4 q_p}{dz^4} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{d^2 q_p}{dz^2} - \frac{\sigma_{\mu_e}^2 H_0^2 \sin^2 \alpha}{\rho(1 + m^2 \sin^2 \alpha)} (1 - im \sin \alpha) q_p - \frac{v}{k} q_p
\]
(5.2.15)

The boundary conditions are

\[
q_p = 0, \quad \text{at} \quad z = 0
\]
(5.2.16)

\[
q = 0, \quad \text{at} \quad z = l
\]
(5.2.17)

\[
\frac{d^2 q_p}{dz^2} = 0, \quad \text{at} \quad z = 0
\]
(5.2.18)

\[
\frac{d^2 q}{dz^2} = 0, \quad \text{at} \quad z = l
\]
(5.2.19)

The interfacial conditions are

\[
\begin{align*}
q &= q_p \\
\nu \frac{dq}{dz} &= \nu \frac{dq_p}{dz} \\
\nu \frac{d^2 q}{dz^2} &= \nu \frac{d^2 q_p}{dz^2} \quad \text{at} \quad z = h
\end{align*}
\]
(5.2.20)
We introduce the non-dimensional variables
\[ z^* = \frac{z}{l}, \quad q^* = \frac{q}{l}, \quad q_p^* = \frac{q_p}{l}, \quad p^* = \frac{\rho l^2}{\rho v^2}, \quad h^* = \frac{h}{l}. \]

The governing non-dimensional equations are (dropping asterisks)
\[
S \frac{d^4 q}{dz^4} - \frac{d^2 q}{dz^2} + \left( \frac{M^2 \sin^2 \alpha (1 - \text{Im} \sin \alpha)}{1 + m^2 \sin^2 \alpha} \right) q = P \quad (5.2.21)
\]
and
\[
S \frac{d^4 q_p}{dz^4} - \frac{d^2 q_p}{dz^2} + \left( \frac{M^2 \sin^2 \alpha (1 - \text{Im} \sin \alpha)}{1 + m^2 \sin^2 \alpha} \right) + D^{-1} \right) q_p = P \quad (5.2.22)
\]
where,
\[ M^2 = \frac{\sigma \mu_e^2 H_0 l^2}{\rho v} \] is the Hartmann number,
\[ m = \omega_c \tau_e \] is the hall Parameter,
\[ D^{-1} = \frac{l^2}{k} \] is the inverse Darcy parameter,
\[ S = \frac{\eta}{\rho l^2 v} \] is the Couple stress parameter,
\[ P = -\frac{\partial p}{\partial x} \] is the imposed pressure gradient.

Corresponding boundary conditions are
\[ q_p = 0, \quad \text{at} \quad z = 0 \quad (5.2.23) \]
\[ q = 0, \quad \text{at} \quad z = 1 \quad (5.2.24) \]
\[ \frac{d^2 q_p}{dz^2} = 0, \quad \text{at} \quad z = 0 \quad (5.2.25) \]
\[ \frac{d^2 q}{dz^2} = 0, \quad \text{at} \quad z = 1 \quad (5.2.26) \]
The interfacial conditions are

\[ q = q_p \quad \text{at} \quad z = h \quad \text{(5.2.27)} \]

\[ \frac{dq}{dz} = \beta \frac{dq_p}{dz} \quad \text{at} \quad z = h \quad \text{(5.2.28)} \]

\[ \frac{d^2q}{dz^2} = \beta \frac{d^2q_p}{dz^2} \quad \text{at} \quad z = h \quad \text{(5.2.29)} \]

\[ \frac{d^3q}{dz^3} = \beta \frac{d^3q_p}{dz^3} \quad \text{at} \quad z = h \quad \text{(5.2.30)} \]

where \( \beta = \frac{v_{eff}}{v} \)

\[ q = Ae^{m_1 z} + Be^{-m_2 z} + Ce^{-m_3 z} + De^{-m_4 z} + \frac{P}{M_2^2 \sin^2(1 - \text{Im} \sin(\alpha))} \frac{1}{1 + m_2^2 \sin^2(\alpha)} \quad \text{(5.2.31)} \]

\[ q_p = Ee^{m_1 z} + Fe^{-m_2 z} + Ge^{-m_3 z} + He^{-m_4 z} + \frac{P}{M_2^2 \sin^2(1 - \text{Im} \sin(\alpha))} \frac{1}{1 + m_2^2 \sin^2(\alpha)} + D^{-1} \quad \text{(5.2.32)} \]

Where the constants \( A, B, \ldots, H \) are mentioned in the appendix.

The shear stresses on the upper plate and lower plate are given by

\[ \tau_U = Am_1 e^{m_1} + Bm_2 e^{m_2} - Cm_1 e^{-m_3} - Dm_2 e^{-m_4} \]

and

\[ \tau_L = (E - G)m_3 + (F - H)m_6 \]

To determine the mass flux by the formula

\[ Q_x + iQ_y = \int_h^l q \, dz \]

the mass flux

\[ Q = \sqrt{Q_x^2 + Q_y^2} \]

\[ Q_x + iQ_y = \frac{A}{m_1} [e^{m_1} - e^{-m_1}] + \frac{B}{m_2} [e^{m_2} - e^{-m_2}] - \frac{C}{m_3} [e^{-m_3} - e^{-m_3}] - \frac{D}{m_4} [e^{-m_4} - e^{-m_4}] \]
5.3. **Results and Discussion:**

The study informs the facts that the behaviour of the flow field with fixed angle of inclination, the shear stresses and the mass flux with reference to the variations in the governing parameters $M$ the Hartmann number, $D^{-1}$ the inverse Darcy parameter, $S$ the couple stress parameter and $m$ the hall parameter, in either cases of small and larger thickness of porous bed. The velocity components for different variations of governing parameters have been plotted in figures (1-16). Observe that, in case of small thickness of porous bed ($h=0.2$), fixing the other parameters an increase in the intensity of the magnetic field $M$ reduces the fluid speed in the clean fluid region and enhances in the porous region. This is in according as in a fact that the magnetic field reduces a retarding force of the clean fluid flow, i.e., the magnitude of the velocity component $u$ reduces in the entire fluid region and $v$ reduces in the clean fluid region and enhances in the porous region (Fig. 1 & 5). The resultant velocity reduces in the clean fluid region, also enhances with $M$ in the porous region. Keeping the thickness of the porous bed small we notice that lower the permeability lesser the fluid speed in the both clean and porous regions. It is evident from the fact that the individual velocity components $u$ and $v$ and the resultant velocity reduce their magnitude for increase in $D^{-1}$ (Fig. 2 & 6). Both the velocity components $u$ and $v$ enhances with increase in the couple stress parameter $S$ in the porous region and hence the resultant velocity also enhances with increase in $S$ in the porous region, where as in the porous region the magnitude of the velocity components $u$ and $v$ enhances in $0.3 \leq z \leq 0.4$ and reduces $0.5 \leq z \leq 0.9$, the resultant velocity also increases in the clean fluid region with increase in $S$ (Fig. 3 & 7). The effect of the hall parameter on the flow field may be observed from figures (4 &8). It is interesting to note that an increase in hall parameter $m$ the magnitude of the velocity component $u$ enhances and $v$ reduces in the porous medium. The magnitude of the velocity component $u$ enhances in the clean fluid region and $v$ reduces in the region $0.3 \leq z \leq 0.5$ and increases in $0.6 \leq z \leq 0.9$ with
However the resultant velocity indicates the retardation in the clean fluid and acceleration in the porous bed for increase in $m$. When the thickness of the porous bed slightly increases (h=0.3...etc). Although it affects the individual velocity components $u$ and $v$. The influence of the thickness of the porous bed threw smaller values does not affect the flow behaviour in general.

Fig (9-12 & 13-16) correspond to the behaviour of the velocity components $u$ and $v$ when the thickness of the porous bed is large (h=0.5). An increase in M enhances $u$ and $v$ retards in the clean fluid region, where as in the porous region $u$ increases and $v$ retards in $0.1 \leq z \leq 0.5$, also the reversal behavior shows for $u$ and $v$ in the range $0.6 \leq z \leq 0.7$. The resultant fluid speed reduces in the clean fluid while it accelerates in the porous bed for increase in the intensity of the magnetic field (Fig 9 & 13). The fluid moves with reduced velocities in porous beds with lesser permeability irrespective of the thickness of the beds (Fig 10 & 14).

In contrast to the earlier case the velocity component $u$ increases in the entire fluid region and the magnitude of velocity component $v$ enhances maximum in the clean fluid region while it reduces in the porous region, their resultant experiences enhancement for increase in the couple stress parameter $S$ (Fig 11 & 15). The behaviour of the flow with reference to the hall parameter independent of the thickness of the porous bed and is evident from (Fig 12 & 16).

The magnitude of the velocity component $u$ increases in the entire fluid region and the velocity component $v$ increases in the clean fluid region while it reduces in the porous region, the resultant fluid speed enhances in the clean fluid and retards in the porous region for an increase in the hall parameter $m$.

The shear stresses are calculated on upper and lower plates and are tabulated in tables (I-IV). We notice that on the upper plate $\tau_z$ reduces with increase in the couple stress parameter $S$ irrespective of the thickness of the porous bed. Lesser the permeability of the medium lower the stresses irrespective thickness of the porous bed. The variation of $\tau_z$ with reference to $M$
reducing in case of small thickness while increasing in case of large thickness of the porous bed. Similar variation of \( \tau_x \) is observed with increase in the hall parameter \( m \) (table. I). The magnitude of \( \tau_x \) increases with increase in couple stress parameter \( S \) in case of small thickness while reduces with \( S \) in case of the large thickness of porous bed. Likewise reduces with increase \( D/l \) for all thickness of the porous bed. With reference to \( M \) the behaviour of \( \tau_x \) similar to that of \( \tau_v \) with \( \tau_v \) reducing in case of small thickness of the bed enhancing with \( M \) in case of large thickness of the bed. Reversal behaviour is observed with reference to variation in \( m \) (table. II). On the lower plate \( \tau_x \) and \( \tau_v \) exhibits similar behaviour with reference to increasing the couple stress parameter \( S \) as well as the Hartmann number \( M \). Lower the permeability of the medium the magnitude of \( \tau_v \) reduces where as \( \tau_x \) exhibits as increasing when the thickness of the porous bed is sufficiently large. An increase in the hall parameter reduces \( \tau_x \) while enhancing \( \tau_v \) irrespective of the thickness of the porous bed (tables. III and IV). The mass flux has been evaluated and tabulated in the table V. We find that the mass flux increases with increase in \( S, D/l \) and \( m \) where as reduces with \( M \) fixing the other parameters.
I. When the velocity profiles for the thickness of the porous bed is small

**FIG. 1: The velocity profile \( u \) for different \( M \) with \( D' = 1000, S = 1 \)**

![Graph showing velocity profile for different \( M \) with \( D' = 1000, S = 1 \)]

**FIG. 2: The velocity profile \( u \) for different \( D' \) with \( M = 2, S = 1 \)**

![Graph showing velocity profile for different \( D' \) with \( M = 2, S = 1 \)]
FIG. 3: THE VELOCITY PROFILE \( u \) FOR DIFFERENT \( S \) WITH \( D^I = 1000, M=2, m=1 \)

FIG. 4: THE VELOCITY PROFILE \( u \) FOR DIFFERENT \( M \) WITH \( D^I = 1000, M=2, S=1 \)
Fig. 5: The velocity profile \( v \) for different \( M \) with 
\( D' = 1000, S = 1, m = 1 \)

Fig. 6: The velocity profile \( v \) for different \( D' \) with 
\( M = 2, S = 1, m = 1 \)
FIG. 7: The velocity profile $v'$ for different $S$ with $D'=1000, M=2, m=1$.

FIG. 8: The velocity profile $v'$ for different $m$ with $D'=1000, M=2, S=1$. 

MHD THREE DIMENSIONAL FLOWS OF COUPLE STRESS FLUID THROUGH A PARALLEL PLATE CHANNEL
II. When the velocity profiles for the thickness of the porous bed is large

**Fig. 9:** The velocity profile $u$ for different $M$ with $D' = 1000, S = 1$

**Fig. 10:** The velocity profile $u$ for different $D'$ with $M = 2, S = 1$
**Fig. 11:** The velocity profile $u$ for different $S$ with $D' = 1000$, $M = 2$, $m = 1$.

**Fig. 12:** The velocity profile $u$ for different $m$ with $D' = 1000$, $M = 2$, $S = 1$. 
Fig. 13: The velocity profile ν for different M with
\( D'1=1000, S=1, m=1 \)

Fig. 14: The velocity profile ν for different D'1 with
\( M=2, S=1, m=1 \)
**Fig. 15:** The velocity profile $v'$ for different $S$ with $D'=1000, M=2, m=1$

**Fig. 16:** The velocity profile $v'$ for different $M$ with $D'=1000, M=2, S=1$
### Table I: The shear stresses ($\tau_r$) on the upper plate.

<table>
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<th>I</th>
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<th>III</th>
<th>IV</th>
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<th>VI</th>
<th>VII</th>
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### Table II: The shear stresses ($\tau_r$) on the upper plate.

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### MHD Three Dimensional Flows of Couple Stress Fluid Through a Parallel Plate Channel
### Table III: The shear stresses ($\tau_r$) on the lower plate.

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<td>0.035562</td>
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<td>0.045586</td>
<td>0.033652</td>
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### Table IV: The shear stresses ($\tau_\theta$) on the lower plate.

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<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
</tr>
</thead>
<tbody>
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<td>0.2</td>
<td>-0.00025</td>
<td>-0.00021</td>
<td>-0.00018</td>
<td>-0.00019</td>
<td>-0.00012</td>
<td>-0.00021</td>
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<td>0.4</td>
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<td>-0.00026</td>
<td>-0.00025</td>
<td>-0.00026</td>
<td>-0.00028</td>
<td>-0.00023</td>
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<td>-0.00038</td>
<td>-0.00036</td>
<td>-0.00028</td>
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### Table V: Mass flux (Q)

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<th>IV</th>
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<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
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<table>
<thead>
<tr>
<th>$h$</th>
<th>M</th>
<th>D$^{-1}$</th>
<th>S</th>
<th>m</th>
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</thead>
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<td>1</td>
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<tr>
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<td>5</td>
<td>1000</td>
<td>1</td>
<td>1</td>
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<td>1</td>
<td>1</td>
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<tr>
<td>0.8</td>
<td>2</td>
<td>1000</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

---

**MHD Three Dimensional Flows of Couple Stress Fluid Through a Parallel Plate Channel**
5.4 Conclusions:

When thickness of the Porous bed is small:

1. The resultant velocity reduces in the clean fluid region, also enhances with $M$ in the porous region.
2. Lower the permeability lesser the fluid speed in the both clean and porous regions.
3. The resultant velocity also enhances with $S$ in the entire flow region.
4. The resultant velocity indicates the retardation in the clean fluid and acceleration in the porous bed for increase in $m$.
5. The behavior of the velocity components in the clean fluid region as well as porous region with variations in $M$, $D^{-1}$, $S$ and $m$ remains unaffected when the thickness of the porous bed slightly increases.

When thickness of the Porous bed is Large:

1. The resultant fluid speed reduces in the clean fluid while it accelerates in the porous bed for increase in the intensity of the magnetic field.
2. The fluid moves with reduced velocities in porous beds with lesser permeability irrespective of the thickness of the beds.
3. Irrespective thickness of the porous bed, lower the permeability of the porous bed lesser the magnitude of the velocity components $u$ and $v$ as well as the resultant velocity.
4. The resultant velocity experiences enhancement for increase in the couple stress parameter $S$.
5. The resultant fluid speed enhances in the clean fluid and retards in the porous region for an increase in the hall parameter $m$.

Stress and mass flux:

1. On the upper plate $t$, reduces with increase in the couple stress parameter $S$ irrespective of the thickness of the porous bed.
2. Lesser the permeability of the medium lower the stresses irrespective thickness of the porous bed.

3. The variation of $\tau$, with reference to $M$ reducing in case of small thickness while increasing in case of large thickness of the porous bed.

4. Similar variation of $\tau$, is observed with increase in the hall parameter $m$.

5. The magnitude of $\tau$, increases with increase in couple stress parameter $S$ in case of small thickness while reduces with $S$ in case of the large thickness of porous bed.

6. Reduces with increase $D_1^{-1}$ for all thickness of the porous bed.

7. With reference to $M$ the behavior of $\tau$, similar to that of $\tau$ with $\tau$, reducing in case of small thickness of the bed enchasing with $M$ in case of large thickness of the bed.

8. Reversal behavior is observed with reference to variation in $m$.

9. On the lower plate $\tau$, and $\tau$ exhibits similar behavior with reference to increasing the couple stress parameter $S$ as well as Hartmann number $M$.

10. Lower the permeability of the medium the magnitude of $\tau$, reduces where as $v$, exhibits as increasing when the thickness of the porous bed is sufficiently large, an increase in hall parameter reduces $\tau$, while enhancing $\tau$, irrespective of the thickness of the porous bed.

11. The mass flux has been evaluated and tabulated in the table V. we find that the mass flux increases with increase in $S$, $D_1^{-1}$ and $m$ where as reduces with $M$ fixing the other parameters.
REFERENCES:


